PAUTA P1-CONTROL 1

Balance de masa estanque 1:

$$\rho \cdot A_1 \frac{dh_1}{dx} = \rho F - \rho \frac{h_1}{R_1}$$

$$A_1 \frac{dh_1}{dx} = F - \frac{h_1}{R_1}$$

Balance de masa estanque 2:

$$\rho \cdot A_2 \frac{dh_2}{dx} = \rho \frac{h_1}{R_1} - \rho \frac{h_2}{R_2}$$

$$A_2 \frac{dh_2}{dx} = \frac{h_1}{R_1} - \frac{h_2}{R_2}$$

Usando variables de desviación:

$$h_1' = h_{1} - h_1^{ee}$$

$$h_2' = h_{2} - h_2^{ee}$$

$$F' = F - F^{ee}$$

Reemplazando:

$$A_1 \frac{d{h_1}'}{dx} = F' - \frac{{h_1}'}{R_1}$$

$$A_2 \frac{d{h_2}'}{dx} = \frac{{h_1}'}{R_1} - \frac{{h_2}'}{R_2}$$

Aplicando *LaPlace:*

$$A_1 \cdot s \cdot \overline{h_1'} = \bar{F}' - \frac{\overline{h_1}'}{R_1}$$

$$A_2 \cdot s \cdot \overline{h_2'} = \frac{\overline{h_1'}}{R_1} - \frac{\overline{h_2'}}{R_2}$$

Reordenando…

$$A_1 \cdot s \cdot \overline{h_1'} \cdot R_1 + \overline{h_1}' = \bar{F}' \cdot R_1$$

$$G_1 = \frac{\overline{h_1}'}{\bar{F}'} = \frac{R_1}{A_1 \cdot R_1 \cdot s + 1}$$

$$A_2 \cdot s \cdot \overline{h_2'} \cdot R_2 + \overline{h_2'} = \frac{R_2}{R_1} \cdot \overline{h_1'}$$

$$G_2 = \frac{\overline{h_2'}}{\overline{h_1'}} = \frac{\frac{R_2}{R_1}}{A_2 \cdot R_2 \cdot s + 1}$$

Identificando términos se tiene:

$$Kp_1 = R_1$$

$$\tau p_1 = A_1 \cdot R_1$$

$$Kp_2 = R_2/R_1$$

$$\tau p_2 = A_2 \cdot R_2$$

Por otro lado

$$\frac{\overline{h_2'}}{\bar{F}'} = G_1 \cdot G_2 = \frac{R_2}{(A_1 \cdot R_1 \cdot s + 1) \cdot (A_2 \cdot R_2 \cdot s + 1)}$$

$$\frac{\overline{h_2'}}{\bar{F}'} = \frac{R_2}{(A_1 \cdot R_1 \cdot A_2 \cdot R_2) \cdot s^2 + (A_1 \cdot R_1 + A_2 \cdot R_2) \cdot s + 1}$$

Que corresponde a un segundo orden, donde

$$Kp = R_2$$

$$\tau = \sqrt{A_1 \cdot R_1 \cdot A_2 \cdot R_2}$$

$$\xi = \frac{(A_1 \cdot R_1 + A_2 \cdot R_2)}{2\sqrt{A_1 \cdot R_1 \cdot A_2 \cdot R_2}}$$

Como la respuesta es críticamente amortiguada => $\xi = 1$

$$\frac{(A_1 \cdot R_1 + A_2 \cdot R_2)}{2\sqrt{A_1 \cdot R_1 \cdot A_2 \cdot R_2}} = 1$$

$$(A_1 \cdot R_1 + A_2 \cdot R_2) = 2\sqrt{A_1 \cdot R_1 \cdot A_2 \cdot R_2} \,/(\,)^2$$

$$\left({A_1}^2 \cdot {R_1}^2 + 2 \cdot A_1 \cdot R_1 \cdot A_2 \cdot R_2 + {A_2}^2 \cdot {R_2}^2\right) = 4 \cdot A_1 \cdot R_1 \cdot A_2 \cdot R_2$$

$$\left({A_1}^2 \cdot {R_1}^2 - 2 \cdot A_1 \cdot R_1 \cdot A_2 \cdot R_2 + {A_2}^2 \cdot {R_2}^2\right) = 0$$

$$(A_1 \cdot R_1 - A_2 \cdot R_2)^2 = 0$$

$$A_1 \cdot R_1 = A_2 \cdot R_2$$

$$\frac{R_1}{R_2} = \frac{A_2}{A_1} = \frac{1}{2} = 0{,}5$$

b)

$$\tau p_1 = A_1 \cdot R_1$$

$$\tau p_2 = A_2 \cdot R_2$$

$$\tau = A_1 \cdot R_1 = \tau p_1 = \tau p_2$$

Se sabe que la solución para un sistema críticamente amortiguado corresponde a:

$$y'(t) = Kp\left[1 - e^{-\frac{t}{\tau}}\left(1 + \frac{t}{\tau}\right)\right]$$

$$h_2{}'(t = 1) = 0{,}5 \cdot Kp = Kp\left[1 - e^{-\frac{1}{\tau}}\left(1 + \frac{1}{\tau}\right)\right]$$

$$0{,}5 = e^{-\frac{1}{\tau}}\left(1 + \frac{1}{\tau}\right)$$

Así, $\tau = 0{,}5958\ min$

c)

$$0{,}9 = \left[1 - e^{\frac{-t}{0{,}5958}}\right]$$

$$t = 1{,}3719\ min$$