CONTROL 1, MA2A2 - 2008

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Sea $I_\delta = \iint_{\partial B(\vec{p},\delta)} (\phi \nabla\psi - \psi\nabla\phi)\cdot d\vec{s} \underset{\underset{P3,(a)}{\uparrow}}{=} \iint_{\partial B(\vec{p},\delta)} \left(-\phi(\vec{r})\frac{\vec{r}-\vec{p}}{\|\vec{r}-\vec{p}\|^3} - \frac{1}{\|\vec{r}-\vec{p}\|}\nabla\phi(\vec{r})\right)\cdot d\vec{s}$

Parametrización: $\vec{r} = \vec{p} + \delta\,\hat{r}(\theta,\varphi) = \vec{p} + (\delta \operatorname{sen}\varphi\cos\theta, \delta\operatorname{sen}\varphi\operatorname{sen}\theta, \delta\cos\varphi)$

0.4 para $\theta\in[0,2\pi)$, $\varphi\in[0,\pi]$.

Diferencial de superficie: $d\vec{s} = \hat{r}\, dA = \hat{r}\,\delta^2\operatorname{sen}\varphi\, d\theta\, d\varphi$

luego

$$I_\delta = I_\delta^1 + I_\delta^2$$

con

$$I_\delta^1 = -\int_0^{\pi}\int_0^{2\pi} \phi(\vec{p}+\delta\hat{r})\frac{\delta\hat{r}}{\delta^3}\cdot\hat{r}\,\delta^2\operatorname{sen}\varphi\, d\theta\, d\varphi$$

0.3

$$= -\int_0^{\pi}\int_0^{2\pi}\phi(\vec{p}+\delta\hat{r})\operatorname{sen}\varphi\, d\theta\, d\varphi \underset{\delta\to 0}{\longrightarrow} -\int_0^{\pi}\int_0^{2\pi}\phi(\vec{p})\operatorname{sen}\varphi\, d\theta\, d\varphi$$

$$= -4\pi\,\phi(\vec{p}).$$

y

$$|I_\delta^2| \le \int_0^{\pi}\int_0^{2\pi}\frac{1}{\delta}\|\nabla\phi(\vec{p}+\delta\vec{r})\|\,\delta^2\operatorname{sen}\varphi\, d\theta\, d\varphi$$

0.3

$$\le \max_{\vec{r}\in B(\vec{p},\delta_0)}\|\nabla\phi(\vec{r})\|\cdot\delta\, 4\pi \underset{\delta\to 0}{\longrightarrow} 0.$$

Así

$$I_\delta \underset{\delta\to 0}{\longrightarrow} -4\pi\,\phi(\vec{p}) + 0$$