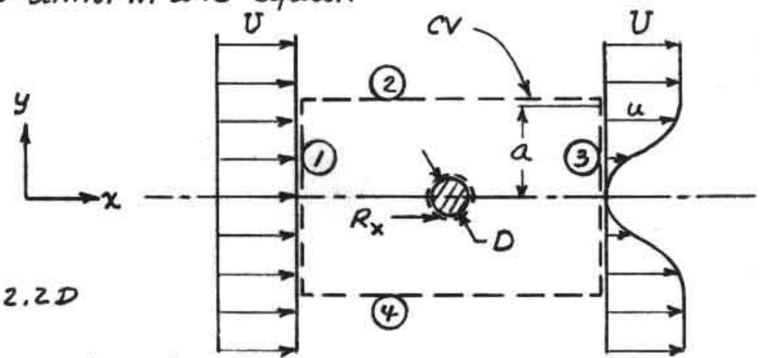
Given: Experimentally measured velocity profile for flow over a cylinder. Pressures are uniform and equal.

$U = 50$ m/s
$\rho = 1.2$ kg/m^3
$D = 30$ mm

$u = U\sin\left(\frac{\pi y}{2a}\right)$; $0 \le |y| \le a$

$u = U$ $y > a$; $a = 2.2D$



Find: Drag force on cylinder per unit width.

Solution: Apply continuity and x component of momentum using CV shown.

Basic equations:

$$0 = \underbrace{\frac{\partial}{\partial t}\int_{CV} \rho \, d\forall}_{=0(1)} + \int_{CS} \rho \vec{V}\cdot d\vec{A}$$

$$F_{S_x} + \underbrace{F_{B_x}}_{=0(2)} = \underbrace{\frac{\partial}{\partial t}\int_{CV} u\rho \, d\forall}_{=0(1)} + \int_{CS} u\rho \vec{V}\cdot d\vec{A}$$

Assumptions: (1) Steady flow
(2) No net pressure force ; $F_{S_x} = R_x$
(3) $F_{B_x} = 0$
(4) Uniform flow at inlet section 1
(5) Incompressible flow
(6) $u = U$ at sections 2 and 4

From continuity

$$0 = \{-|\rho U w\, 2a|\} + \dot{m}_2 + \left\{\left|2\int_0^a \rho u w\, dy\right|\right\} + \dot{m}_4 \; ; \; \dot{m}_{2+4} = 2\int_0^a \rho (U-u) w\, dy$$

From momentum

$$R_x = U\{-|\rho U w\, 2a|\} + U\dot{m}_2 + 2\int_0^a \rho u^2 w\, dy + U\dot{m}_4$$

$$= U\dot{m}_{2+4} + 2\int_0^a \rho(u^2 - U^2) w\, dy = 2\int_0^a \rho\left[U(U-u) + u^2 - U^2\right] w\, dy$$

$$R_x = 2\int_0^a \rho u (u - U) w\, dy \quad (R_x \text{ is force of cylinder } \underline{\text{on}} \text{ CV})$$

$$F_D = -R_x = 2\int_0^a \rho u (U-u) w\, dy = 2\rho w U^2 \int_0^a \frac{u}{U}\left(1 - \frac{u}{U}\right) dy \; ; \; \frac{u}{U} = \sin\left(\frac{\pi}{2}\frac{y}{a}\right)$$

$$\frac{F_D}{w} = 2\rho U^2\left(\frac{2a}{\pi}\right)\int_0^{\pi/2}(\sin\theta - \sin^2\theta)\, d\theta = \frac{4}{\pi}\rho U^2 a\left[-\cos\theta - \frac{\theta}{2} + \frac{\sin 2\theta}{4}\right]_0^{\pi/2} = \frac{4}{\pi}\rho U^2 a\left[1 - \frac{\pi}{4}\right]$$

$$\frac{F_D}{w} = \frac{4}{\pi} \times 1.20\,\frac{\text{kg}}{\text{m}^3} \times (50)^2\frac{\text{m}^2}{\text{s}^2} \times (2.2)\,0.03\,\text{m}\left(1 - \frac{\pi}{4}\right) \times \frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}} = 54.1 \text{ N/m}$$

$\frac{F_D}{w}$

{ Plus sign means force on cylinder acts to right. }