CÁLCULO DIFERENCIAL E INTEGRAL

Problema 1. Considere S_k la suma parcial dada por:

$$\frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + ... + \frac{k}{(k+1)!}$$

Analice cuando S_k converge y calcule su valor.

Problema 2. Calcule el valor de las siguientes series:

(i) $\displaystyle\sum_{k\geq 1} \frac{4}{(4k-3)(4k+1)}$

(ii) $\displaystyle\sum_{k\geq 1} \frac{2k+1}{k^2(k+1)^2}$

Problema 3. Analice la veracidad de las siguientes implicancias: Sea $(A_k) \geq 0$

(i) $\sum A_k^2$ Converge $\Rightarrow \sum A_k$ Converge

(ii) $\sum A_k^2$ Converge $\Rightarrow \sum (-1)^k A_k$ Converge

(iii) $\sum A_k$ Converge $\Rightarrow \sum (-1)^k A_k$ Converge

(iv) $\sum (-1)^k A_k$ Converge $\Rightarrow \sum A_k$ Converge

(v) $\sum |A_k|$ Converge $\Rightarrow \sum (-1)^k A_k$ Converge

(vi) $\sum (-1)^k A_k$ Converge $\Rightarrow \sum A_k^2$ Converge

Problema 4. Analice la convergencia de:

(i) $\displaystyle\sum_{n\geq 1} \frac{\text{sen}(n^2)}{\sqrt{n^4+4}}$

(ii) $\displaystyle\sum_{n\geq 1} (-1)^{n+1}[\sqrt{n+1} - \sqrt{n}]$

(iii) $\displaystyle\sum_{n\geq 0} \frac{\text{sen}^2(n)}{3^n}$

(iv) $\displaystyle\sum_{n\geq 1} \frac{n + 2\log(n)}{n^2}$

(v) $\displaystyle\sum_{n\geq 0} (\frac{n+1}{n+3})^{n^2}$

(vi) $\displaystyle\sum_{n\geq 1} \frac{e^{pn}}{n!}$

(vii) $\displaystyle\sum_{n\geq 0} \arctan(\frac{1}{n^2+n+1})$

(viii) $\displaystyle\sum_{n\geq 0} \frac{n!}{n^n}$

(ix) $\displaystyle\sum_{n\geq 0} \frac{2^n n!}{n^n}$

(x) $\displaystyle\sum_{n\geq 2} \frac{1}{n\log(n) + (\log n)^{3/2}}$

(xi) $\displaystyle\sum_{n\geq 0} \frac{n^2}{\sqrt{2^n}}$

(xii) $\displaystyle\sum_{n\geq 2} \frac{(-1)^n}{n\ln(n)}$

Problema 5. Pruebe que si $x_n \geq 0$:

$$\sum x_n \text{ Converge } \Rightarrow \sum \sqrt{x_{n+1}x_n} \text{ Converge}$$

Problema 6. Sea $g : [0,1] \to \mathbb{R}^+$ continua y creciente en $[0,1]$, con $g(0) = 0$.
Pruebe que la serie $\displaystyle\sum_{n\geq 1} g(\frac{1}{n})$ converge si y solo si la integral $\displaystyle\int_0^1 \frac{g(x)}{x^2}dx$ converge.

Problema 7. Sea $\sum u_k$ con $u_k \geq 0$ tal que:

$$\text{lím} \frac{\ln(1/u_k)}{\ln(u_k)} = \lambda > 1$$

Pruebe que $\sum u_k$ converge.