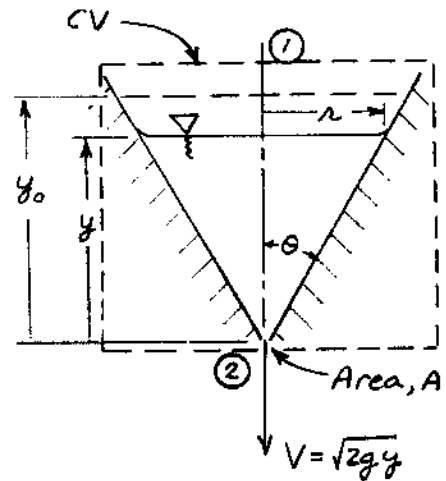


Given: Funnel of liquid draining through a small hole of diameter $d = 5 \text{ mm}$ (area, A) as shown; y_0 is initial depth.



Find: (a) Expression for time to drain
(b) Expression for result in terms of
initial volume V_0 , and
initial volume flow rate
 $Q_0 = AV_0 = A\sqrt{2gy_0}$

Plot: t as a function of y_0 ($0.1 \leq y_0 \leq 1 \text{ m}$) with angle θ as a parameter for $15^\circ \leq \theta \leq 45^\circ$.

Solution

Apply conservation of mass using CV shown.

Basic equation: $0 = \frac{\partial}{\partial t} \int_{CV} \rho dV + \int_{CS} \rho \vec{V} \cdot d\vec{A}$

Assumptions: (1) Incompressible flow
(2) Uniform flow at each section
(3) Neglect p_{atm} compared to p_{H_2O} .

Then,

$$0 = \frac{\partial}{\partial t} \int_{CV} \rho dV + \frac{\partial}{\partial t} \int_{CS} \rho dV + \int_{CS} \rho \vec{V} \cdot d\vec{A} + \int_{CS} \rho \vec{V} \cdot d\vec{A}$$

For the CV,

$$dV = A_s dy = \pi r^2 dy = \pi (y \tan \theta)^2 dy ; V = \pi \tan^2 \theta \frac{y^3}{3}$$

Thus

$$0 = \rho_{H_2O} \frac{\partial}{\partial t} \left(\pi \tan^2 \theta \frac{y^3}{3} \right) + \rho_{H_2O} A \sqrt{2gy}$$

$$0 = \pi \tan^2 \theta y^2 \frac{dy}{dt} + A \sqrt{2g} y^{1/2}$$

Separating variables, $y^{3/2} dy = - \frac{\sqrt{2g} A}{\pi \tan^2 \theta} dt$

Integrating from y_0 at $t=0$ to 0 at t ,

$$\int_{y_0}^0 y^{3/2} dy = \frac{2}{5} (-y_0^{5/2}) = - \frac{\sqrt{2g} A}{\pi \tan^2 \theta} t$$

or

$$t = \frac{2}{5} \frac{\pi \tan^2 \theta y_0^{5/2}}{\sqrt{2g} A}$$

t