Macroeconomía y Costos de Ajuste
Cátedras 11 y 12
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III. Prices

1. Motivation
2. Partial equilibrium models
3. Microeconomic Evidence
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5. General Equilibrium Models

1. A generic model
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5. Midrigan (2006)
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7. Gertler and Leahy (2006)
8. Kehoe and Midrigan (2007)

5.1. A generic model

1. Consumers
2. Firms
3. Equilibrium
4. Computing the equilibrium

We follow Midrigan (2006), simplifying to the case of a one-product firms.

Basics

- s_t: event realized at time t
- $s^t = (s_0, s_1, ..., s_t)$: history of events upto t
- $\pi(s^t)$: probability of s^t as of time 0
- Continuum of identical consumers of mass 1
- Continuum of monopolistically competitive firms of mass 1, they differ in their productivity levels
- Each firm sells one good, indexed by $z \in [0, 1]$

Consumers

At time 0 the representative consumer chooses plans $\{c(z, s^t), n(s^t)\}$ to maximize

$$\sum_{t=0}^{\infty} \beta^t \sum_{s^t} \pi(s^t) U(c(s^t), n(s^t))$$

subject to

$$\int_0^1 p(z, s^t) c(z, s^t) dz = w(s^t) n(s^t) + \Pi(s^t),$$

and where $c(s^t)$ denotes the consumption aggregator

$$c(s^t) = \left(\int_0^1 c(z, s^t)^{(\theta-1)/\theta} dz \right)^{\theta/(\theta-1)}$$

Comments on previous slide

Comments on previous slides formulation:

- Preferences are defined over leisure and a continuum of imperfectly substitutable goods indexed by z
- Consumers sell part of their time endowment to the labor market and invest their wealth in one-period shares of firms. Of course, in equilibrium identical consumers hold equal shares of all the economy's firms
- n denotes the supply of labor, w the nominal wage rate, Π profits received by the consumer, $p(z, s^t)$ the nominal price of good z
- θ: elasticity of substitution across goods

Firms

Firms produce output using a technology linear in labor:

$$y(z, s^t) = a(z, s^t) l(z, s^t),$$

where the firm's productivity evolves according to:

$$\log a(z, s^t) = \rho_a \log a(z, s^{t-1}) + \epsilon(z, s_t)$$

and $\epsilon \in [\epsilon_{\min}\ ,\ \epsilon_{\max}]$ is a random variable uncorrelated across firms, goods and the time period.

Firms operate along their consumer demand schedules, derived as solution to the consumer problem discussed above:

$$c(z, s^t) = \left(\frac{p(z, s^t)}{P(s^t)} \right)^{-\theta} c(s^t)$$

with the price index P is defined as:

$$P(s^t) = \left(\int_0^1 p(z, s^t)^{1-\theta} dz \right)^{1/(1-\theta)} .$$

Firms

Every time a firm resets its price it pays a fixed cost of ξ in units of labor

The t-period stochastic discount factor is:

$$q(s^t) \equiv \beta^t \frac{U_c(c(s^t), n(s^t))}{U_c(c(s^o), n(s^0))},$$

where U_c denotes the marginal utility of consumption

The firm's problem therefore is:

$$\max \sum_{t=0}^{\infty} \sum_{s^t} \pi(s^t) q(s^t) \Pi(z, s^t)$$

where

$$\Pi(z, s^t) = \left(\frac{p(z, s^t)}{P(s^t)} \right)^{-\theta} \left(\frac{p(z, s^t)}{P(s^t)} - \frac{w(s^t)}{a(z, s^t) P(s^t)} \right) c(s^t) - \xi \frac{w(s^t)}{P(s^t)} \mathcal{I} \left\{ p(z, s^t) \neq p(z, s^{t-1}) \right\}$$

Money

Money is introduces by assuming that nominal spending must be equal to the money stock:

$$\int_0^1 p(z,s^t)c(z,s^t)dz = M(s^t).$$

The log-money supply growth rate

$$\mu(s^t) \equiv \frac{M(s^t)}{M(s^{t-1})}$$

evolves according to an AR(1) process

$$\log \mu(s^t) = \rho_\mu \log \mu(s^{t-1}) + \eta(s^t),$$

with η an i.i.d. $N(0,\sigma_\eta^2)$ innovation.

Equilibrium

An equilibrium is a collection of prices and allocations: $p(z, s^t)$, $w(s^t)$, $P(s^t)$, $\Pi(s^t)$, $c(z, s^t)$, $c(s^t)$, $n(s^t)$, $l(z, s^t)$, $y(z, s^t)$ such that

1. taking prices (including Π) as given, consumers demand $c(z, s^t)$ and supply labor $n(s^t)$
2. taking other firms' prices and the price level as given, firms choose their price and labor demand $p(z, s^t)$, $l(z, s^t)$
3. goods markets clear
4. labor markets clear
5. money market clears
6. actual profits equals Π_t assumed by consumer

Computation of Equilibrium

- Normalize all variables by the money stock:

$$\bar{P}(s^t) = \frac{P(s^t)}{M(s^t)}.$$

This helps get a *bounded* state-space:

- Define the firm's last period's normalized price:

$$\bar{p}_{-1}(z, d^t) = \frac{p(z, z, s^{t-1})}{M(s^t)} \in \mathcal{P}$$

- Support for the distribution of productivity shocks:

$$\mathcal{A} = \left[\frac{\epsilon_{\min}}{1-\rho}, \frac{\epsilon_{\max}}{1-\rho}\right].$$

Computation of Equilibrium

- Aggregate state of the economy is an infinite dimensional object consisting of
  - $\mu(s^t)$
  - joint distribution of last period's firm prices and technology levels:

$$\phi : 2^{\mathcal{P}} \times 2^{\mathcal{A}} \to [0, 1]$$

  - law of motion:

$$\phi' = \Gamma(\mu, \phi)$$

Computation of Equilibrium

Let $V^a(a,\mu,\phi)$ and $V^n(p_{-1},a,\mu,\phi)$ denote a firm's value of adjusting and not adjusting its nominal price.

We have:

$$V^a(a,\mu,\phi) = \max_{p}\left\{\left(\frac{\bar{p}_{-1}}{\bar{P}} - \frac{\bar{w}}{a\bar{P}}\right)\left(\frac{\bar{p}}{\bar{P}}\right)^{-\theta} c - \xi\frac{\bar{w}}{\bar{P}} + \beta\int \frac{U'_c}{U_c} V(p'_{-1},a',\mu',\phi')dF(\epsilon,\eta)\right\}$$

$$V^n(p_{-1},a,\mu,\phi) = \left(\frac{\bar{p}_{-1}}{\bar{P}} - \frac{\bar{w}}{a\bar{P}}\right)\left(\frac{\bar{p}_{-1}}{\bar{P}}\right)^{-\theta} c + \beta\int \frac{U'_c}{U_c} V(p'_{-1},a',\mu',\phi')dF(\epsilon,\eta)$$

where $V = \max(V^a, V^n)$ is the firm's value function and p is the nominal price the firm chooses whenever it adjusts.

Computation of Equilibrium

The laws of motion of the state vector is:

$$\phi' = \Gamma(\mu, \phi)$$

with

$$\begin{aligned} \log a' &= \rho_a \log a + \epsilon, \\ \log \mu' &= \rho_\mu \log \mu + \eta \end{aligned}$$

and

$$p'_{-1} = \begin{cases} \bar{p}/\mu & \text{if adjust,} \\ \bar{p}_{-1}/\mu & \text{otherwise} \end{cases}$$

The unknowns are the functions V^a, V^n, c, $\bar{w}$, $\bar{P}$ and Γ.

Computation of Equilibrium

To solve numerically:

- allow aggregate variables to dends only on finite number of moments of ϕ (Krusell-Smith trick)
- replace unknown functions with a linear combination of orthogonal polynomials
- solve for the unknown coefficients for these polynomials by requiring that the set of six functional equations (2 Bellman equations plus 4 equilibrium conditions) be exactly satisfied at a finite number of nodes along the state-space

5.2. Dotsey, King and Wolman (1999)

Assumptions:

- Only source of heterogeneity: stochastic adjustment cost (i.i.d. across firms and over time, from distribution $G(\omega)$, in units of labor). I.e., no productivity shocks (idiosyncratic or aggregate).
- Aggregate shock: money supply, follows a random walk with positive drift, all realizations are positive.
- Assumes (does not derive) money demand of the form:

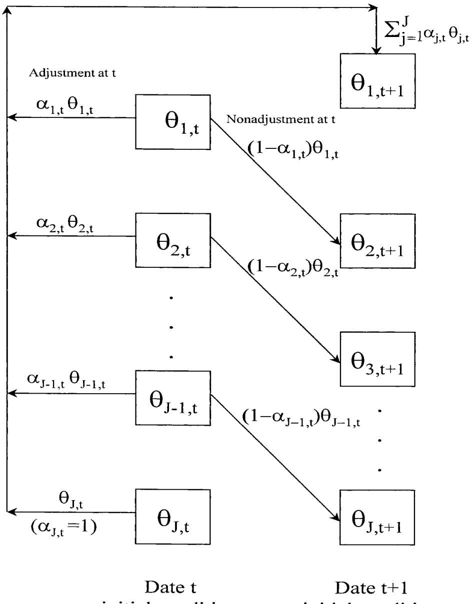
$$\log(M_t/P_t) = \log C_t - \eta R_t.$$

- Fixed supply of capital

Dotsey, King and Wolman (1999): Results

- All firms that adjust their price in period t choose the same price: P_t^*.
- All firms that last adjusted j periods ago (vintage j) charge the same nominal (and real) price at time t: P_{t-j}^*
- Firms in vintage j adjust if and only if their current adjustment cost draw is below a certain threshold, $\omega_{j,t}$.
- Older vintages are more likely to adjust: for given t, $\omega_{j,t}$ is increasing in j
- All firms adjust after J periods

Cross-section dynamics



DKW: Bellman equations

Denote:

- θ_{jt}: fraction of firms that last adjusted their price j periods ago, as of the beginning of period t, $j = 1, \ldots, J$.
- α_{jt}: fraction of firms in vintage j that adjust at time t, $j = 1, \ldots, J$. $\alpha_{Jt} = 1$.
- $\bar{\alpha} \equiv 1 - \alpha$
- $v_{0,t}$: value of a price-adjusting firm at t
- $v_{j,t}$: value of firm in vintage j at time t, $j = 0, 1, \ldots, J - 1$
- $\Xi_{j,t}$: total expected resources associated with adjustment of the j-th vintage in period t

DKW: Bellman equations

We then have:

$$\begin{aligned} v_{0,t} &= \max_{P_t^*}[\pi_{0t} + \beta \mathrm{E}_t \frac{\lambda_{t+1}}{\lambda_t}\{\bar{\alpha}_{1,t+1} v_{1,t+1} + \alpha_{1,t+1} v_{0,t+1} - \Xi_{1,t+1}\}] \\ v_{j,t} &= \pi_{jt} + \beta \mathrm{E}_t \frac{\lambda_{t+1}}{\lambda_t}\{\bar{\alpha}_{j+1,t+1} v_{j+1,t+1} + \alpha_{j+1,t+1} v_{0,t+1} - \Xi_{j+1,t+1}\} \end{aligned}$$

The adjustment cost threshold for vintage *j* satisfies

$$v_{0,t} - v_{j.t} = w_t \xi \implies \alpha_{j,t} = G((v_{0,t} - v_{j.t})/w_t).$$

Also:

$$\Xi_{j,t} = w_t \int_0^{G^{-1}(\alpha_{j,t})} \xi g(\xi) d\xi.$$

State-Dependent Adjustment Hazard

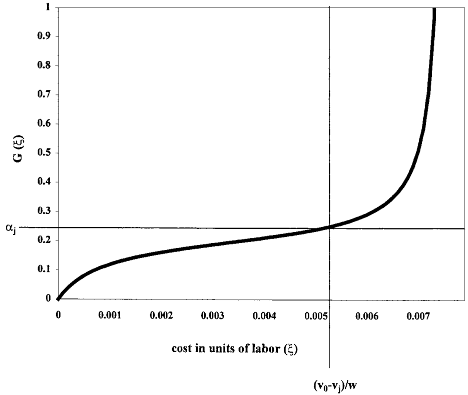


FIGURE II
The Distribution of Fixed Costs, and Determination of the Marginal Firm

Aggregate State Vector and Computation

To describe the economy at time t, we need:

- price distribution: P^*_{t-j}, $j = 1, \dots, J$; $\theta_{j,t}$, $j = 1, \dots, J$.
- exogenous variables that describe the money supply process.

Even though the state-space is finite and relatively small ($J = 8$ in the benchmark case), DKW linearize the model around the steady-state with no uncertainty in the growth of money supply. This rules out non-linear effects (i.e., time-varying IRFs) by assumption.

Effect of Money Supply Shock

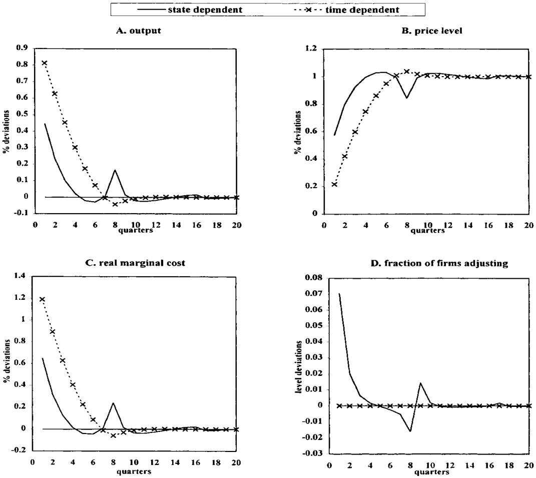


FIGURE IV
Baseline Experiment: Response to a Permanent Money Supply Shock

Effect of Money Supply Shock

- There's an echo effect, which would disappear if you had another source of heterogeneity (e.g., firm-specific productivity shocks)
- State- and time-dependent models calibrated so that the fraction of adjusters is the same in both cases. Hence we understand well why prices adjust faster in the state-dependent model (positive extensive margin).
- IRF of inflation upon impact in state-dependent model is 3 times as large as in the time-dependent version. We'll see next that this is no accident.

A Rule of Thumb

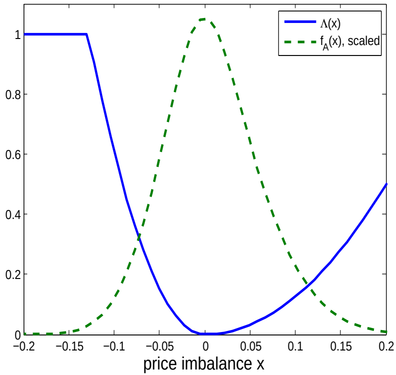
Back to the general framework in Caballero and Engel (2007), even though what follows is only in the working paper version.

$$\begin{aligned}
\mathcal{F} &= \int \Lambda(x) f_A(x) dx + \int x\Lambda'(x) f_A(x) dx \\
&= \int_{\{x:\Lambda(x)>0\}} \Lambda(x) \left[1 + \frac{x\Lambda'(x)}{\Lambda(x)}\right] f_A(x) dx \\
&= \int_{\{x:\Lambda(x)>0\}} \Lambda(x) \left[1 + \eta(x)\right] f_A(x) dx \\
\mathcal{A} &= \int_{\{x:\Lambda(x)>0\}} \Lambda(x) f_A(x) dx
\end{aligned}$$

If $\eta(x) \cong \eta$ in the region with most of the mass of $f_A(x)$, then:

$$\mathcal{F} \cong (\eta + 1)\mathcal{A}.$$

In the region where $f_A(x)$ has most of its mass: $\eta(x) \cong 2$.



A useful benchmark/rule-of-thumb:

$$\mathcal{F} \sim 3\mathcal{A}.$$

5.3. Danziger (1999)

Underappreciated paper

Assumptions:

- Heterogeneity: idiosyncratic productivity shocks (random walk, uniform distribution with large variance)
- Aggregate shocks: money supply (random walk with positive drift) and productivity shock (also random walk with drift)
- Adjustment cost: a fraction of profits

Danziger (1999): Results

- Proves existence of an equilibrium in Markov strategies with two-sided *Ss* pricing policies (the (L, C, U) policies we discussed earlier) in the firm's log-markup.
- Introduces a useful trick that simplifies the state space notably: distribution of productivity shocks is irrelevant
- Money is (almost) neutral

Bellman Equation

At time t a firm's owner chooses his nominal price p_t to maximize:

$$V_t = \mathrm{E}_t \sum_{s=1}^{\infty} e^{-\rho s}(1 - \mathcal{A}_s)\left(\frac{M_s}{p_s} - \frac{w_s M_s}{q_s p_s^2}\right)$$

where $\rho > 0$ is the owner's discount rate, w_t the market wage, q_t the firm's total productivity, M_t the money supply, and $\mathcal{A}_t$ the fraction of firms adjusting. The above expression incorporates market clearing for each monopolistically competitive good.

The firm owner's utility is independent of the price level P_t, because demand is proportional to P_t (adjustment cost a fraction of profits used here), and the marginal product is constant.

Equilibrium

Only Markov strategies are considered, i.e.:

$$p_t = p_t(q_t, w_t, M_t, p_{t-1}).$$

The economy is in a Markov-perfect equilibrium if, at each t:

1. Each firm owner's Markov price strategy maximizes his expected discounted utility assuming that all other firm owners follow their Markov price strategy
2. Aggregate demand equals aggregate supply of labor.

Equilibrium

Denote:

- $q_{it} = a_{it} b_t$: firm i's productivity, equal to the product of its idiosyncratic and aggregate components, both of which follow geometric random walks
- ξ_{it}: firm i's markup at the beginning of period t, before price adjustments:

$$\xi_{it} = \frac{q_{it} p_{i,t-1}}{w_t}.$$

Equilibrium

Danziger proves the existence of an equilibrium with:

- w_t proportional to $b_t M_t$:

$$w_t = \omega b_t M_t.$$

- ξ_t follows a two-sided *Ss* policy:

$$p_t = \begin{cases} p_{t-1}, & \text{if } \xi_t \in (s, S), \\ \omega l M_t / a_t, & \text{otherwise.} \end{cases}$$

The Crucial Trick

- Assume the distribution of $\log \xi_t$ at the beginning of period t has mass $\mathcal{A}$ at $\log I$ and mass $1 - \mathcal{A}$ at a uniform on $(\log s, \log S)$
- The aggregate monetary shock takes place, shifting the above distribution by Δm to the left
- Idiosyncratic shocks with a cross-section that is uniform over an interval much wider than $\log S - \log s$ take place next. The resulting distribution is a weighted average of two densities:
    - a uniform on $[-\log \bar{\alpha} + \log I, \log \bar{\alpha} + \log I]$ corresponding to firms with $\xi = I$ at the beginning of the period.
    - a sum of two independent uniforms remaining firms

A basic probability result

Assume X and Y are independent random variables, X uniform on $[0, a]$ and Y uniform on $[0, b]$, with $b > a$. Let $Z = X + Y$. Then the density of Z, $f(z)$, satisfies:

$$f(z) = \begin{cases} z/ab, & 0 < z < a, \\ 1/b, & a < z < b, \\ (a+b-z)/ab, & b < z < a+b. \end{cases}$$

The Crucial Trick

Back to where we were two slides ago.

From the probability result it follows that, after adjusting, the log-markup distribution of firms:

- with initial markup equal to I will be a convex combination of a uniform on $(\log s, \log S)$ and a mass point at $\log I$.
- in the group initially uniformly distributed on the inaction range will be a convex combination of a mass point at the return point and a uniform on the inaction range.

The Crucial Trick

- A simple calculation shows that the fraction of firms adjusting within each group (and therefore overall) is equal to $(2 \log \bar{\alpha} - \log(S/s))/2 \log \bar{\alpha}$.
- Hence, as long as

$$\mathcal{A} = \frac{2 \log \bar{\alpha} - \log(S/s)}{2 \log \bar{\alpha}}$$

we have that the beginning-of-period distribution is invariant, no matter what is the realization of the aggregate shock.

Money is (almost) neutral

- See Figure 2 in the paper
- For a formal proof: use the expression we derived for $\mathcal{E}$ in the case of two sided policies to prove that $\mathcal{E} = 1 - \mathcal{A}$, and hence $\mathcal{F} = 1$. This derivation ignores non-linearities due to Dixit-Stiglitz price indices, which explains the "almost" neutrality statement.

5.4. Golosov and Lucas (2007)

Assumptions:

- Cf. generic model (Sec. 5.1).
- Monetary growth rate constant in benchmark case: $\rho_\mu = 0$ and $\sigma_\eta = 0$
- Large idiosyncratic (productivity) shocks play an important role

Optimal Policy

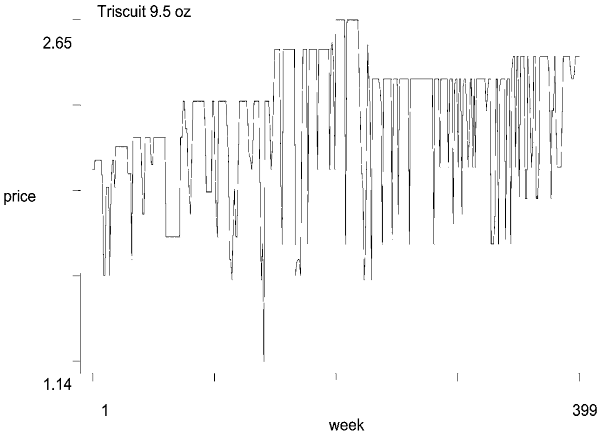
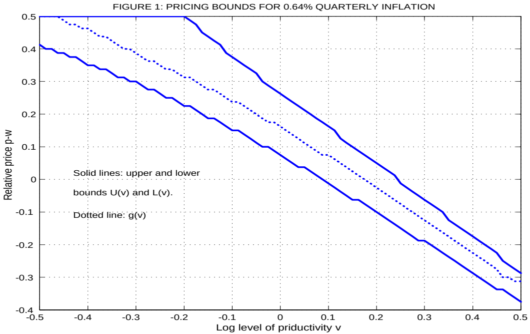


FIG. 2.—Price of Triscuits (9.5 oz.) in Dominick's Finer Foods supermarket in Chicago. Source: Chevalier et al. (2000).

Remove sales from the data, since sales are not part of the model.

Optimal Policy



Inaction range smaller when productivity high:

1

- Golosov-Lucas: getting prices right is more valuable when productivity is high
- Really due to having an adjustment cost that does not depend on the level of production

Calibration

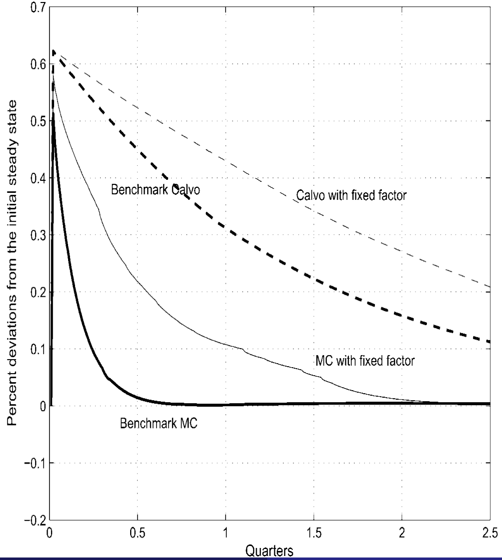
MENU COSTS AND PHILLIPS CURVES

TABLE 1
CALIBRATED PARAMETER VALUES
Baseline Values: $(\eta, \sigma_v^2, k) = (.55, .011, .0025)$

Moment	Data (1)	Model (2)	$\eta = .65$ (3)	$\sigma_v^2 = .015$ (4)
Quarterly inflation rate	.0064	.0064	.0064	.0064
Standard deviation of inflation	.0062	0	0	0
Frequency of change	.219	.239	.232	.273
Mean price increase	.095	.097	.094	.104
Standard deviation of new prices	.087	.090	.080	.108

NOTE.—Col. 2 is based on the baseline values. Cols. 3–5 are based on the same values, except for the at the head of each column.

Impulse Response Function



Impulse Response Function

- Considerably less monetary non-neutrality that in “equivalent” Calvo model
- Equivalent Calvo model: same fraction of adjusters, ...
- Intuition provided by Golosov-Lucas: selection effect
- Correct intuition: positive extensive margin, difference particularly large with fixed *Ss* rules

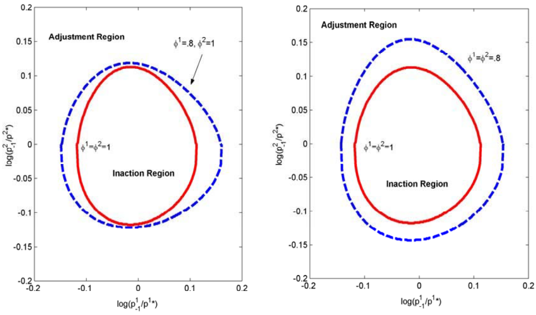
5.5. Midrigan (2006)

Summary:

- Multiproduct firm, after paying the fixed cost can adjust the price of all goods
- Also assume persistence in monetary growth rate: $\rho_\mu > 0$
- Obtain an adjustment hazard much closer to Calvo than in Golosov-Lucas, hence an IRF that similar to that in the equivalent Calvo model
- Distribution of idiosyncratic shocks: heavier tails than normal to match the distribution of price changes

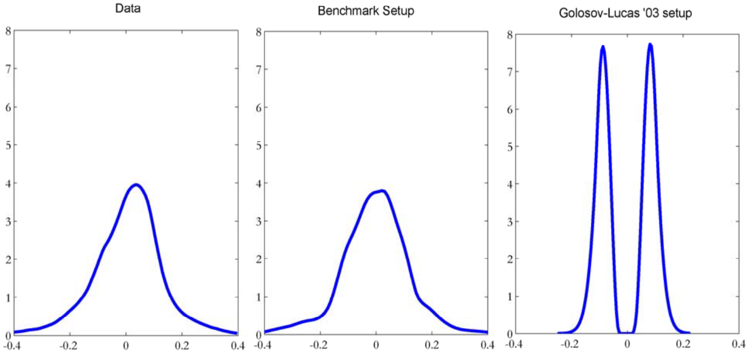
Optimal Policy

Figure 3: Inaction (Ss) regions for multi-product firms



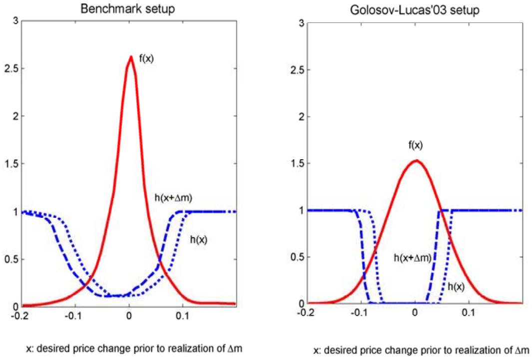
Distribution of Price Changes

Figure 4: Distribution of non-zero price changes: Model vs. Data



Midrigan vs. Golosov-Lucas

Figure 7: Adjustment hazard and ergodic density of desired price changes



5.6. Carvalho (2006), Nakamura-Steinsson (2006)

- Heterogeneous across price-setters can increase aggregate inertia
- In the simplest setting: half-life of a shock is determined by the median adjustment cost
- Carvalho: Calvo setting
- Nakamura-Steinsson: *Ss*-setting

5.7. Gertler-Leahy (2006)

- Find an *Ss*-type model where the New Keynesian Phillips Curve equation is a “good” approximation
- Use the Danziger trick to keep the state-space simple and derive explicit approximations
- Islands with local labor markets, only a fraction of islands receive idiosyncratic shocks in a given period
- Set up such that firms do not adjust when their island received no idiosyncratic shock

5.8. Kehoe and Midrigan (2007)

- First paper to incorporate sales into the model
- Two adjustment technologies:
  - higher fixed cost to change price forever
  - lower fixed cost to change price for one period
- Dominick's scanner price data:
  - prices change a lot, but spend most of the time at one price