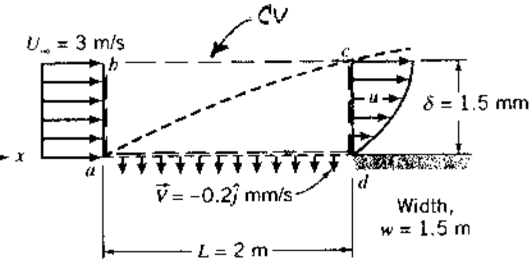
Given: Steady incompressible flow of air on porous surface shown in Fig. P4.38. Velocity profile at downstream end is parabolic. Uniform suction is applied along ad.

Find: (a) Volume flow rate across cd.
(b) Volume flow rate through porous surface (ad).
(c) Volume flow rate across bc.

Solution: Apply conservation of mass to CV shown.



Basic equation:

$$0 = \cancel{\frac{\partial}{\partial t}}\int_{CV} \rho \, d\forall + \int_{CS} \rho \vec{V} \cdot d\vec{A} \qquad (=0(1))$$

Assumptions: (1) Incompressible flow
(2) Parabolic profile at section cd: $\frac{u}{U_\infty} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$

Then $$0 = \int_{CS} \vec{V} \cdot d\vec{A} = Q_{ab} + Q_{bc} + Q_{cd} + Q_{da} \qquad (1)$$

$$Q_{cd} = \int_{cd} \vec{V} \cdot d\vec{A} = \int_0^\delta u w \, dy = w U_\infty \delta \int_0^1 \frac{u}{U} d\left(\frac{y}{\delta}\right) = w U_\infty \delta \int_0^1 \left[2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2\right] d\frac{y}{\delta}$$

$$= w U_\infty \delta \left[\left(\frac{y}{\delta}\right)^2 - \frac{1}{3}\left(\frac{y}{\delta}\right)^3\right]_0^1 = \frac{2}{3} w \delta U_\infty$$

$$Q_{cd} = \frac{2}{3} \times 1.5 \, m \times 0.0015 \, m \times 3 \frac{m}{s} = 4.50 \times 10^{-3} \, m^3/s \text{ (out of CV)}$$ Q_{cd}

Flow across ad is uniform, so

$$Q_{ad} = \vec{V} \cdot \vec{A} = v\hat{j} \cdot wL(-\hat{j}) = -vwL$$

$$Q_{ad} = -0.2 \frac{mm}{s} \times 1.5 \, m \times 2 \, m \times \frac{m}{1000 \, mm} = 6.00 \times 10^{-4} \, m/s \text{ (out of CV)}$$ Q_{ad}

Finally, from Eq. 1,

$$Q_{bc} = -Q_{ab} - Q_{cd} - Q_{da} \qquad (2)$$

But $$Q_{ab} = \vec{U}_\infty \cdot \vec{A}_{ab} = U_\infty \hat{i} \cdot w\delta(-\hat{i}) = -w\delta U_\infty$$

$$Q_{ab} = -1.5 \, m \times 0.0015 \, m \times 3 \frac{m}{s} = -6.75 \times 10^{-3} \, m^3/s \text{ (into CV)}$$

Substituting into Eq. 2,

$$Q_{bc} = \left[-\left(-6.75 \times 10^{-3}\right) - 4.50 \times 10^{-3} - 6.00 \times 10^{-4}\right] m^3/s$$

$$Q_{bc} = 1.65 \times 10^{-3} \, m^3/s \text{ (out of CV)}$$ Q_{bc}