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Who invented the great numerical algorithms?

A discussion over coffee.

Ivory tower or coal face?

NAG AGM, September 1999

Predictions for Scientific Computing Fifty Years From Now

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This essay is adapted from a talk given June 17, 1998 at the conference "Numerical Analysis and Computers — 50 Years of Progress" held at the University of Manchester in commemoration of the 50th anniversary of the Mark 1 computer.

Fifty years is a long, long time in any technological field. In our own field of scientific computing or numerical analysis, think back to 1950. Around the world, numerical problems in 1950 were solved with slide rules and on paper, or with mechanical calculators that had little in common with today's computers. Some of the algorithms we use today were in existence then, but on the whole, the last fifty years have changed numerical computing beyond recognition. The next fifty will do it again.

My remarks consist of twelve predictions. I did not aim for these to orbit around a unifying theme, but that is nevertheless what happened.

1. We May Not Be Here

In the 20th century, everything technological seems to be changing exponentially. This raises a problem. Exponentials do not go on for ever; something happens to them. Now in my opinion, many of the exponentials we are sitting on have not yet started to level off. Here at the beginning of the third millennium, biology is just beginning its great explosion, and although electronics got a head start of a few decades, it is hardly slowing down yet.

this one? — in the very first century of radio, television, light bulbs, telephones, phonographs, lasers, refrigerators, automobiles, airplanes, spacecraft, computers, nuclear power, nuclear weapons, plastics, antibiotics, and genetic engineering?

I believe that the explanation of our special position in history may be that it is not so special after all, because history tends not to last very long. This argument has been called the *Copernican Principle* by J R Gott of Princeton University.

There is a second line of evidence, sometimes known as *Fermi's paradox*, that also suggests that technological civilisations are short-lived. The human race is not an outpost of a galactic society; it is a domestic product. How can we explain this if technological civilisations last tens of thousands of years? An ages-old technological civilisation will expand across its galaxy, simply because it can. (Don't ask why, for expanding is what life does. If one species doesn't, another will replace it.) Yet in 100,000 years of expanding at one hundredth the speed of light, a civilisation can spread one thousand light years, a distance encompassing millions of stars. Is it plausible that technological civilisations are so rare as to arise on only one star among millions?

I believe that the explanation of the emptiness out there may be that technological civilisations perish before they start to spread across their galaxy — or that they start spreading, then perish in a cataclysm so great as to take the galaxy with them.

Suddenly the problem of predicting fifty years of scientific computing begins to look easy! Let's get down to it.

SOME MAJOR DEVELOPMENTS IN SCIENTIFIC COMPUTING

(29 of them)

Before 1940

Newton's method
least-squares fitting
Gaussian elimination
Gauss quadrature
Adams formulae
Runge-Kutta formulae
finite differences

1940-1970

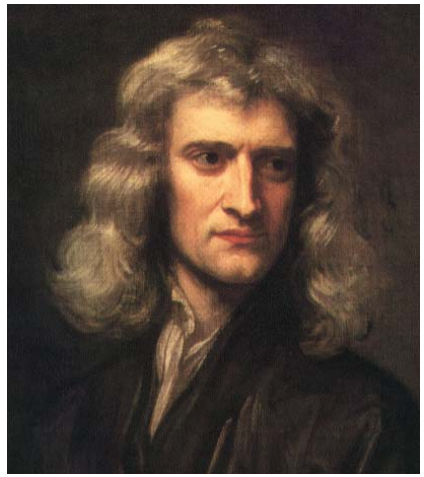
floating-point arithmetic
splines
Monte Carlo methods
simplex algorithm
conjugate gradients & Lanczos
Fortran
stiff ODE solvers
finite elements
orthogonal linear algebra
QR algorithm
Fast Fourier Transform
quasi-Newton iterations

1970-2000

preconditioning
spectral methods
MATLAB
multigrid methods
IEEE arithmetic
nonsymmetric Krylov iterations
interior point methods
fast multipole methods
wavelets
automatic differentiation

Before 1940

Newton's Method for nonlinear eqs.



Heron, al-Tusi 12c, Al Kashi 15c, Viète 1600, Briggs 1633…

Isaac Newton 1642-1727
Mathematician and physicist
Trinity College, Cambridge, 1661-1696
(BA 1665, Fellow 1667,
Lucasian Professor of Mathematics 1669)
De analysi per aequationes numero terminorum infinitas **1669** (published 1711)
After 1696, Master of the Mint

Joseph Raphson 1648-1715
Mathematician at Jesus College, Cambridge
Analysis Aequationum universalis **1690**
Raphson's formulation was better than Newton's ("plus simple" - Lagrange 1798)
FRS 1691, M.A. 1692
Supporter of Newton in the calculus wars—*History of Fluxions,* 1715

Thomas Simpson 1710-1761
1740: *Essays on Several Curious and Useful Subjects…*
1743-1761: Royal Military Academy, Woolwich
Important!—first to treat non-polynomial equations, first to treat systems of eqs.

Least-squares fitting

Carl Friedrich Gauss 1777-1855

Mathematics, astronomy, geodesy, magnetism

1792-1795: Braunschweig Collegium Carolinum

1795, but not published until 1809
($\rightarrow$ big fight with Legendre)

(During this time as a teenager in Braunschweig he also discovered the binomial theorem, quadratic reciprocity, arithmetic-geometric mean…)

1807-1855: University of Göttingen

Adrien-Marie Legendre 1752-1833

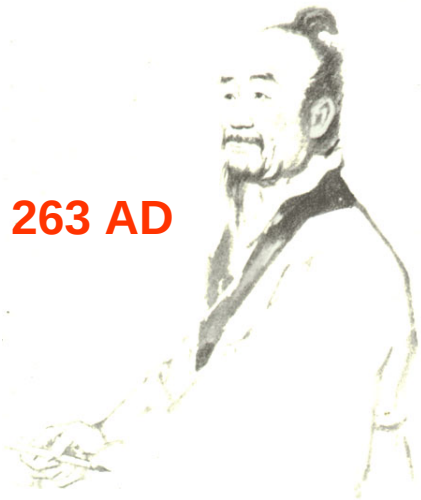
1791-1833: Académie des Sciences, Paris

1805 “Sur la méthode des moindres carrés”
applications to orbits of comets

Gaussian elimination for linear systems of eqs.

Liu Hui c. 220 – c. 280

Chinese mathematician
discusses elimination in his commentaries on
The Nine Chapters on the Mathematical Art **263 AD**

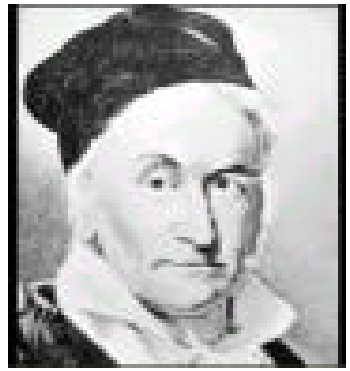


Joseph Lagrange 1736-1813

Symmetric quadratic forms **1759**

Carl Friedrich Gauss 1777-1855

Symmetric systems, normal eqs. **1809**



Carl Gustaf Jacob Jacobi 1804-1851

1826-1844: Univ. of Königsberg
General systems **1857** (posthumous)

Gauss quadrature for numerical integration

Carl Friedrich Gauss 1777-1855

"Methodus nova integralium valores per approximationem inveniendi", *Comment. Soc. Reg. Sient. Götting. Recent.* **1814**

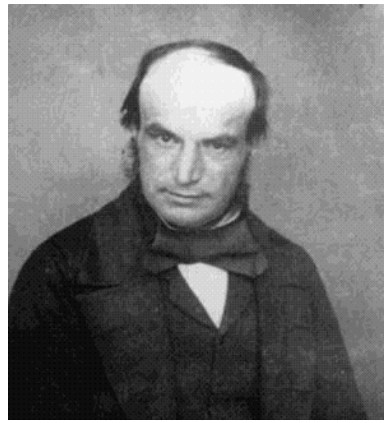
Gauss did it by continued fractions and hypergeometric functions. Today's more familiar interpretation via orthogonal polynomials was developed by **Jacobi** (1804-1851) in **1826**.

Adams formulae for ODEs

Leonhard Euler 1707-1783

1727-1741: St. Petersburg Academy
1768: *Institutiones Calculi Integralis*
1741-1766: Berlin Academy
1766-1783: St. Petersburg Academy

John Couch Adams 1819-1892



astronomer and mathematician; predicted existence of Neptune
1839-1892: Cambridge Univ.—Senior Wrangler 1843
1855?: work on multistep methods
1858-1892: Lowndean Professor of Astronomy and Geometry
Declined knighthood and Astronomer Royal post

Francis Bashforth 1819-1912

influential ballistics expert
1840-1843: Cambridge Univ.—Second Wrangler 1843
1864-1872: Professor of Applied Mathematics, Royal Military Academy, Woolwich
1883: paper describing Adams methods (for calculating shapes of drops).

Runge-Kutta formulae for ODEs

Like Adams formulas, these are a generalization of Euler.

Carl David Tolme Runge 1856-1927

1895 *Math. Anal.*, “Über die numerische Auflösung…”

Karl Heun 1859-1929

PhD. 1881 Göttingen, Prof. Theoretical Mechanics Karlsruhe

1900 *Zeit. Math. Phys.*, “Neue Methode zur approximativen Integration…”

Martin Wilhelm Kutta 1867-1944

1901 general R-K theory, *Zeit. Math. Phys.*, “Beitrag zur näherungsweisen Integration…”

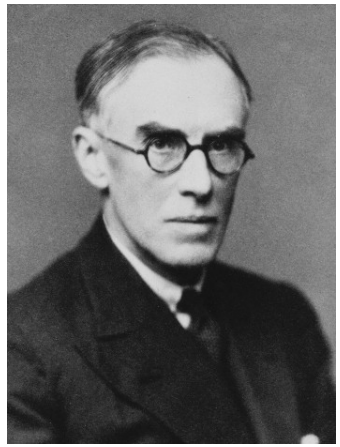
also Moulton 1926 and von Mises 1930,
and in the modern era, John Butcher.

Finite differences for PDEs

Lewis Fry Richardson 1881-1953
Richard Southwell 1888-1970

Richard Courant 1888-1972
Kurt Friedrichs 1901-1982
Hans Lewy 1904-1988

John von Neumann 1903-1957
Peter Lax ≈1926-



1940 – 1970

Floating point arithmetic

Konrad Zuse 1910-1995

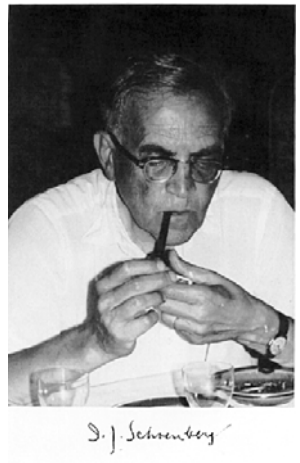
Civil engineer by training
Worked on computers beginning in 1934
“Zuse Apparatebau” company founded in Berlin 1940
Z1 computer, completed in Berlin **1936**
much further developed: Z3 computer, 1941
22-bit floating point binary arithmetic
(14 bits for fraction, 8 for exponent)
1Hz, programmable, stored data but not program
Machine was destroyed in 1945 air raids

Zuse was also an artist.

Splines

Paul de Faget de Casteljau 1930-
French mathematician/physicist
1958-1992: Citroën; unpublished work in **1958**

Pierre Bezier 1910-1999
1933-1975: engineer at Renault
1960: beginning of CADCAM work, Bezier curves



Isaac Jacob ("Iso") Schoenberg 1903-1990
Born in Romania (Landau's son-in-law). To USA in 1930.
Chicago, Harvard, Princeton, Swarthmore, Colby…
1941-1966: University of Pennsylvania
1943-1945: Army Ballistic Research Laboratory
1946: two papers on splines
1966-1973: University of Wisconsin

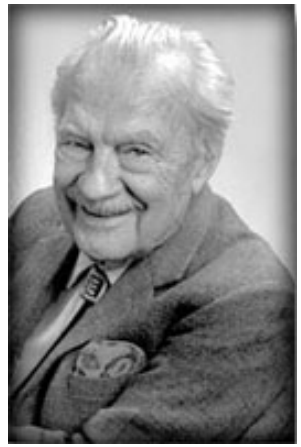
Carl de Boor 1937-
Born in what became East Germany. To USA in 1959.
1960-1964: General Motors
1962: first of many publications on splines
Purdue, Michigan…
1972- University of Wisconsin

Monte Carlo simulation methods

Stanislaw Ulam 1909-1984
Born in Poland, to USA in 1935, pure mathematician by training
Princeton, Harvard, Wisconsin, USC
1943-1965: Los Alamos (key figure in hydrogen bomb)
1965-1984: Dept. of Mathematics, U. of Colorado

John von Neumann 1903-1957
Born in Hungary, to USA in 1930, pure mathematician by training
Manhattan Project, Los Alamos, Atomic Energy Comm.
1930-1957: Princeton University & Inst. Advanced Study

Nicholas Metropolis 1915-1999
Greek-American, physicist by training
Oscillated between U. of Chicago and Los Alamos
1932, 1941, 1945, 1948, 1957, 1965



1947: Invention by Ulam & von N. for applications in neutron diffusion

1949: publication of “The Monte-Carlo Method” by Ulam & Metropolis

Also Fermi, Richtmyer, …

Simplex algorithm for linear programming

Leonid Kantorovich 1912-1986

1934-1960 Professor of Mathematics, Leningrad State U.
1939: *Mathematical Methods in the Organization and Planning of Production*
1975: Nobel Prize in Economics

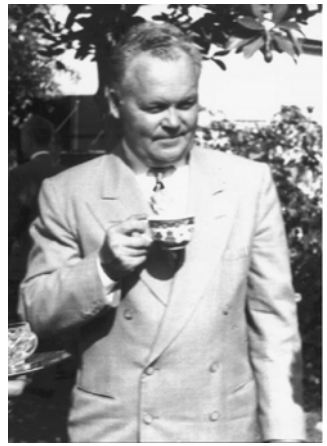
George Dantzig 1914-2005

1941-1946: Head of Combat Analysis Branch, US Air Force Statistical Control
1944: War Department Exceptional Civilian Service Medal
1946: receives PhD at UC Berkeley
1947: Simplex algorithm
1948: Koopmans coins expression “linear programming”
1947-1952: Mathematical Advisor, US Defense Department
1952-1960: RAND Corporation
1960-1966: UC Berkeley
1966- : Stanford University

Conjugate gradient and Lanczos iterations

Cornelius Lanczos 1893-1974

Born in Hungary: Fejér, Einstein, …
1931-1949: Purdue and Boeing
1949-1952: Inst. Numer. Anal., NBS, UCLA
1952-1972: Dublin Inst. Adv. Study, Ireland



Magnus Hestenes 1906-1991

late 1920s-1947: University of Chicago
1947-1973: UCLA
1949-1952: Inst. Numer. Anal., NBS, UCLA

Eduard Stiefel 1909-1978

eminent in geometry and physics as well as computation
Swiss Federal Institute of Technology

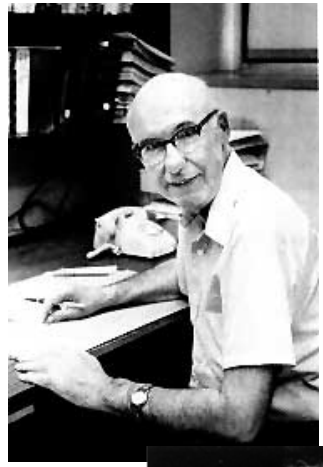
1952: landmark papers by Lanczos and Hestenes & Stiefel

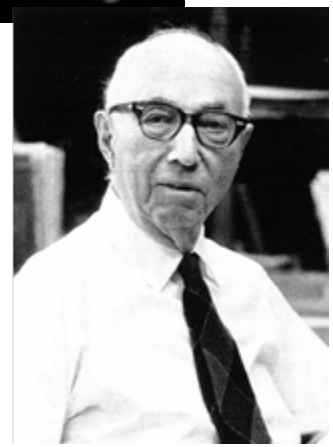
Fortran

John Backus 1924-

grew up in Delaware, USA
not an outstanding student; disorganized early career
with some years in US Army
1949: AB in Mathematics, Columbia University
1950-1991 IBM
1954: first paper about Fortran; programming team is built
1957: Fortran released by IBM
1976: National Medal of Science
1977: Turing Award

Stiff ODE solvers





Charles Francis Curtiss 1921-
1948-1960: Dept. Chemistry, George Washington U.
1960- : Dept. Chemistry, U. Wisconsin

Joseph Oakland Hirschfelder 1911-1990
1937-1981: Dept. Chemistry, U. Wisconsin
1943-1946: group leader, Los Alamos
1946: Chief Phenomenologist, Bikini Bomb Test
1952: "Integration of stiff equations" with Curtiss, *PNAS*
Nat. Academy of Science; Nat. Medal of Science 1976

Germund Dahlquist 1925-2005
Royal Institute of Technology, Sweden
1963: "A special stability problem for linear multistep methods…", *BIT*

C. Willlliam Gear 1935-
1956-1990: University of Illinois
1965, 1966 and others: visits to Argonne National Lab
1967: first paper on stiff solvers
1971: *Numerical Initial-Value Problems in ODEs*
1990-2000: NEC

Finite elements for PDE

Richard Courant 1888-1972
1943 ”Variational methods…”
(landmark paper, but attracted no notice till later)

Finite elements grew out of the aeronautical engineering of the 1950s.
Additional names include Martin, Turner, Irons, Kelsey, Topp.

John H. Argyris 1913-2004

Born in Greece; much of career at U. of Stuttgart, Germany
1960 *Energy Theorems and Structural Analysis*

Ray W. Clough ≈1921-
1950s: Boeing?
1960 “The finite element in plane stress analysis”
1970- : Professor of Structural Engineering, UC Berkeley
eminent authority in earthquake engineering
1994: National Medal of Science

Other key early figures include Babushka & Zienkewicz

Orthogonal linear algebra

Wallace Givens 1911-1993

1950s and1960s: Argonne National Laboratory
Later, professor at U. of Tennessee
1958: introduction of Givens rotations

Alston Householder 1904-1993

1946-1969: Oak Ridge National Laboratory
1958: 4-page paper introducing Householder reflections
1964: *The Theory of Matrices in Numerical Analysis*

Gene Golub 1932-

Professor at Stanford since mid-1960s.
Key early contributions to many topics including SVD and least-squares
1965: "Numerical methods for solving linear least-squares problems"

QR algorithm for matrix eigenvalues

Heinz Rutishauser 1918-1970
ETH Zurich
1958 LR algorithm

V. N. Kublanovskaya

Steklov Institute of Mathematics, St. Petersburg
1961 “On some algorithms for the solution of the… eigenvalue problem”

J. G. F. Francis

Late 1950s: National Research Development Corporation, London
Assistant of Christopher Strachey
1961 ”The QR transformation…” I & II, *Computer J.*

James H. Wilkinson 1919-1986

Undergraduate in Mathematics at Cambridge
1940-1946: war work related to numerics and ballistics
1946: Turing’s assistant on Pilot Ace Computer
1946-1986: National Physical Laboratory
1965: *The Algebraic Eigenvalue Problem*
1969: FRS
1970: Turing Award

Fast Fourier Transform

Gauss 1805 (unpublished) age 28, 2 years before Fourier!
Runge 1903 Stumpff 1939
Thomas 1948 Danielson & Lanczos 1942 Good 1958
Wheeler… Gentleman…

Modern birth due to Tukey & Garwin & Sande in 1963, leading to **1965** Cooley-Tukey paper in *Mathematics of Computation*

John Tukey 1915-2000

Princeton University, founder of Statistics Dept.
(also Bell Labs and consultant to U.S., govt. & industry)

Richard Garwin 1928-

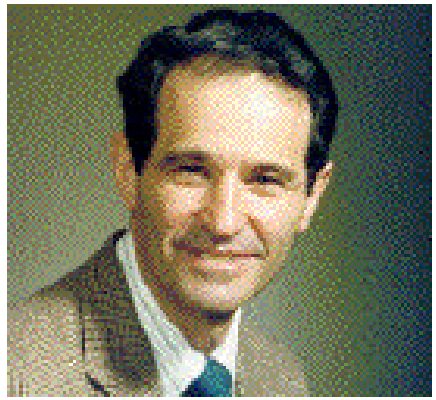
Watson Scientific Lab, Columbia U. (later at TJ Watson)
Well known physicist with major involvement in H-bomb

James W. Cooley 1926-

IBM TJ Watson Research Center. U. Rhode Island.

Quasi-Newton iterations for optimization

The field was launched between 1959 and 1970.



William Davidon 1927-
1954 PhD in Physics, U. Chicago
1959: “variable metric” report at Argonne National Lab.
(It was finally published in 1991, first issue of *SIOPT*)
1961-1991: Prof. of Physics and Maths, Haverford

Michael Powell 1936-
1959-1976 Harwell A.E.R.E.
1976- DAMTP, University of Cambridge
1983 FRS

Charles Broyden 1933-
1955-1965: English Electric
1965: “good” and “bad” Broyden methods
U. College Aberystwyth, U. of Essex
1985?-2003 University of Bologna

Roger Fletcher 1939-
1969-1973 Harwell A.E.R.E.… U. of Leeds
1963: Davidon-Fletcher-Powell paper
1971-2005 University of Dundee
2003 FRS

1970 – 2000

Preconditioning for iterative solution of linear systems

Many people contributed to the discovery of preconditioning, including Evans, Varga, Wachspress, Golub, Concus and O'Leary. Yet there was a particular preconditioner that made the idea famous and is still one of the most effective today: incomplete factorization.

Henk van der Vorst ≈1944-

Since 1970s: Universities of Delft and Utrecht, Netherlands

1977: original paper on incomplete LU factorization

Spectral methods for PDE

Important work in 1950s and 1960s by Lanczos, Clenshaw, Elliott, Fox and Mason et al.
Contributions also from Kreiss and Oliger and others.
These methods were made famous by:

Steve Orszag 1940?-

1966?-1984: Applied Mathematics, MIT
1971: series of major papers on spectral methods in fluid mechanics
1984-1998: Prof. of Applied Mathematics, Princeton U.
1998- Prof. of Mathematics, Yale U.

David Gottlieb 1944-

From Israel; came to USA in 1972
1972-1976: MIT and ICASE (NASA Langley)
1977: spectral methods book by D.G. and S.A.O.
1976-1985: Dept. of Applied Mathematics, Tel-Aviv U.
1985- : Prof. of Applied Mathematics, Brown U.

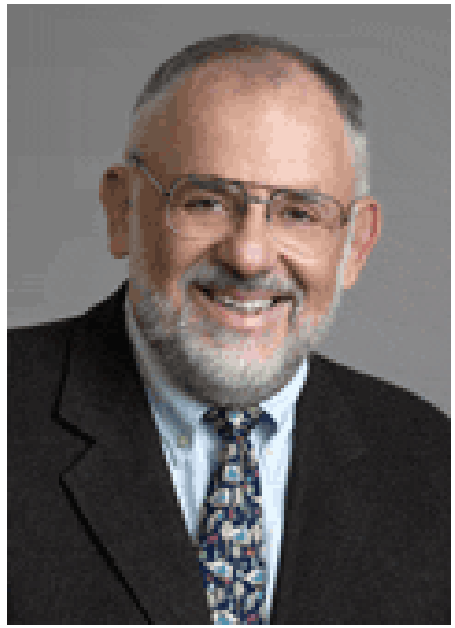
MATLAB

Cleve Moler 1939-

Author of EISPACK, LINPACK, four textbooks

high school Utah, BA Caltech, PhD Stanford
1965-1973: University of Michigan
1973-1984: University of New Mexico
strong links with Argonne National Laboratory
1977: creation of first version of Matlab
1984: Jack Little founds MathWorks
1985: first Matlab sale
1984-1989: Moler employed at Intel and Ardent
1989: joins MathWorks as Chief Scientist

N.B.: Matlab is more than half as old as Fortran!



Multigrid Methods for PDE

R. P. Fedorenko

1961: invention of 2-grid and later multigrid method.
This work extended also by N. S. Bakhvalov, 1966.

Achi Brandt 1938-

1963- : Applied Mathematics, Weizmann Institute, Israel
1973: first paper on multigrid methods
1977: 57-page paper in *Mathematics of Computation*

Wolfgang Hackbusch 1948-

1976: Independent rediscovery of multigrid
1982- : Professor of Applied Maths., U. Kiel
199?-: director of a Max-Planck Inst. In Leipzig

IEEE arithmetic

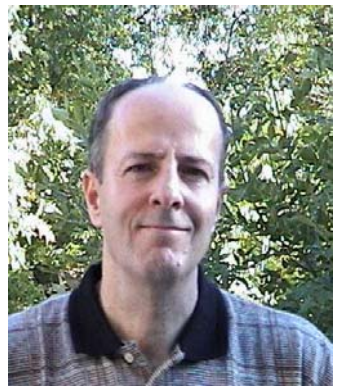
William (“Velvel”) Kahan 1933-

late 1960s?- : Dept. of Mathematics, UC Berkeley
1977: the draft IEEE floating-point standard released
1985: adoption of the standard after much wrangling
1989: Turing Award

Nonsymmetric Krylov iterations for large matrix problems

Many contributors including Arnoldi, Elman, Schultz, Freund, Gutknecht.

P. K. W. Vinsome 19??-
Shell Petroleum Co.
1976 paper on Orthomin



Youcef **Saad** ≈1950-
Yale University, U. of Minnesota
1986: GMRES paper with Schultz

Henk van der Vorst ≈1944-
Professor of Mathematics at U. of Utrecht
1986: BiCGSTAB paper—most cited maths paper in 1990s
1996: Jacobi-Davidson paper with Sleijpen

Dan Sorensen ≈1947-
Argonne National Laboratory
Rice University
1992: implicitly restarted Arnoldi
1996: *ARPACK User's Guide* with Lehoucq and Yang

Interior Point Methods for optimization

Earlier work by Carroll (1961) and Khachiyan (1979) and also by Fiacco & McCormick (1968), Margaret Wright (1976) and others on barrier methods.

Narendra Karmarkar 1957?-

1978: BTech in Elect. Engr., IIT Bombay
1982?: PhD, U. C. Berkeley
1983-? AT&T Bell Labs
1984 :"A new polynomial time algorithm for linear programming," *Combinatorica*
Now lives in India

Fast Multipole Method for *N*-body simulation and more

Related earlier work by Barnes & Hut & others

Vladimir Rokhlin 1952–

Born in USSR; to USA in late 1970s
1976-1985: Exxon Production Research Co.
1983: PhD in Applied Mathematics, Rice U.
1985 "Rapid solution of integral equations…"
1985- Prof. of Computer Science, Yale U.

Leslie Greengard 1958–

From Boston, New York, New Haven
1987 M.D. and Ph.D. (Comp. Sci.) Yale U.
1987 "A fast algorithm for particle simulations", with Rokhlin
1989– Prof. of Mathematics, Courant Inst., NYU

Both VR and LG have eminent fathers.

Wavelets

Jean Morlet 19??-

Geophysicist at Elf Aquitaine / Oric

Work beginning 1975 leads to major publication **1982**

Also Alex Grossmann 1984, Stephane Mallat 1989,
Yves Meyer 1986

Ingrid Daubechies 1954-

Training in physics and mathematics

From Belgium; came to USA in 1987

1975-1987: Vrije Universiteit Brussel

1987-1994: AT&T Bell Labs; Rutgers U.

Big change in 1980s physics $\rightarrow$ mathematics

1988: “Orthonormal bases of compactly supported wavelets”

1993- : Princeton University

MacArthur Prize and many others

Automatic differentiation

Many antecedents including Beda (1959), Wengert (1964), Speelpenning (1980), Kedem (1980), Rall (1981), Baur and Strassen (1984)… more recently Bischof & Carle (ADIFOR, 1991) and others.

A central figure in the modern rebirth of these ideas (in particular the use of “reverse mode”) has been

Andreas Griewank 1950-

Argonne National Laboratory
Institute for Scientific Computing, TU Dresden
Humboldt-University Berlin

The Inventors

Adams
Argyris
Backus
Bashforth
Bezier
Brandt
Broyden
Clough
Cooley
Courant
Curtiss
Dahlquist
Dantzig
Daubechies
Davidon
de Boor
de Casteljau
Euler
Fedorenko
Fletcher
Francis
Friedrichs
Garwin
Gauss
Gear
Givens
Golub
Gottlieb
Greengard
Griewank
Hackbusch
Hestenes
Heun
Hirschfelder
Householder
Liu
Jacobi
Kahan
Kantorovich
Karmarkar
Kublanovskaya
Kutta
Lagrange
Lanczos
Lax
Legendre
Lewy
Metropolis
Moler
Morlet
von Neumann
Newton
Orszag
Powell
Raphson
Richardson
Rokhlin
Runge
Rutishauser
Saad
Schoenberg
Sorensen
Southwell
Stiefel
Tukey
Ulam
van der Vorst
Vinsome
Wilkinson
Zuse

Who was an engineer?

Adams	**Argyris**	**Backus**	Bashforth
Bezier	Brandt	Broyden	**Clough**
Cooley	Courant	Curtiss	Dahlquist
Dantzig	Daubechies	Davidon	de Boor
de Casteljau	Euler	Fedorenko	Fletcher
Francis	Friedrichs	Garwin	Gauss
Gear	Givens	Golub	Gottlieb
Greengard	Griewank	Hackbusch	Hestenes
Heun	Hirschfelder	Householder	Liu
Jacobi	Kahan	Kantorovich	Karmarkar
Kublanovskaya	Kutta	Lagrange	Lanczos
Lax	Legendre	Lewy	Metropolis
Moler	Morlet	von Neumann	Newton
Orszag	Powell	Raphson	Richardson
Rokhlin	Runge	Rutishauser	Saad
Schoenberg	Sorensen	**Southwell**	Stiefel
Tukey	Ulam	van der Vorst	Vinsome
Wilkinson	**Zuse**		

Who was a physicist?

Adams	Argyris	Backus	Bashforth
Bezier	Brandt	Broyden	Clough
Cooley	Courant	Curtiss	Dahlquist
Dantzig	**Daubechies**	**Davidon**	de Boor
de Casteljau	Euler	Fedorenko	Fletcher
Francis	Friedrichs	**Garwin**	Gauss
Gear	Givens	Golub	Gottlieb
Greengard	Griewank	Hackbusch	Hestenes
Heun	Hirschfelder	Householder	Liu
Jacobi	Kahan	Kantorovich	Karmarkar
Kublanovskaya	Kutta	Lagrange	Lanczos
Lax	Legendre	Lewy	**Metropolis**
Moler	**Morlet**	von Neumann	**Newton**
Orszag	Powell	Raphson	Richardson
Rokhlin	Runge	Rutishauser	Saad
Schoenberg	Sorensen	Southwell	Stiefel
Tukey	Ulam	van der Vorst	**Vinsome**
Wilkinson	Zuse		

Who was a chemist?

Adams	Argyris	Backus	Bashforth
Bezier	Brandt	Broyden	Clough
Cooley	Courant	**Curtiss**	Dahlquist
Dantzig	Daubechies	Davidon	de Boor
de Casteljau	Euler	Fedorenko	Fletcher
Francis	Friedrichs	Garwin	Gauss
Gear	Givens	Golub	Gottlieb
Greengard	Griewank	Hackbusch	Hestenes
Heun	**Hirschfelder**	Householder	Liu
Jacobi	Kahan	Kantorovich	Karmarkar
Kublanovskaya	Kutta	Lagrange	Lanczos
Lax	Legendre	Lewy	Metropolis
Moler	Morlet	von Neumann	Newton
Orszag	Powell	Raphson	Richardson
Rokhlin	Runge	Rutishauser	Saad
Schoenberg	Sorensen	Southwell	Stiefel
Tukey	Ulam	van der Vorst	Vinsome
Wilkinson	Zuse		

Who was a mathematician?

Adams	Argyris	Backus	**Bashforth**
Bezier	**Brandt**	**Broyden**	Clough
Cooley	**Courant**	Curtiss	**Dahlquist**
Dantzig	**Daubechies**	**Davidon**	**de Boor**
de Casteljau	**Euler**	**Fedorenko**	**Fletcher**
Francis	**Friedrichs**	Garwin	**Gauss**
Gear	**Givens**	**Golub**	**Gottlieb**
Greengard	**Griewank**	**Hackbusch**	**Hestenes**
Heun	Hirschfelder	**Householder**	**Liu**
Jacobi	**Kahan**	**Kantorovich**	**Karmarkar**
Kublanovskaya	**Kutta**	**Lagrange**	**Lanczos**
Lax	**Legendre**	**Lewy**	Metropolis
Moler	Morlet	**von Neumann**	**Newton**
Orszag	**Powell**	**Raphson**	**Richardson**
Rokhlin	**Runge**	**Rutishauser**	**Saad**
Schoenberg	**Sorensen**	Southwell	**Stiefel**
Tukey	**Ulam**	**van der Vorst**	Vinsome
Wilkinson	Zuse		

(Including computer scientists and statisticians, since very hard to distinguish)

Who was a professor?

Adams
Bezier
Cooley
Dantzig
de Casteljau
Francis
Gear
Greengard
Heun
Jacobi
Kublanovskaya
Lax
Moler
Orszag
Rokhlin
Schoenberg
Tukey
Wilkinson

Argyris
Brandt
Courant
Daubechies
Euler
Friedrichs
Givens
Griewank
Hirschfelder
Kahan
Kutta
Legendre
Morlet
Powell
Runge
Sorensen
Ulam
Zuse

Backus
Broyden
Curtiss
Davidon
Fedorenko
Garwin
Golub
Hackbusch
Householder
Kantorovich
Lagrange
Lewy
von Neumann
Raphson
Rutishauser
Southwell
van der Vorst

Bashforth
Clough
Dahlquist
de Boor
Fletcher
Gauss
Gottlieb
Hestenes
Liu
Karmarkar
Lanczos
Metropolis
Newton
Richardson
Saad
Stiefel
Vinsome

(Including English academics like Raphson with titles other than professor)

Who had major involvement with government or industry?

Adams	**Argyris**	**Backus**	**Bashforth**
Bezier	Brandt	**Broyden**	**Clough**
Cooley	**Courant**	Curtiss	Dahlquist
Dantzig	**Daubechies**	**Davidon**	**de Boor**
de Casteljau	Euler	Fedorenko	**Fletcher**
Francis	Friedrichs	**Garwin**	**Gauss**
Gear	**Givens**	Golub	Gottlieb
Greengard	**Griewank**	Hackbusch	**Hestenes**
Heun	**Hirschfelder**	**Householder**	Liu
Jacobi	**Kahan**	**Kantorovich**	**Karmarkar**
Kublanovskaya	Kutta	Lagrange	**Lanczos**
Lax	Legendre	Lewy	**Metropolis**
Moler	**Morlet**	**von Neumann**	Newton
Orszag	**Powell**	Raphson	Richardson
Rokhlin	Runge	Rutishauser	Saad
Schoenberg	**Sorensen**	**Southwell**	Stiefel
Tukey	**Ulam**	van der Vorst	**Vinsome**
Wilkinson	**Zuse**		

(i.e., near the time of their big contributions)

Who was British?

Adams
Bezier
Cooley
Dantzig
de Casteljau
Francis
Gear
Greengard
Heun
Jacobi
Kublanovskaya
Lax
Moler
Orszag
Rokhlin
Schoenberg
Tukey
Wilkinson
Argyris
Brandt
Courant
Daubechies
Euler
Friedrichs
Givens
Griewank
Hirschfelder
Kahan
Kutta
Legendre
Morlet
Powell
Runge
Sorensen
Ulam
Zuse
Backus
Broyden
Curtiss
Davidon
Fedorenko
Garwin
Golub
Hackbusch
Householder
Kantorovich
Lagrange
Lewy
von Neumann
Raphson
Rutishauser
Southwell
van der Vorst
Bashforth
Clough
Dahlquist
de Boor
Fletcher
Gauss
Gottlieb
Hestenes
Liu
Karmarkar
Lanczos
Metropolis
Newton
Richardson
Saad
Stiefel
Vinsome

Who was born in the USA?

Adams
Bezier
Cooley
Dantzig
Bezier
Francis
Gear
Greengard
Heun
Jacobi
Kublanovskaya
Lax
Moler
Orszag
Rokhlin
Schoenberg
Tukey
Wilkinson

Argyris
Brandt
Courant
Daubechies
Euler
Friedrichs
Givens
Griewank
Hirschfelder
Kahan
Kutta
Legendre
Morlet
Powell
Runge
Sorensen
Ulam
Zuse

Backus
Broyden
Curtiss
Davidon
Fedorenko
Garwin
Golub
Hackbusch
Householder
Kantorovich
Lagrange
Lewy
von Neumann
Raphson
Rutishauser
Southwell
van der Vorst

Bashforth
Clough
Dahlquist
de Boor
Fletcher
Gauss
Gottlieb
Hestenes
Liu
Karmarkar
Lanczos
Metropolis
Newton
Richardson
Saad
Stiefel
Vinsome

Who ended up in the USA?

Adams	Argyris	**Backus**	Bashforth
Bezier	Brandt	Broyden	**Clough**
Cooley	**Courant**	**Curtiss**	Dahlquist
Dantzig	**Daubechies**	**Davidon**	**de Boor**
de Casteljau	Euler	Fedorenko	Fletcher
Francis	**Friedrichs**	**Garwin**	Gauss
Gear	**Givens**	**Golub**	**Gottlieb**
Greengard	Griewank	Hackbusch	**Hestenes**
Heun	**Hirschfelder**	**Householder**	Liu
Jacobi	**Kahan**	Kantorovich	Karmarkar
Kublanovskaya	Kutta	Lagrange	Lanczos
Lax	Legendre	**Lewy**	**Metropolis**
Moler	Morlet	**von Neumann**	Newton
Orszag	Powell	Raphson	Richardson
Rokhlin	Runge	Rutishauser	**Saad**
Schoenberg	**Sorensen**	Southwell	Stiefel
Tukey	**Ulam**	van der Vorst	Vinsome
Wilkinson	Zuse		

How old were they?

eligible for the Fox Prize!

Adams 36
Bezier 50
Cooley 39
Dantzig 33
de Casteljau 28
Francis ?
Gear 32
Greengard 29
Heun 41
Jacobi 22,40
Kublanovskaya
Lax 35
Moler 38
Orszag 31
Rokhlin 33
Schoenberg 43
Tukey 50
Wilkinson 46

Argyris 47
Brandt 35
Courant 40,57
Daubechies 34
Euler 59
Friedrichs 27
Givens 47
Griewank 40
Hirschfelder 41
Kahan 44
Kutta 34
Legendre 53
Morlet ?
Powell 27
Runge 45
Sorensen 45
Ulam 38
Zuse 26

Backus 30
Broyden 32
Curtiss 31
Davidon 32
Fedorenko ?
Garwin 37
Golub 33
Hackbusch 28
Householder 54
Kantorovich 27
Lagrange 23
Lewy 24
von Neumann 44
Raphson 42
Rutishauser 28
Southwell 52
van der Vorst33,42

Bashforth 64
Clough 39
Dahlquist 38
de Boor 25
Fletcher 24
Gauss 18,32,37
Gottlieb 33
Hestenes 46
Liu 43
Karmarkar 27
Lanczos 59
Metropolis 33
Newton 27
Richardson 35
Saad 36
Stiefel 43
Vinsome ?

CONCLUSIONS

- The inventors were/are almost all academic mathematicians
- Most were extremely eminent
- Their great discoveries came at all ages
- About half had major involvements with government or industry

  (That's *big* industry—AT&T, IBM, Boeing, etc.—
  and big government labs like Argonne, Harwell, NPL)
- Most were seriously involved with applications
- It's hard to disentangle the effects of WWII

What will be the first great numerical algorithm of the 21st century?