IQ57A- Dinámica y Control de Procesos
Control- Semestre Primavera 2008

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Problema 3

(a) Obtenga el modelo linealizado del sistema

Ecuaciones del sistema:

$$\frac{d\xi_1}{dt} = 10(6 - \xi_1) - 2\xi_1\sqrt{\xi_2} \qquad (1)$$
$$\frac{d\xi_2}{dt} = 80\mu - 10\xi_2 \qquad (2)$$
$$\frac{d\xi_3}{dt} = \xi_1\sqrt{\xi_2} + 0{,}1\xi_2 - 10\xi_3 \qquad (3)$$
$$\frac{d\xi_4}{dt} = 100\xi_1\sqrt{\xi_2} - 10\xi_4 \qquad (4)$$

Usando la expansión de Taylor:

$$f(X_1, X_2) = f(X_1^{ee}, X_2^{ee}) + \frac{\partial f(X_1^{ee}, X_2^{ee})}{\partial X_1}(X_1 - X_1^{ee}) + \frac{\partial f(X_1^{ee}, X_2^{ee})}{\partial X_2}(X_2 - X_2^{ee}) \qquad (5)$$

Se tiene lo siguiente:

$$\xi_1\sqrt{\xi_2} = \xi_1^{ee}\sqrt{\xi_2^{ee}} + \sqrt{\xi_2^{ee}}(\xi_1 - \xi_1^{ee}) + \frac{\xi_1^{ee}}{2\sqrt{\xi_2^{ee}}}(\xi_2 - \xi_2^{ee}) \qquad (5.1)$$

Llamando:

$$\alpha = \sqrt{\xi_2^{ee}}$$
$$\beta = \frac{\xi_1^{ee}}{2\sqrt{\xi_2^{ee}}}$$

La linealizacion queda:

$$\xi_1\sqrt{\xi_2} = \alpha\xi_1 + \beta(\xi_2 - \alpha) \qquad (5.2)$$

Reemplazando en las ecuaciones se tiene:

$$\frac{d\xi_1}{dt} = 10(6 - \xi_1) - 2[\alpha\xi_1 + \beta(\xi_2 - \alpha)] \qquad (1.1)$$
$$\frac{d\xi_2}{dt} = 80\mu - 10\xi_2 \qquad (2.1)$$
$$\frac{d\xi_3}{dt} = \alpha\xi_1 + \beta(\xi_2 - \alpha) + 0{,}1\xi_2 - 10\xi_3 \qquad (3.1)$$
$$\frac{d\xi_4}{dt} = 100[\alpha\xi_1 + \beta(\xi_2 - \alpha)] - 10\xi_4 \qquad (4.1)$$

Definiendo las siguientes variables de desviación:

$$\xi'_i = \xi_i - \xi_i^{ee}$$
$$\mu' = \mu - u^{ee}$$

Las ecuaciones quedan:

$$\frac{d\xi'_1}{dt} = -10\xi'_1 - 2[\alpha\xi'_1 + \beta\xi'_2] = -\xi'_1(10 + 2\alpha) - 2\beta\xi'_2 \quad (1.3)$$
$$\frac{d\xi'_2}{dt} = 80\mu' - 10\xi'_2 \quad (2.3)$$
$$\frac{d\xi'_3}{dt} = \alpha\xi'_1 + \beta\xi'_2 + 0{,}1\xi'_2 - 10\xi'_3 = \alpha\xi'_1 + (\beta + 0{,}1)\xi'_2 - 10\xi'_3 \quad (3.3)$$
$$\frac{d\xi'_4}{dt} = 100[\alpha\xi'_1 + \beta\xi'_2] - 10\xi'_4 \quad (4.3)$$

Aplicando la transformada de Laplace:

$$s\overline{\xi'_1} = -\overline{\xi'_1}(10 + 2\alpha) - 2\beta\overline{\xi'_2} \quad (1.4)$$
$$s\,\overline{\xi'_2} = 80\overline{\mu'} - 10\overline{\xi'_2} \quad (2.4)$$
$$s\overline{\xi'_3} = \alpha\overline{\xi'_1} + (\beta + 0{,}1)\overline{\xi'_2} - 10\overline{\xi'_3} \quad (3.4)$$
$$s\overline{\xi'_4} = 100\left[\alpha\overline{\xi'_1} + \beta\overline{\xi'_2}\right] - 10\overline{\xi'_4} \quad (4.4)$$

De (1.4)

$$\boxed{G_{12} = \frac{\overline{\xi'_1}}{\overline{\xi'_2}} = \frac{{-2\beta}/{(10+2\alpha)}}{{s}/{(10+2\alpha)}+1}} \quad (1.5)$$

De (2.4)

$$\boxed{G_{\mu 2} = \frac{\overline{\xi'_2}}{\overline{\mu}} = \frac{8}{0{,}1s+1}} \quad (2.5)$$

De (3.4)

$$\overline{\xi'_3} = \frac{\alpha/10}{0{,}1s + 1}\overline{\xi'_1} + \frac{(\beta + 0{,}1)/10}{0{,}1s + 1}\overline{\xi'_2}$$

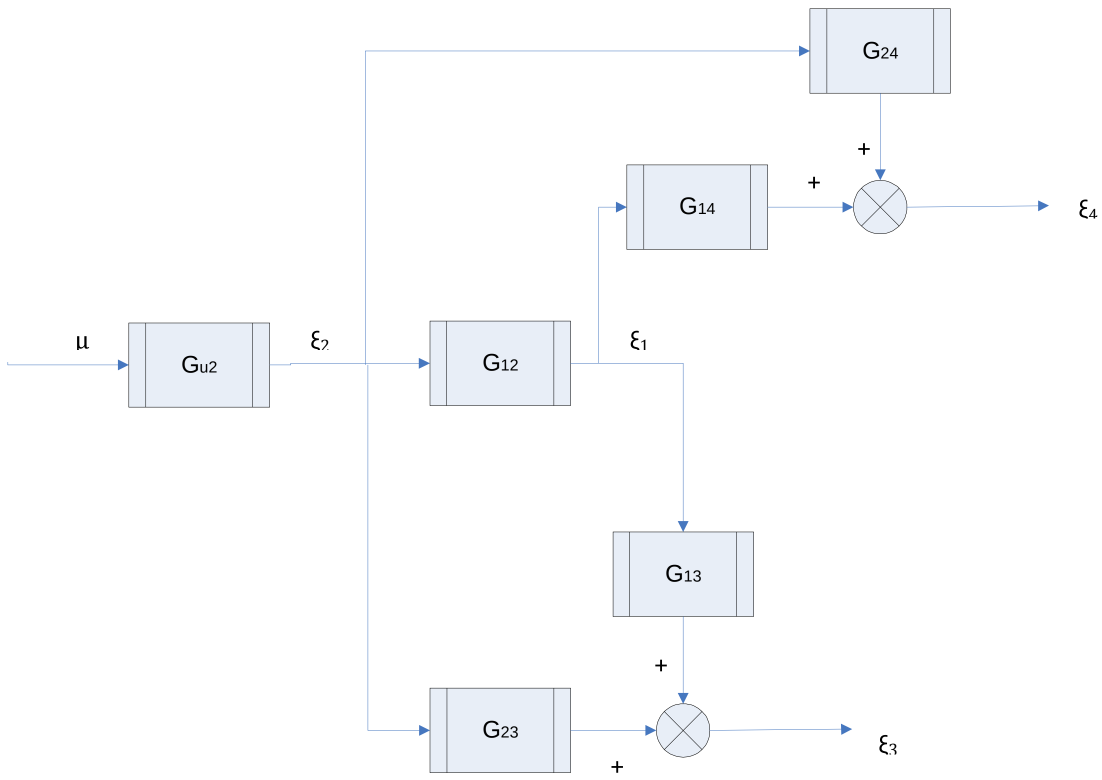
$$\boxed{\overline{\xi'_3} = G_{13}\overline{\xi'_1} + G_{23}\overline{\xi'_2}} \quad (3.5)$$

De (4.4)

$$\overline{\xi'_4} = \frac{10\alpha}{0{,}1s + 1}\overline{\xi'_1} + \frac{10\beta}{0{,}1s + 1}\overline{\xi'_2}$$

$$\boxed{\overline{\xi'_4} = G_{14}\overline{\xi'_1} + G_{24}\overline{\xi'_2}} \quad (4.5)$$

b) Dibuje un diagrama de bloques del sistema completo indicando las variables de entrada y salida



c) Usando el modelo linealizado encuentre la función de transferencia entre $\overline{\mu'}(s)$ $y\, \overline{\xi'_3}(s)$

De(1.5) (2.5) (3.5):

$$\overline{\xi'_3} = G_{13}\overline{\xi'_1} + G_{23}\overline{\xi'_2}$$

$$G_{\mu 2} = \frac{\overline{\xi'_2}}{\bar{\mu}} \rightarrow \overline{\xi'_2} = G_{\mu 2}\bar{\mu}$$

$$G_{12} = \frac{\overline{\xi'_1}}{\overline{\xi'_2}} \rightarrow \overline{\xi'_1} = G_{12}\overline{\xi'_2}$$

Reemplazando

$$\overline{\xi'_3} = G_{13}G_{12}\overline{\xi'_2} + G_{23}\overline{\xi'_2} = (G_{13}G_{12} + G_{23})\overline{\xi'_2} = (G_{13}G_{12} + G_{23})G_{\mu 2}\bar{\mu}$$

$$\frac{\overline{\xi'_3}}{\bar{\mu}} = \left(\frac{\alpha/10}{0{,}1s+1} * \frac{-2\beta/(10+2\alpha)}{s/(10+2\alpha)+1} + \frac{(\beta+0{,}1)/10}{0{,}1s+1}\right) * \frac{8}{0{,}1s+1}$$