

# Macroeconomía y Costos de Ajuste

## Cátedras 9 y 10

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# III. Prices

- 1 Motivation
- 2 Partial equilibrium models
- 3 Microeconomic Evidence
- 4 Distributional Dynamics
- 5 General equilibrium models

# 1. Motivation

- ① Monopoly
- ② Monopolistic Competition
- ③ New Keynesian Phillips Curve

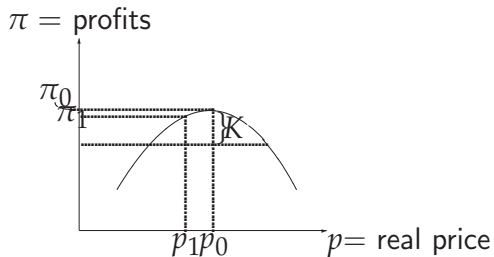
# Introduction

- One of the main questions macroeconomics deals with is why money matters, that is, why do nominal variables (such as money) have real effects.
- Lucas and others showed that under rational expectations money is neutral. The Keynesian response initially centered on rigidities in labor contracts (Fischer, Taylor).
- The problem with this approach is that it implies countercyclical wages, which are not observed in practice.

# Introduction

- A second problem is that labor relations are usually long term, so that the wage observed at a given moment in time may not be a good indicator of true labor compensation.
- A second generation of models in the Keynesian tradition (Mankiw (1985), Akerloff y Yellen (1985) y Parkin (1985)) departed from the assumption of perfect competition, focussing on price rigidities instead of wage rigidities. In this literature the cost of adjusting prices is crucial to explain the output-inflation tradeoff (Phillips curve).
- Precursors of fixed adjustment costs Ss literature: Barro (1972), Sheshinski and Weiss (1977, 1983)

## 1.1. Monopoly



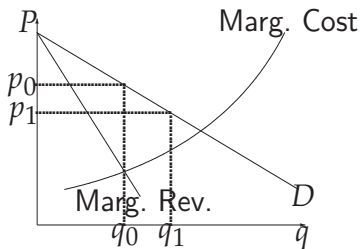
- A monopolist faces a fixed cost  $K$  of adjusting her price:
- Intuition:
  - since adjusting prices is costly, the monopolist doesn't adjust until the difference between her profits with and without adjusting exceeds  $K$ .
  - also, when a monopolist faces a price (somewhat) below her optimal price, she maximizes profits by increasing the quantity produced (since marginal revenues is above marginal cost at the monopoly price).
  - it follows that inflation (in the model: a reduction in the real price) leads to an increase in output.

# Formal Derivation

- $t = 0$ : Monopolist charges (log) real price  $p_0$  and obtains profits  $\pi_0$ .
- $t = 1$ : Monetary (or aggregate demand) shock takes place, leading to a real-log-price  $p_1 < p_0$ .
- Since  $\pi(p_0) - \pi(p_1) < K$ , it is optimal for the monopolist not to adjust nominal prices (and increase output).
- Hence a positive monetary shock increases output.



# Formal Derivation

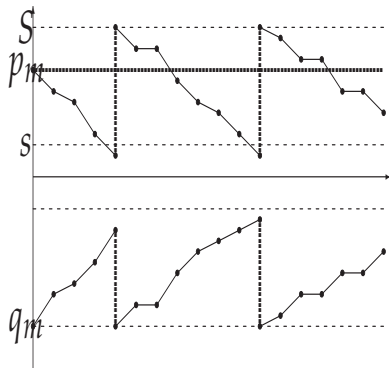


- 1 From the envelope theorem we have that, if  $p_0 - p_1 = \varepsilon$ , then  $\pi(p_0) - \pi(p_1) = O(\varepsilon^2)$ . Hence small ('menu') costs of adjusting (of order  $\varepsilon^2$ ) have large effects (of order  $\varepsilon$ ) both on  $p$  and on  $q$ .
- 2 As mentioned in Section I of the course, recent papers (Levy et al. 1997; Zbaracki et al., 2004) measure the actual costs of adjusting prices finding that they are much larger than presumed. Hence the argument made in the previous point isn't necessary to justify the importance of frictions in price adjustment.
- 3 A central element in the result above is that we abandoned the assumption of perfect competition. Otherwise a monetary expansion doesn't lead to an increase in output.

# Limitations

- When prices finally adjust, output falls substantially (see the figure on the next page).
- This motivates taking aggregation and dynamics seriously, moving from a monopoly to monopolistic competition.

# Limitations



## 1.2. Monopolistic Competition

- Blanchard and Kiyotaki (1987, AER)
- What follows: simplified version in Ch. 6 (secs. 4, 5 and 6) of Romer (2005, 3rd ed)
- Incorporates money into standard Dixit-Stiglitz model
- Next: static model, useful for intuition
- Later: dynamic models

# Labor supply

- Large number of individuals, each one sets the price of one good and is the sole producer of that good
- Labor is the only input into production
- Individuals do not produce their own good directly; instead there is a competitive labor market where they can sell their labor and hire workers
- Awkwardness of above assumptions not needed (see Blanchard and Kiyotaki's paper), yet it simplifies the derivation considerably

# Demand for goods

- Demand for each good is log-linear:

$$q_i = y - \eta(p_i - p), \quad (1)$$

where

- $q_i$ : log of producer  $i$ 's output
- $p_i$ : log of producer  $i$ 's price
- $y$ : average of the  $q_i$ 's
- $p$ : average of the  $p_i$ 's
- $\eta$ : output elasticity of demand, assumed larger than one to ensure interior solution

# Demand for goods

- Thus sellers have market power to set their price above marginal cost.
- Hence, if they cannot adjust their price to respond to a positive demand shock, they are willing to increase their production to satisfy demand at the current price
- Production function:

$$Y_i = L_i$$



# Individual's Optimization Problem

- Individual  $i$  maximizes:

$$U_i = C_i - \frac{1}{\gamma} L_i^\gamma,$$

where

- $C_i$ : individual's nominal income divided by the price level
  - Wage elasticity of labor supply (derived below):  $1/(\gamma - 1)$ .
- It follows that individual  $i$  maximizes:

$$U_i = \frac{(P_i - W)Q_i + WL_i}{P} - \frac{1}{\gamma} L_i^\gamma. \quad (2)$$

where  $W$  denotes the nominal wage and the remaining notation is obvious.

# Aggregate Demand

- We assume:

$$y = m - p. \quad (3)$$

- Many interpretations:

- 1 A useful shortcut, what we need is that  $y$  be decreasing in  $p$ . In this context  $m$  is a generic variable affecting aggregate demand, not money.
- 2 Blanchard and Kiyotaki (1987) replace  $C$  in the utility function by a Cobb-Douglas aggregator of  $C$  and  $M/P$  to derive (3).
- 3 Rotemberg (1987) derives (3) from a cash-in-advance constraint.

- Substituting (1) into (2) gives us

$$U_i = \frac{(P_i - W)Y(P_i/P)^{-\eta} + WL_i}{P} - \frac{1}{\gamma}L_i^\gamma. \quad (4)$$

- Choice variables:  $P_i$  and  $L_i$

- FOC w.r.t.  $P_i$  leads to:

$$\frac{P_i}{P} = \frac{\eta}{\eta - 1} \frac{W}{P}. \quad (5)$$

- That is, a producer with market power sets price as a markup over marginal cost, with the size of the markup decreasing in the elasticity parameter  $\eta$ .

- FOC w.r.t.  $L_i$  leads to:

$$L_i = \left( \frac{W}{P} \right)^{1/(\gamma-1)} . \quad (6)$$

- Thus labor supply is an increasing function of the real wage, the elasticity is  $1/(\gamma - 1)$ .

# Equilibrium

- Symmetry  $\Rightarrow$  in equilibrium  $L_i = Y_i = Y$ .
- Hence, from (6):

$$\frac{W}{P} = Y^{\gamma-1}.$$

- Substituting this expression into (5) yields each producer's **desired** relative price as a function of output:

$$p_i^* - p = c + (\gamma - 1)y, \quad (7)$$

with  $c = \log \eta / (\eta - 1)$ .

- We return to this expression shortly

# Equilibrium

- Since producers are symmetric, each one will take the price level  $P$  as given, and will charge  $P$ . Hence, from (7) we have that the equilibrium level of output is:

$$Y = \left( \frac{\eta - 1}{\eta} \right)^{1/(\gamma-1)}. \quad (8)$$

- Finally, combining the above expression with (3) yields the equilibrium price level:

$$P = \frac{M}{\left( \frac{\eta-1}{\eta} \right)^{1/(\gamma-1)}}. \quad (9)$$

# Implications

- A planner would choose  $L_i$  so as to maximize

$$L_i - \frac{1}{\gamma} L_i^\gamma$$

which yields  $L_i = 1$  and therefore  $Y = 1$ .

- The equilibrium value of  $Y$  we obtained is less than one.
- Since producers have market power they produce less than the socially optimal amount.
- That is, from (5) we have a real wage equal to  $(\eta - 1)/\eta$  and therefore less than one, while the marginal product of labor is equal to one.
- This reduces the amount of labor supplied and thus causes equilibrium output to be less than optimal.



# Implications

- From (8) we have that:

$$y_{\text{opt}} - y_{\text{eq}} = \nu \log[\eta/(\eta - 1)],$$

where  $\nu \equiv 1/(\gamma - 1)$  denotes the wage elasticity of labor supply.

- Thus the gap between equilibrium and optimal output is larger when:
  - producers have more market power
  - labor supply is more responsive to the real wage

# Recessions, booms and welfare

- Usually periods with high output are viewed as good times while periods with low output are viewed as bad times.
- Once we incorporate nominal rigidities, the model will deliver output fluctuations arising from incomplete nominal adjustments in the face of monetary shocks.
- If the equilibrium in the absence of shocks is optimal, both times of high output and times of low output are departures from the optimum and therefore undesirable.
- But since equilibrium output is less than optimal, a boom brings output closer to the social optimum, whereas a recession pushes it farther away.
- Thus recessions and booms have asymmetric effects on welfare.

# Aggregate Demand Externality

- Suppose the economy is initially in equilibrium and consider a marginal reduction of all prices.
- $M/P$  rises and so does aggregate output.
- This affects the representative individual in two ways:
  - the real wage increases. Since the individual is neither a net purchaser nor a net supplier of labor, at the margin this does not affect his or her welfare.
  - because aggregate output increases, the demand curve for the individual's good,  $Y(P_i/P)^{-\eta}$ , shifts outward. Since the individual is selling at a price that exceeds marginal cost, this change increases welfare.
- Pricing decisions have externalities which operate through the overall demand of goods: **aggregate demand externality**.

# Imperfect competition alone is not enough

- Final implication.
- Imperfect competition alone does not imply monetary nonneutrality.
- It follows from (8) and (9) that a change in the money stock leads to proportional changes in the nominal wage and all nominal prices. Output and the real wage are unchanged.

# Some Important Expressions

- We saw that price-setters' optimal relative price is increasing in aggregate output:

$$p_i^* - p = c + (\gamma - 1)y,$$

with  $c = \log \eta / (\eta - 1)$ .

- In the model this arises because the real wage increases with output.
- In more general settings it could also arise from:
  - increases in the costs of other inputs
  - diminishing returns
  - costs of adjusting output
- Substituting  $y = m - p$  in the expression above yields

$$p_i^* = c + (1 - \phi)p + \phi m$$

where  $\phi \equiv \gamma - 1$ .

# Are Small Frictions Enough?

- Consider an economy like the one described above
- Initially: the economy is at its flexible-price equilibrium
- Aggregate demand is determined after prices are set
- Assume that aggregate demand turns out to be lower than expected
- At this point each firm can lower its price by paying a menu cost ... or it can stick to its original price
- Major simplification: prices set afresh at the start of each period

# Are Small Frictions Enough?

- Since the economy is large, each firm takes other firms' actions as given
- Constant nominal prices are an equilibrium if, when all other firms hold their prices fixed, the maximum gain to a representative firm from changing its price is less than the menu cost of price adjustment.
- Romer asks how large the menu-cost needs to be, as a fraction of revenue, for “not changing prices” to be a Nash equilibrium.
- He considers a 3% fall in  $M$  with other prices unchanged.
- He presents a quantitative example showing that menu costs must be huge for nominal prices to be an equilibrium.

# Are Small Frictions Enough?

- Romer assumes  $\nu = 0.1$  and  $\eta = 5$  (markup of 25%).
- Let's look at a firm that considers changing its price, assuming the remaining firms won't change their price.
- Since the labor market clears and the labor-supply elasticity is low, the negative aggregate demand shock must translate into a major decrease in the real wage
- Thus, the firm considering whether to lower its price has big incentives to do so, since it can then increase its production substantially and make a higher profit than if it sticks to its pre-shock price.
- This means that unchanged nominal prices cannot be an equilibrium unless menu-costs are unrealistically high (or wages respond less to aggregate demand).



# Combining Nominal and Real Rigidities

- In the previous example, menu costs would have large effects if the wage responded little to market conditions, that is, if we had a **real rigidity** (in wages).
- More generally, greater real rigidities correspond to a lower value (closer to zero) of  $\phi$  in

$$p_{it}^* - p_t = c + \phi y_t$$

# Combining Nominal and Real Rigidities

- Real rigidity alone does not cause monetary disturbances to have real effects: as we saw before, if prices can adjust fully, money is neutral regardless of the degree of real rigidity.
- But real rigidities magnify the effects of nominal rigidities: the greater the degree of real rigidity, the larger the range of prices for which nonadjustment is an equilibrium.
- There are many possible sources of real rigidities, we look at some of them next, but omit others such as those coming from financial markets

# Real Rigidities and Labor Markets

- Lack of labor mobility leads to real rigidities
  - less than complete labor mobility  $\Rightarrow$  firm must pay higher real wage as it hires more workers  $\Rightarrow$  firm cuts price less and increases production less in response to a fall in aggregate output
- Consider models that break the link between the elasticity of labor supply and the response of the cost of labor to demand disturbances: efficiency wages, search, matching, implicit contracts, ...

# Real Rigidities and Countercyclical Markups

- Countercyclical markups and real rigidities go hand-in-hand, since they imply prices respond less to aggregate demand.
- Possible sources of countercyclical markups:
  - thick markets effects that make it easier for firms to disseminate information and for consumers to acquire it when aggregate output is high
  - combination of long-term relationships between customers and firms and capital market imperfections: part of benefits of cutting price come in the future...
  - shifts in the composition of demand toward goods with more elastic demand
  - increased competition due to entry
  - higher sales increase firms incentives to deviate from implicit collusion by cutting prices

## 1.3. The New Keynesian Phillips Curve

- Main source: Woodford's 2003 book
- Also see Gali's 2008 book
- Review that follows based on Gali and Gertler (1999)
- See Chap. 6.8 in Romer for an alternative derivation
- Contributions:
  - uses measure of real marginal cost (instead of output gap)
  - allows for subset of firms that sets prices according to backward looking rule of thumb
  - identify and estimate structural parameters from the model
- Two key parameters:
  - average duration an individual price is fixed
  - fraction of firms that use the rule of thumb

# Main Results

- Real marginal costs are statistically significant and quantitatively important determinant of inflation, as predicted by the theory
- Forward looking behavior is very important: 70-80% of U.S. firms exhibit it
- Backward looking behavior is statistically significant yet of limited quantitative importance
- The average duration a price is fixed is considerable (5 quarters), compared with 3-4 quarters suggested by Nakamura-Steinson's work

# Notation

- All variables in logs
- $1 - \theta$ : Calvo-probability of adjustment
- $p_t$ : price-deviation from zero inflation steady-state
- $p_t^*$ : optimal reset price (in Calvo model)
- $mc^n$ : nominal marginal cost
- $mc \equiv mc^n - p$ : real marginal cost
- $\beta$ : discount factor
- Continuum of firms
- $\pi_t \equiv p_t - p_{t-1}$ .

# Deriving the NKPC

- A monopolistic competition setting
- Dixit-Stiglitz model with money in the utility function and Calvo price-setting
- Can derive formally (won't do, see Woodford or Gali's book) that the static target (in the sense of Section I) equals the nominal marginal cost and therefore:

$$p_t^* = (1 - \beta\theta) \sum_{k \geq 0} (\beta\theta)^k E_t[mc_{t+k}^n]. \quad (10)$$



# Deriving the NKPC

- From the Calvo model we saw in Section I:

$$p_t = \theta p_{t-1} + (1 - \theta)p_t^* \quad (11)$$

- From (10):

$$p_t^* = (1 - \beta\theta)(p_t + mc_t) + \beta\theta E_t[p_{t+1}^*]. \quad (12)$$

- Subtracting  $p_{t-1}$  from both sides of (11) and (12):

$$\pi_t = (1 - \theta)(p_t^* - p_{t-1}), \quad (13)$$

$$p_t^* - p_{t-1} = \pi_t + (1 - \beta\theta)mc_t + \beta\theta E_t[p_{t+1}^* - p_t]. \quad (14)$$

# Deriving the NKPC

- Getting rid of  $p_t^* - p_{t-1}$  and  $E_t[p_{t+1}^* - p_t]$  in (14) by expressions derived from (13) and rearranging terms leads to:

$$\pi_t = \gamma mc_t + \beta E_t[\pi_{t+1}], \quad (15)$$

with  $\gamma = (1 - \beta\theta)(1 - \theta)/\theta$ .

- Expression above: **New Keynesian Phillips Curve**.

# Deriving the NKPC

- Iterating (15) forward leads to:

$$\pi_t = \gamma \sum_{k \geq 0} \beta^k E_t[mc_{t+k}]. \quad (16)$$

- Finally note that from (15) we also have:

$$p_t = \frac{1}{1 + \beta} p_{t-1} + \frac{\beta}{1 + \beta} E_t[p_{t+1}] + \frac{\gamma}{1 + \beta} mc_t. \quad (17)$$

# Phillips Curve and Output Gap

- Traditional empirical work on the Phillips curve has emphasized some output gap measure instead of marginal cost
- Let:
  - $y_t$ : log-output
  - $y_t^*$ : log-natural-output (output with fully flexible prices)
  - $x_t \equiv y_t - y_t^*$ : output gap
- Then, under certain conditions we have that:

$$mc_t = \kappa x_t$$

- And from (15) we obtain:

$$\pi_t = \gamma \kappa x_t + \beta E_t[\pi_{t+1}], \quad (18)$$

- Similar (but not the same, see below) as the traditional Phillips Curve.

# Interaction of Nominal and Real Rigidities

- The smaller  $\gamma\kappa$ , the slower the adjustment of the price level
- The coefficient  $\kappa$  usually measures the importance of **real** rigidities: real wage rigidity
- For example, in Blanchard and Kiyotaki:

$$\kappa = \frac{\beta - 1}{1 + \sigma(\beta - 1)},$$

with

- $\sigma$ : elasticity of substitution among goods
- $\beta - 1 \geq 0$ : elasticity of marginal utility w.r.t. leisure

# Interaction of Nominal and Real Rigidities

- $\kappa$  is decreasing in  $\sigma$ , increasing in  $\beta$
- If  $\kappa$  reflects mainly disutility of labor, it is likely to be large, yet may be “pulled down” by a small value of  $\gamma$  (large  $\theta$ , infrequent adjustment).
- The slope of the Phillips curve is determined by the interaction of real and nominal rigidities

# Implications

- Staggering (and the New Keynesian Phillips curve) imply **price stickiness**.
  - Follows from (17).
- There is **no** inflation stickiness: inflation is fully forward looking.
  - Follows from (15).

# Implications

- Hence price stickiness does not imply inflation stickiness.
  - Consistent with the evidence?
- A key difference between the Phillips curves derived and the traditional Phillips curve is that:
  - we have:  $E_t[\pi_{t+1}]$ , hence no inflation stickiness
  - in the past:  $E_{t-1}[\pi_t]$ , usually assumed to equal  $\pi_{t-1}$ ; hence do have inflation stickiness



- Assume  $\beta \cong 1$ , lag (18) one period, and add  $\pi_t$  on both sides to obtain:

$$\pi_t = \lambda \kappa x_{t-1} + \pi_{t-1} + \varepsilon_t,$$

with  $\varepsilon_t \equiv \pi_t - E_{t-1}[\pi_t]$ .

- When estimating the equation above using quarterly (quadratically) detrended data, the coefficient on  $x_{t-1}$  has the wrong sign!
- Fundamental problem:
  - derived Phillips curve implies that inflation should lead the output gap over the cycle
  - a rise in current inflation should signal a subsequent rise in the output gap
  - the data show exactly the opposite: output gap leads inflation

- If you use marginal cost measures instead of output gap, you get the right sign in the estimated Phillips curve
  - Gali and Gertler (1999) for the US
  - Gali, Gertler and López-Salido (2001) for Europe
- If you add a fraction of backward-looking agents, they turn out to be significant (around 20-30%)

## 2. Partial Equilibrium Models

- 1 Neutrality Result: Caplin-Spulber (1987)
- 2 Beyond Neutrality: Caplin-Leahy (1991)
- 3 Estimating hazard models: Caballero and Engel (1993)

## 2.1. Caplin-Spulber Neutrality Result

- Caplin and Spulber (QJE, 1987)

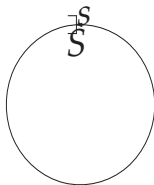
### Assumptions

- Continuous time.
- Fixed adjustment costs.
- Non-negative monetary shocks.
- Continuous path of money.
- No idiosyncratic shocks (only monetary shocks).
- Income and substitution effects cancel.
- Continuum of 'identical' firms (identical, except for the price they may charge at a given moment in time).

# Neutrality Result

- From Section 3 we have conditions on monetary shocks that ensure that optimal policies are of the one-sided( $S, s$ ) type, in  $z_{i,t} \equiv p_{i,t} - m_t$  (we have changed the sign convention, here  $z$  is the negative of our gap variable in previous lectures, i.e.,  $z = p - p^*$ ).
- Caplin and Spulber (CS) depart from previous work by assuming that initially ( $t = 0$ ) the x-section of  $z$ 's is uniform on  $[S, s]$ .
- Next we show that this implies that the x-section of  $z_{i,t}$  at any  $t > 0$  remains uniform.

# Neutrality Result



# Neutrality Result

- Between  $t$  and  $t + \Delta t$  (time is continuous):
  - A fraction  $\psi = \frac{\Delta m}{S-s}$  of firms adjust their price.
  - Each one adjusts by  $A = (S - s)$ .
  - The impact of adjustments on aggregate prices is:  
$$\Delta p_A = \psi \times A = \Delta m$$
- Hence, the change in aggregate output is:

$$\begin{aligned}\Delta y &= \Delta m - \Delta p \\ &= \Delta m - \Delta p_A \\ &= 0.\end{aligned}$$

- It follows that money is neutral.

# Conclusion

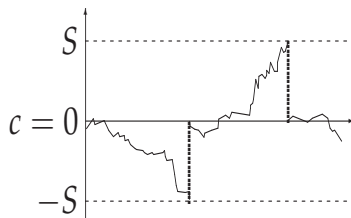
*Even though at the micro level we have significant rigidities (most firms are inactive in any given time period), aggregate behavior is indistinguishable from that of an economy with no frictions, where firms adjust their prices continuously.*



## 2.2. Beyond Neutrality: Caplin-Leahy (1991)

- $p_i^*$ : optimal log-price without frictions.
- $m_t$ : log-money  $\rightsquigarrow \text{BM}(0, \sigma^2)$ .
- $p_i^* = m_t$  (no idiosyncratic shocks).
- $z_{i,t} = p_{i,t} - p_{i,t}^* = p_{i,t} - m_t$ .
- Hence we may assume that the optimal policy is  $(L, c, U)$  in  $z_{i,t}$ , with  $c = 0$  (see figure on the next slide).

# Optimal Policy



- The additional assumption that is crucial in this paper is that the initial  $x$ -section of  $z_i$ 's is uniform on a subinterval of  $[-S, S]$  of length  $S$ , i.e., that covers half of the inaction range. For concreteness we'll assume the initial  $x$ -section uniform on  $[-S/2, S/2]$ .
- As in earlier models, aggregate output is given by:

$$y_t = m_t - p_t,$$

with

$$p_t = \int_0^1 p_{i,t} di,$$

where we assume a continuum of firms indexed by  $i \in [0, 1]$ .

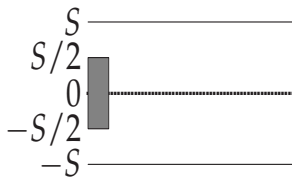
- Hence:

$$y_t = m_t - p_t = \int_0^1 p_{i,t}^* di - \int_0^1 p_{i,t} di = - \int_0^1 z_{i,t} di,$$

so that aggregate output is inversely proportional to the mean of the x-section of deviations.

- Derivations are much simpler if we assume that the sample paths of  $m_t$  are not only continuous, but also differentiable (strictly speaking this isn't true for a BM and Caplin-Leahy derive the results rigorously).

- The initial x-section is depicted below:



- Since adjustments in  $p_i$  are of size  $\pm S$ , we have that at all times the x-section of the  $z_i$ 's is uniform on a subinterval of  $[-S, S]$  of length  $S$ .
- And, since  $y_t = - \int_0^1 z_{i,t} di$ , we have that aggregate output equals (minus) the mid point of the x-section of the  $z_i$ .
- Hence the largest value  $y_t$  can take is  $S/2$ , when  $z_{i,t} \rightsquigarrow U[-S, 0]$ .  
When this happens, **all** firms charge a price **below** their frictionless optimal price, so that demand for their product is above trend and  $y_{i,t} > 0$ . The opposite situation happens when the  $z_{i,t} \rightsquigarrow U[0, S]$  and  $y_t = -S/2$ .

- Regarding what happens with the price level,  $p_t$ , or more precisely inflation:

$$\Delta p_t \equiv p_{t+dt} - p_t$$

we consider three possibilities:

- Case 1**

- $z_{i,t} \rightsquigarrow U[A, A + S]$  with  $-S < A < 0$ .
- $z_{i,t} \rightsquigarrow U[-S, 0]$  and  $\Delta m_t \equiv m_{t+dt} - m_t < 0$
- $z_{i,t} \rightsquigarrow U[0, S]$  and  $\Delta m_t > 0$

In all of these cases:

- nobody* adjusts prices,  $\Delta p_t = 0$ ,  $\Delta y_t = \Delta m_t$ .

- Case 2

- $z_{i,t} \rightsquigarrow U[-S, 0]$  and  $\Delta m_t > 0$ .

In this case, those with deviation between  $-S + \Delta m_t$  and  $-S$  adjust and (see Panel (a) in the figure on the next page):

$$\begin{aligned}\Delta p_t &= \frac{\Delta m_t}{S} \times S = \Delta m_t \\ \Delta y_t &= 0.\end{aligned}$$

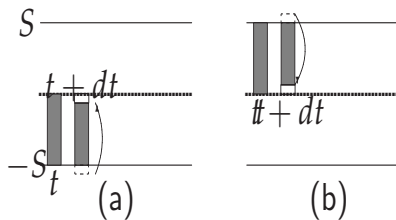
- Case 3

- $z_{i,t} \rightsquigarrow U[0, S]$  and  $\Delta m_t < 0$ .

In this case (see Panel (b) in the figure on the next page), those with deviation between  $S - \Delta m_t$  and  $S$  adjust, and:

$$\Delta p_t = \Delta m_t, \quad \Delta y_t = 0.$$





# Conclusion

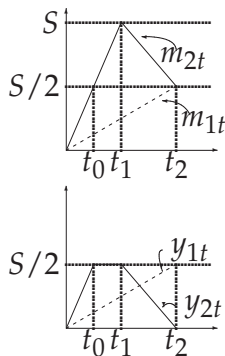
- 1. By contrast with Caplin and Spulber, in this case money is *not* neutral in the short run. There are times where  $y_t$  responds one-to-one to  $m_t$ .
  - For example, assume that initially  $y = 0$  and a monetary expansion occurs so that  $m$  grows to  $S/2$ . Then  $y$  will grow all the way to  $S/2$  and there will be no inflation.
  - If  $m$  continues growing after this,  $y$  stops growing and we have inflation.
  - More generally, on average, the initial effect of a monetary expansion is an increase in output, followed eventually by inflation.

# Conclusion

- 2.  $\lim_{\tau \rightarrow \infty} E_t[dy_{t+\tau}/dm_{t+\tau}] = 0$ , since as  $\tau$  grows, the probability assigned to the scenarios where  $E_t[dy_{t+\tau}/dm_{t+\tau}] = 1$  and  $-1$  approach each other.
- 3. Both conclusions above can be summarized as stating that money is neutral in the long run but not in the short run.

- How the economy arrives at a given situation matters:
- See the figure on the next page. The upper panel shows two possible monetary sample paths that end up with the same value of  $m$ . The lower panel shows the corresponding sample paths of aggregate output, where in both cases we assume that the initial x-section of deviations,  $z$ , is uniform on  $[-S/2, S/2]$ . This example illustrates that **how** money reaches a given value has important implications for the output path.

# History Matters



## 2.3. Estimating Hazard Models

- A. Motivation
- B. Application: US prices

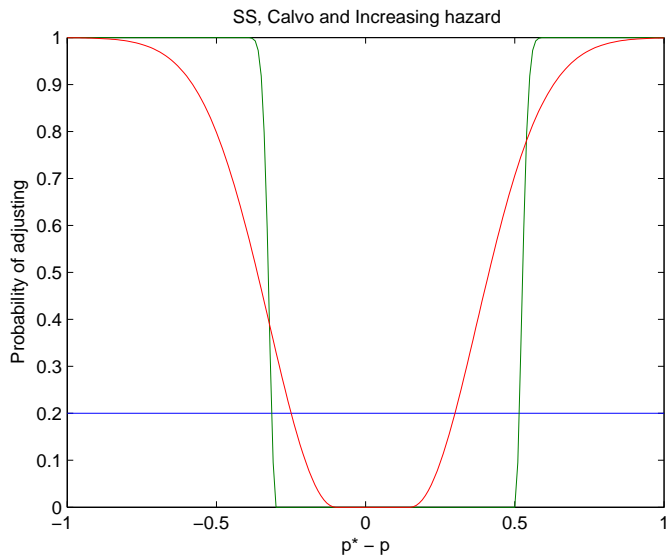
# A. Motivation

- Calvo: constant probability of adjusting
- Ss: probability of adjusting either zero or one
- Increasing hazard: probability increases **smoothly** with distance from return point
- Essence of Ss-type models:

*Tolerate less well larger departures from dynamic target*

- Caballero and Engel (AER, 1992; QJE, 1993) in the context of labor

# Motivation





# Simple families of increasing hazards

- Simplest:

$$\Lambda(x) = \min(1, \lambda_0 + \lambda_2 x^2)$$

- Could have different slopes for positive and negative  $x$
- More elegant:

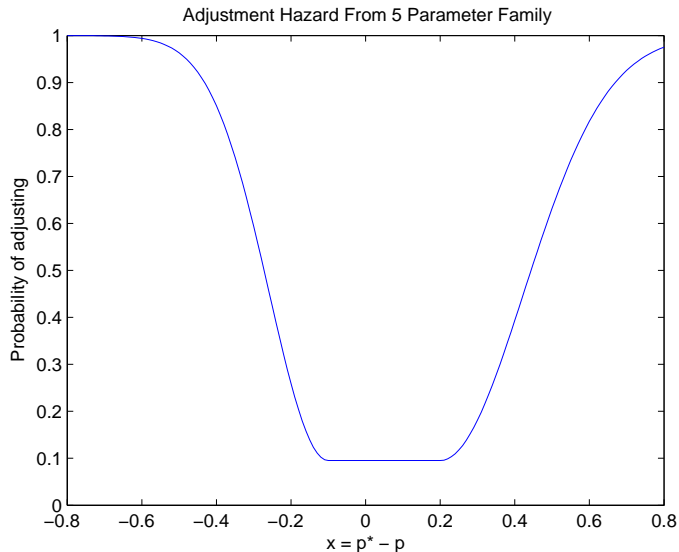
$$\Lambda(x) = 1 - e^{-\lambda_0 - \lambda_2 x^2}.$$

- Again, could have different slopes for positive and negative  $x$
- More general:

$$\Lambda(x) = \begin{cases} 1 - e^{-\lambda_0 - \lambda_{2L}(x-x_L)^2}, & x < x_L, \\ 1 - e^{-\lambda_0}, & x_L < x < x_R, \\ 1 - e^{-\lambda_0 - \lambda_{2R}(x-x_R)^2}, & x > x_R. \end{cases}$$

- Micro foundations: later when covering investment

# A Five Parameter Family of Hazards



# An Expression for the Aggregate

- Given:
  - a  $x$ -section  $f(x, t)$  at time  $t$ , immediately before adjustments take place
  - adjustment hazard  $\Lambda(x)$
- We have that the contribution to inflation from firms with  $x \in [x_0, x_0 + \Delta x]$  is approximately equal to:

$$-x_0 \Lambda(x_0) f(x_0, t) \Delta x$$

- Adding up over  $x$  this leads to the following expression for aggregate inflation:

$$\Pi_t = - \int x \Lambda(x) f(x, t) dx.$$

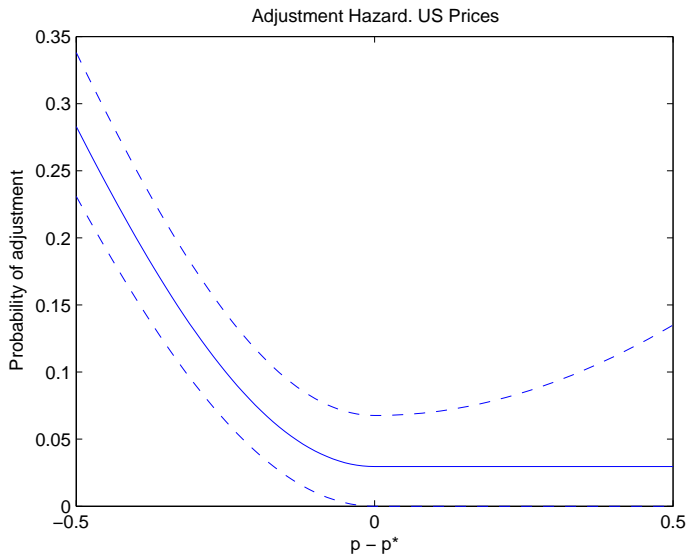
## B. Application to U.S. Prices

- Caballero and Engel (EER, 1993)
- Estimate increasing hazard model for U.S. inflation
- Annual data, 1955-1989
- Main findings:
  - reject constant hazard (Calvo) in favor of increasing hazard
  - fraction of firms adjusting varies over time
  - $IRF_0$  varies over time
  - asymmetries in  $IRF_0$  for positive and negative shocks

# Results

	Constant Hazard	Non-Constant Hazard	
		Constr.: $\lambda_{2R} = \lambda_{2L}$	Unconstr.
$\lambda_0$	0.33 (0.17)	0.19 (0.28)	0.03 (0.02)
$\lambda_{2L}$	—	—	1.21 (0.08)
$\lambda_{2R}$		1.14 (0.43)	0.00 (0.15)
SSR $\times 100$	0.262	0.227	0.225

# Results



# Some Useful Indices

- The following vary over time with an increasing hazard model (and remain constant with a linear Calvo-type model):
  - Fraction of firms adjusting
  - Marginal response to an aggregate shock: analogous to  $IRF_0$ , but conditional on history up to time  $t$
  - Asymmetry index: difference in marginal response to shocks above and below average, conditional on cross-section at time  $t$

# Time-Varying Fraction of Adjusters

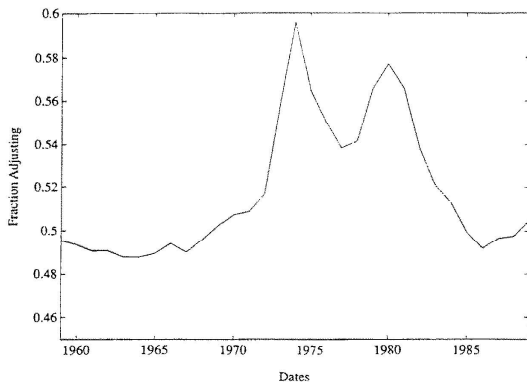


Fig. 2. Number of firms adjusting their prices.



# Time-Varying $IRF_0$

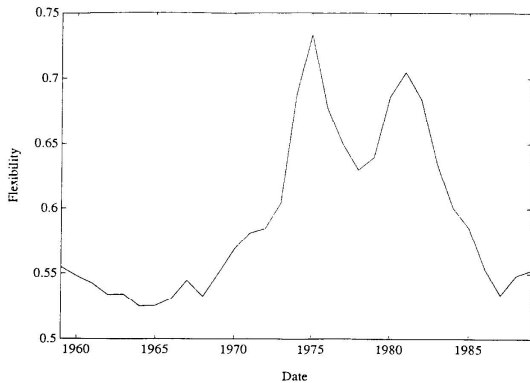


Fig. 3. Flexibility index.

# Time-Varying Asymmetry Index

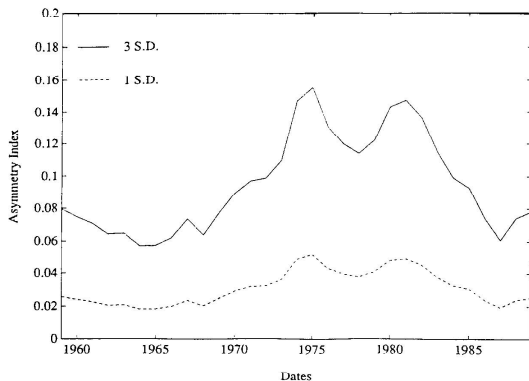


Fig. 4. Asymmetry index.

### 3. Microeconomic Evidence

- 1 Bilal and Klenow
- 2 Nakamura and Steinsson
- 3 Beyond the United States
- 4 Survey evidence

## 3.1. Bils and Klenow (2004)

### Data:

- Monthly prices used to build the CPI
- BLS, 1995–2001
- 70,000–80,000 prices per month from around 22,000 outlets in 88 geographic areas
- Covers 68.9% of consumer spending
- 350+ categories of consumer goods and services: Entry Level Items (ELI)

# Main Findings

- Median frequency of price changes: 4.3 months
- Median frequency of price changes after adjusting for sales ('regular' prices): 5.5 months
- Frequency of price adjustments differs dramatically across goods:
- Compared to the predictions of the Calvo model, actual inflation rates (at the ELI level) are far more volatile and transient

# Sales or no sales?

- Should we consider all prices when estimating median price durations? Or only regular prices?
- If sales always involve a markdown of the same size and take place on the same dates (Columbus Day Sales, Labor Day Sales, etc.) then price-stickiness is related to regular price changes and sales are irrelevant
- If retailers use sales to adjust the real price they charge on average (sales take place more often when the real price is too high) then more frequent sales are associated with less price stickiness
- The truth possibly lies somewhere in between...

# Frequency of Price Changes

Year	Median frequency (%)	Median Duration (months)
1995	21.3	4.2
1996	20.8	4.3
1997	19.9	4.5
1998	21.2	4.2
1999	21.4	4.2
2000	21.7	4.1
2001	22.0	4.0

Earlier studies for specific goods (newspapers, supermarkets): price durations close to one year

Bils and Klenow: difference due mainly to the fact that their estimate are based on a much broader sample of products.

# Differences across goods

- Frequency of price adjustments differs dramatically across goods:
  - newspaper, men's haircuts, taxi fares: change less than 5% of months
  - gasoline, tomatoes, airfare prices change more than 70% of months
- Durable goods show more frequent changes than the overall consumer bundle
- Goods sold in more competitive markets display more frequent price changes ... effect goes away if you leave out foods and energy



# Differences across goods

TABLE 2  
MONTHLY FREQUENCY OF PRICE CHANGES FOR SELECTED CATEGORIES

	Price Quotes with Price Changes (%) (1)	Price Quotes with Price Changes, Excluding Observations with Item Substitutions (%) (2)
All goods and services	26.1 (1.0)	23.6 (1.0)
Durable goods	29.8 (2.5)	23.6 (2.5)
Nondurable goods	29.9 (1.5)	27.5 (1.5)
Services	20.7 (1.5)	19.3 (1.6)
Food	25.3 (1.8)	24.1 (1.9)
Home furnishings	26.4 (1.8)	24.2 (1.8)
Apparel	29.2 (3.0)	22.7 (3.1)
Transportation	39.4 (1.8)	35.8 (1.9)
Medical care	9.4 (3.2)	8.3 (3.3)
Entertainment	11.3 (3.5)	8.5 (3.6)
Other	11.0 (3.3)	10.0 (3.3)
Raw goods	54.3 (1.9)	53.7 (1.7)
Processed goods	20.5 (.8)	17.6 (.7)

# Testing the Calvo Model

Assuming that nominal marginal costs follow a random walk (Bils-Klenow provide evidence suggesting this is a reasonable assumption), Rotemberg's version of the Calvo model (see class handout for Section I) implies that:

$$\pi_{it} = (1 - \lambda_i)\pi_{i,t-1} + \epsilon_{it},$$

where  $\lambda_i$  denotes the Calvo-frequency of price adjustment in ELI  $i$  (they obtain 123 product level price series and match them to the 350+ ELIs) and the  $\epsilon_{it}$  are i.i.d.

Denoting by  $\rho_i$  the estimated first-order autocorrelations for the above regression, the Calvo model predicts:

$$\rho_i \simeq 1 - \lambda_i.$$

# Testing the Calvo Model

As noted in Section I of this course, Bils and Klenow obtain values of  $\rho_i$  much smaller than suggested by the above expression: their average value is  $-0.05$ , with a standard deviation of  $0.02$ . Furthermore, the correlation between the  $\rho_i$  and  $\lambda_i$  they obtain is  $0.26$  (with a standard deviation of  $0.09$ ), which contradicts the negative correlation predicted by the Calvo model.

As discussed in Section I, one possible explanation for the above discrepancy is that with a finite and not very large number of prices in each of the 123 groups of goods considered, the Rotemberg equivalence result does not hold and the first order correlation of the inflation series is significantly smaller than the frequency of price adjustments.

## 3.2. Nakamura and Steinsson

Use longer and more detailed version of data set used by BK, also consider producer prices (we'll focus on consumer prices only)

Obtain the following five facts:

- 1 median duration of regular prices during 1998–2005 lies between 8 and 11 months, depending on how substitutions, sales and missing observations (stockouts) are treated; they provide 8 estimates
- 2 one-third of regular price changes are price decreases
- 3 the frequency of price increases covaries strongly with inflation while the frequency of price decreases and the size of price increases and price decreases do not
- 4 the frequency of price change is highly seasonal: highest in 1st quarter, lowest in 4th quarter
- 5 price-change hazard: initially downward sloping, then flat (except for a spike at 12 months)

# Explaining the Difference between BK and NS

- Median frequency of regular price adjustments: Bils-Klenow obtain 5.5 months, Nakamura-Steinsson obtain between 8.0 and 11.0 months, depending on how stockouts, sales and substitutions are dealt with
- An important part of the difference: BK only had data for sales on some sectors and extrapolated to the remaining sectors (the 'uniformity assumption')
- Excluding sales more than doubles the median duration of consumer prices even though only 20% of price changes are due to sales. This happens because sales are concentrated in a few sectors (food, apparel) and these sectors have frequencies of price changes close to the median.

# Explaining the Difference between BK and NS

- Klenow-Kryvtsov (2007) acknowledge that BK underestimated price durations because of the above, but argue that this brings their median duration of regular prices up to 7.2 months for 1988–2005, not to the 8-11 month range.
- The remaining difference is explained by the fact that there are many ways of accounting for stockouts, sales and substitutions. Also, KK consider a subsample of three major cities in their analysis, which is only 20% of total data set.
- My personal reading: which of the 8 estimates in NS you use, taking values between 8.0 and 11.0 months, will depend on the model you have in mind

Calculating price adjustment frequencies is complicated by three issues:

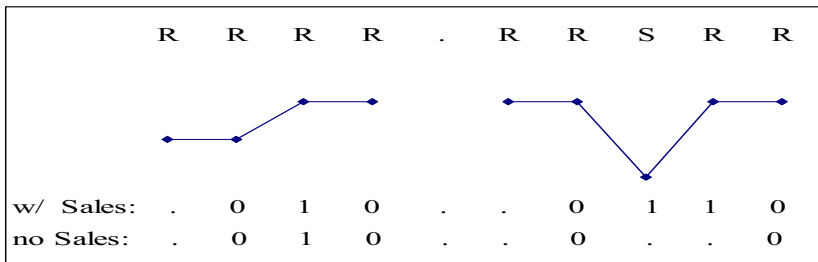
- 1 Missing values as a consequence of stockouts
- 2 Products out of the CPI research database are sometimes substituted out of the database and new products introduced in their stead
- 3 When you want to exclude sales, it is not obvious how to do it

# Contiguous Observations Method

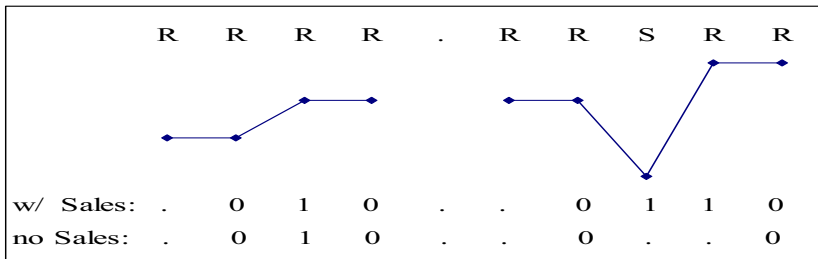
- Calculate frequencies, infer price durations from frequencies (all methods do this)
- Treat sales as missing observations
- Only consider contiguous observations where none of them is missing, when computing adjustment frequency:
  - if both prices are the same conclude there was no price change
  - if both prices differ, conclude there was a price change



# Contiguous Observations Method



Panel A



Panel B

# Contiguous Observations Method

- In the example on the preceding slide:
  - estimated adjustment frequency all prices:  $3/7$  both for panel A and for panel B
  - estimated adjustment frequency regular prices: 0.2 both for panel A and for panel B
- Sale leads to drop two observations:
  - biases frequency downward in panel B
  - biases frequency upward in panel A
- Estimates obtained by NS using this method for frequency of price changes, when working with all prices, are very similar to BK

# Accounting Issues: Substitutions

- What should be done about substitutions? In the first version of their paper, NS excluded substitution related price changes. This led them to report that regular prices change every 27 months in apparel, where the average item lasts only about 10 months.
- Ignoring substitution related price changes is equivalent to ignoring 80% of prices change at the point of model turnover.
- Of course, assuming that whenever a good is substituted by another good with a different price this corresponds to a price change biases duration estimates in the opposite direction.
- NS provide duration estimates for both scenarios, leading to the 8-11 month range mentioned above

# Frequency/magnitude of increases/decreases

**Table:** Median Frequency and Size of Regular Price Adjustments

Statistic	1988-1997	1998-2005
Median freq. $\Delta p \neq 0$ :	11.1	8.7
Median freq. $\Delta p > 0$ :	7.9	6.1
Median freq. $\Delta p < 0$ :	3.2	2.8
Median price increase:	—	7.3
Median price decrease:	—	10.5

# Distrib. of price-change frequencies across ELIs

Freq. $\Delta p$ :	0.000-0.010	0.011-0.020	0.021-0.030
Prob.:	0.00031	0.01635	0.04858
Freq. $\Delta p$ :	0.031-0.040	0.041-0.050	0.051-0.060
Prob.:	0.09620	0.09074	0.05115
Freq. $\Delta p$ :	0.061-0.070	0.071-0.080	0.081-0.090
Prob.:	0.11521	0.02983	0.07096
Freq. $\Delta p$ :	0.091-0.100	0.101-0.110	0.111-0.120
Prob.:	0.02490	0.03850	0.02435
Freq. $\Delta p$ :	0.121-0.140	0.141-0.170	0.171-0.200
Prob.:	0.03425	0.03452	0.01026
Freq. $\Delta p$ :	0.201-0.250	0.251-0.300	0.301-0.400
Prob.:	0.03043	0.02710	0.08246
Freq. $\Delta p$ :	0.401-0.600	0.601-0.800	0.801-1.000
Prob.:	0.07989	0.01801	0.07629

# The Hazard of Price Changes

- Define the hazard as:

$$\Lambda(t) = \Pr\{T = t | T \geq t\},$$

where  $T$  denotes the number of periods since the last price change

- For  $S_s$ -type models we have that, usually,  $\Lambda(t)$  increases with  $t$ .
- A careful empirical analysis, taking care of heterogeneity in adjustment frequencies across product types, leads to a hazard that is somewhat decreasing for the first few months, and constant thereafter.
- As we will see later in the course, one possible explanation is that there are systematic differences in adjustment costs *within* any given sector.

## 3.3. Beyond the United States

### Inflation Persistence Network (IPN)

- 10 countries: Austria, Belgium, Finland, France, Germany, Italy, Luxembourg, Netherlands, Portugal, Spain.
- One paper per country, summarized in Dhyne et al. (2005, 2006).
- Period covered varies with country, earliest: 1989, latest: 2004.
- Many additional papers going beyond data description.
- Product substitutions are treated as price changes
- Some countries report sales, others do not. When they report, the study considers regular price changes.

# Euro Area: Stylized Facts

- 1 Monthly frequency of price adjustments: 15.1%. Average duration of a price spell: between 4 and 5 quarters.
- 2 Substantial heterogeneity across goods in frequency of price changes (high frequency: energy and unprocessed food products; low frequency: non-energy industrial goods, processed foods, services).
- 3 Cross-country heterogeneity: frequency of price adjustments during 1996-2001 ranges between 10% for Italy and 23% for Luxembourg.
- 4 40% of price changes are price reductions



# Other Countries

- Brazil: Gouvea (2007)
- Chile: Medina, Rappoport and Soto (2007)
- Hungary: Ratfai (2003)
- Mexico: Gagnon (2005)

## 3.4. Survey Evidence

- Blinder (1991, 1994, 1998)
- Fabiani et al (2006)
- Bewley (work in progress)

# Survey Evidence

- What people do or what they say they do?
- Literal or broad interpretations of state-dependent pricing?
- Structured survey or informal interviews?

# Survey Evidence: Euro Area

- Fabiani et al. (2006): 11,000 firms in Euro area
- Prices mostly set following mark up rules
- Price discrimination is common
- One-third of firms follow time-dependent rules
- Two-thirds allow for elements of state-dependent rules
- Majority of firms take into account both past and expected economic developments in their pricing decisions
- Price reviews: between 1 and 3 per year; actual price changes less frequent.
- Price stickiness driven mainly by customer relationships and coordination failure
- Cost shocks have a bigger impact when prices have to be raised, while a fall in demand is more likely to induce a price change than an increase in demand

# Survey Evidence: Bewley

- Important sectoral differences in how prices are set
- Customer relationships very important in some sectors
- Literal menu-costs relevant only for: restaurants, catalogue sales, supermarkets and department stores, specialty stores selling seasonal goods

# What Next?

- We have much more precise numbers on the frequency of price adjustment
- How do these numbers relate to our models with **aggregate** price-stickiness?

## 4. Distributional Dynamics

- Based on Caballero and Engel (2007)
- A simple framework that will help understand this literature's papers
- Emphasizes the importance of cross-sectional dynamics in macro models with micro Ss policies

## 4.1. Motivation

- VAR estimates for the response of US Inflation to monetary policy shocks:
  - peak in roughly two years
- Want a model with “sound” microeconomic underpinnings that generates such a response:
  - nominal rigidities
  - real rigidities, strategic complementarities
  - aggregation
  - suitable for welfare analysis
- Workhorse model in macro these days: Calvo model ... not satisfactory
- Ss-type models have better micro foundations



# Main Issues

- In Section 2 we discussed in detail new evidence on **micro** frequency of price adjustments
- Is the frequency of **micro** price adjustments relevant for **aggregate** price flexibility?
  - not necessarily: Caplin and Spulber (1987)
  - what is so special about Caplin and Spulber?
- Does price flexibility increase monotonically with the fraction of firms adjusting?
- Is it always true that the price level of Ss-type models responds more to monetary shocks than Calvo models?
- Is the “selection effect” (Golosov-Lucas) the correct intuition underlying the difference between Calvo and Ss-type models?

# Main Results

- Caplin and Spulber: no **aggregate** stickiness because there is no **micro** stickiness
- The selection effect is neither necessary nor sufficient for having more aggregate price stickiness in Calvo than in Ss models
- Focus instead on the **extensive margin** (Bar-Ilan and Blinder, 1992)
- Decompose the aggregate price response into the sum of contributions of the extensive and intensive margins:

$$\mathcal{E} > 0 \iff \text{Calvo more sticky than Ss}$$

- This suggests where to look for Ss-type models that match VAR price inertia

# Paper Outline

- 1 Motivation and Overview
- 2 Caplin and Spulber's Neutrality Result
- 3 From Caplin and Spulber to Calvo
- 4 Generalized Ss Models and the Extensive Margin
- 5 Selection or Extensive Margin Effect?
- 6 An Application to U.S. Consumer Prices
- 7 Conclusion

## 4.2. Caplin and Spulber's Neutrality Result

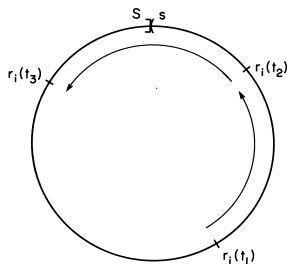
- Caplin and Spulber (QJE, 1987)

### Assumptions

- Continuous time.
- Fixed adjustment costs.
- Non-negative monetary shocks.
- Continuous path of money.
- No idiosyncratic productivity/cost/demand shocks (only monetary shocks).
- Income and substitution effects cancel.
- Continuum of 'identical' firms (identical, except for the price they may charge at a given moment in time).

- Can show that the above assumptions for monetary shocks ensure that optimal policies are of the one-sided  $S$ s type, in  $x_{i,t} \equiv p_{i,t} - p_{it}^*$  with  $p_{it}^* = m_t + S$ .
- Caplin and Spulber (CS) depart from previous work by assuming that initially ( $t = 0$ ) the x-section of  $x$ 's is uniform on  $[s - S, 0]$ .
- Next we show that this implies that the x-section of  $x_{i,t}$  at any  $t > 0$  remains uniform.

# Caplin and Spulber



- Between  $t$  and  $t + \Delta t$  (time is continuous):
  - A fraction  $\psi = \frac{\Delta m}{S-s}$  of firms adjust their price.
  - Each one adjusts by  $A = (S - s)$ .
  - The impact of adjustments on aggregate prices is:  
$$\Delta p_A = \psi \times A = \Delta m$$

- Hence, the change in aggregate output is:

$$\begin{aligned}\Delta y &= \Delta m - \Delta p \\ &= \Delta m - \Delta p_A \\ &= 0.\end{aligned}$$

- It follows that money is neutral.

# Conclusion

The standard in interpretation of Caplin and Spulber's neutrality result is the following:

*Even though at the micro level we have significant rigidities (most firms are inactive in any given time period), aggregate behavior is indistinguishable from that of an economy with no frictions, where firms adjust their prices continuously.*

That is, micro price stickiness with macro price flexibility.

Is this interpretation correct?



Macroeconomists measure the dynamic response to shocks via impulse response functions. Next we do this, beginning with an **individual** firm's IRF:

$$\frac{\Delta p_i(\Delta m, x)}{\Delta m} = \begin{cases} 0, & \text{if } x > s - S + \Delta m, \\ (S - s)/\Delta m, & \text{otherwise.} \end{cases}$$

# Caplin-Spulber Revisited

- **Microeconomic** flexibility index:

$$\mathcal{F}^{\text{micro}} \equiv \int_{s-S}^0 \frac{\Delta p_i(\Delta m, x)}{\Delta m} h_E(x) dx,$$

where  $h_E(x)$ : time average of an individual firm's trajectory of  $x$

- Since  $h_E(x)$  is uniform on  $(s - S, 0]$ :

$$\mathcal{F}^{\text{CS,micro}} = \int_{s-S}^{s-S+\Delta m} \frac{s-s}{\Delta m} \times \frac{1}{S-s} dx = 1.$$

- The histogram of the marginal responses of a firm over time to a monetary shock,  $\Delta p_{it}/\Delta m$ , has a large fraction of observations at zero and a small fraction at  $(S - s)/\Delta m$ . The time-average satisfies:

$$\lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=1}^T \frac{\Delta p_{it}}{\Delta m} = 1.$$

- More generally:

$$\text{IRF}_k^{\text{CS,micro}} = \begin{cases} 1, & \text{for } k = 0 \\ 0, & \text{for } k \geq 1. \end{cases}$$

## Property 1

There is no micro price stickiness in Caplin and Spulber (for stickiness measures based on the IRF).

# Micro and Macro Price Stickiness

Given a cross-section  $f(x)$ :

$$\frac{\Delta p}{\Delta m}(f) \equiv \int \frac{\Delta p_i(x, \Delta m)}{\Delta m} f(x) dx.$$

Averaging over all x-sections  $f(x)$ :

$$\mathcal{F}^{\text{macro}} = \sum_{k=1}^n w_k \frac{\Delta p}{\Delta m}(f_k) = \frac{\Delta p}{\Delta m} \left( \sum_{k=1}^n w_k f_k \right) = \frac{\Delta p}{\Delta m}(f_A)$$

where

$$f_A(x) = \sum_{k=1}^n w_k f_k(x)$$

is the average cross-section.

# Micro and Macro Price Stickiness

Hence:

$$\mathcal{F}^{\text{macro}} \equiv \int \frac{\Delta p_i(\Delta m, x)}{\Delta m} f_A(x) dx.$$

Yet by the Ergodic Theorem:  $h_E(x) = f_A(x)$

## Property 2

In any stationary model:

$$\mathcal{F}^{\text{macro}} = \mathcal{F}^{\text{micro}}$$

# Understanding Caplin and Spulber

Combining Properties 1 and 2:

## Property 3

For Caplin and Spulber:

$$\mathcal{F}^{\text{CS,macro}} = \mathcal{F}^{\text{CS,micro}} = 1.$$

No macro stickiness because you have no micro stickiness.

Valid in general for stickiness measures based on the IRF.

# The Role of the Cross-Section

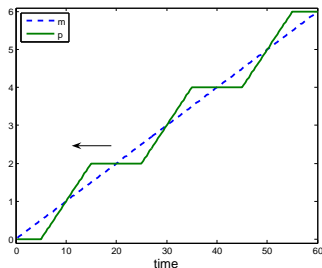
- Price imbalances  $x$ : uniform on **half** the inaction range
- Now the **conditional** IRF varies over time:

$$\mathcal{F}^f \equiv \int \frac{\Delta p_i(\Delta m, x)}{\Delta m} f(x) dx = \begin{cases} 0 & \text{when no firm is adjusting,} \\ 2 & \text{when some firms are adjusting.} \end{cases}$$

- In general: aggregate price flexibility varies over time.

# The Role of the Cross-Section

For example, with constant money growth  $\mu$



Yet, after averaging over all x-sections, continue having:

$$\mathcal{F}^{\text{macro}} = \mathcal{F}^{\text{micro}} = 1.$$



# Incorporating Strategic Complementarities

- Firms wish to coordinate their prices
- Now:

$$p_{it}^* = (1 - a)m_t + ap_t,$$

- $a \in [0, 1/2)$ : strength of complementarities.
- No multiplicities
- We now have:

$$\Delta p = \underbrace{\Delta p^* f(s - S) \Delta t}_{\text{fraction}} \times \underbrace{(S - s)}_{\text{size}},$$

- Next assume constant money growth  $\mu$

# Incorporating Strategic Complementarities

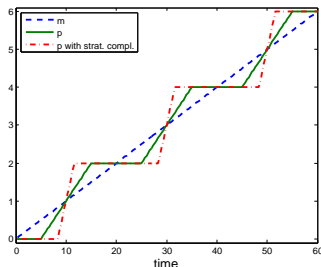
Using both expressions above to solve for  $\Delta p$ :

$$\Delta p = \begin{cases} 0 & \text{when no firm is adjusting,} \\ (2 - 2a)\mu\Delta t/(1 - 2a) & \text{when some firms are adjusting.} \end{cases}$$

Hence:

$$\mathcal{F}(f) = \begin{cases} 0 & \text{when no firm is adjusting,} \\ (2 - 2a)/(1 - 2a) & \text{when some firms are adjusting.} \end{cases}$$

# Strategic Complementarities



Continue having:

$$\mathcal{F}^{\text{macro}} = \mathcal{F}^{\text{micro}} = 1.$$

Valid in general for **one-sided** Ss models (Caballero and Engel, 1993)

## 4.3. From Caplin-Spulber to Calvo

Caplin-Spulber with a positive hazard  $\lambda$  of adjusting

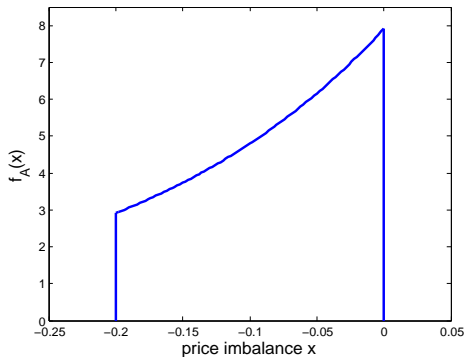
- $\lambda = 0 \Rightarrow$  Caplin-Spulber
- $s \rightarrow \infty \Rightarrow$  Calvo.

To find the average cross-section  $f_A(x)$ :

$$\begin{aligned}f_A(x, t + \Delta t) &= (1 - \lambda \Delta t) f_A(x + \mu \Delta t, t) \\ \Rightarrow f'_A(x) &= \alpha f_A(x) \\ \Rightarrow f_A(x) &= \frac{\alpha e^{\alpha(x+S-s)}}{e^{\alpha(S-s)} - 1},\end{aligned}$$

with  $\alpha = \lambda/\mu$ .

# From Caplin-Spulber to Calvo



# From Caplin-Spulber to Calvo

- We have:

$$\mathcal{A} = \lambda + (1 - \lambda)F_A(s - S + \mu) \cong \lambda + (1 - \lambda)f_A(s - S)\mu$$

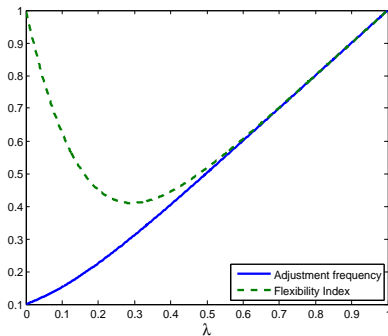
- Hence:  $\mathcal{A}$  is increasing in  $\lambda$

- We also have:

$$\frac{\Delta p}{\Delta m} = \begin{cases} 1, & \text{for Calvo-adjusters,} \\ (S - s)/\Delta m, & \text{for Ss-adjusters} \end{cases}$$

- Hence:  $\mathcal{F} \cong \lambda + (1 - \lambda)f_A(s - S)(S - s)$ .
- And  $\mathcal{F} = 1$  for  $\lambda = 0$  and  $\lambda = 1$ , that is,  $\mathcal{F}$  does not vary monotonically with  $\lambda$

# From Caplin-Spulber to Calvo



# From Caplin-Spulber to Calvo

- Contribution of the intensive margin to  $\mathcal{F}$  increases with  $\lambda$ , one-for-one
- Contribution of the extensive margin to  $\mathcal{F}$  decreases with  $\lambda$ , since the fraction of firms in the neighborhood of the trigger barrier decreases
- The former increases at a constant rate of 1 with  $\lambda$
- The latter decreases at a high rate for small  $\lambda$  and at smaller rates as  $\lambda$  becomes larger
- The extensive margin dominates how  $\mathcal{F}$  varies with  $\lambda$  when  $\lambda$  is small
- The intensive margin dominates how  $\mathcal{F}$  varies with  $\lambda$  when  $\lambda$  is large



## 4.4. Generalized Ss Models and the Extensive Margin

- Discrete time
- $\Delta m_t$  i.i.d.  $(\mu_A, \sigma_A^2)$
- Idiosyncratic productivity/demand shocks  $v_{it}$  i.i.d.  $(0, \sigma_I^2)$
- Then:

$$\begin{aligned}\Delta p_{it}^* &= \Delta m_t + v_{it} \\ x &\equiv p_{i,t-1} - p_{it}^*\end{aligned}$$

# Generalized Ss Models

- Also: idiosyncratic adjustment cost shocks i.i.d.  $G(\omega)$
- Can define an adjustment hazard  $\Lambda(x)$
- In general we have the **increasing hazard** property:

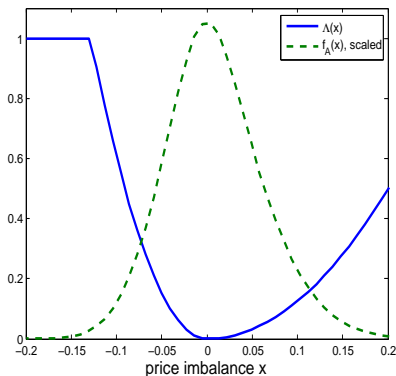
$$\begin{aligned}\Lambda'(x) &< 0 & \text{for } x < 0, \\ \Lambda'(x) &> 0 & \text{for } x > 0.\end{aligned}$$

Firms tolerate less well larger price imbalances.

- Hence:

$$x\Lambda'(x) > 0, \quad \text{for all } x$$

# Generalized Ss Models



We have:

$$\Delta p_t = - \int x \Lambda(x) f(x, t) dx.$$

# A Basic Decomposition

$$\Delta p(\Delta m) = - \int x \Lambda(x) f_A(x + \Delta m) dx = - \int (x - \Delta m) \Lambda(x - \Delta m) f_A(x) dx.$$

$$\Rightarrow \Delta p'(\Delta m) = \int \Lambda(x - \Delta m) f_A(x) dx + \int (x - \Delta m) \Lambda'(x - \Delta m) f_A(x) dx.$$

And since

$$\mathcal{F} \equiv \Delta p'(\Delta m = 0)$$

we obtain

$$\mathcal{F} = \mathcal{A} + \int x \Lambda'(x) f_A(x) dx.$$

# A Basic Decomposition

Contribution of the **extensive margin**:

$$\mathcal{E} \equiv \int x \Lambda'(x) f_A(x) dx.$$

**Property 4:**  $\mathcal{E} = 0$  in the Calvo model, hence  $\mathcal{F}^{\text{Calvo}} = \mathcal{A}$ .

**Property 5:**  $\mathcal{E} > 0$  in incr. hazard models, hence  $\mathcal{F}^{\text{Ss}} > \mathcal{A}$ .

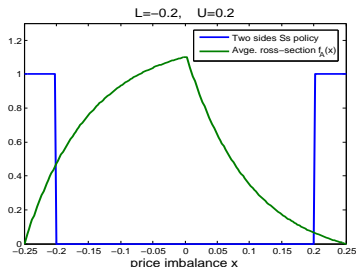
# Extensions

Above derivation assumed  $x\Lambda(x)f(x)$  is continuous.

If this function has discontinuities  $D_1, D_2, \dots$ :

$$\mathcal{F} = \underbrace{\int \Lambda(x)f_A(x)dx}_{\mathcal{A}} + \underbrace{\int x\Lambda'(x)f_A(x)dx}_{\mathcal{E}} + \sum D_k$$

# Two Sided Ss Policies



$$\mathcal{A} = F_A(L) + (1 - F_A(U))$$

$$\mathcal{E} = |L|f_A(L) + Uf_A(U)$$

$|L|f_A(L)$  : adjusted upward instead of not adjusting

$Uf_A(U)$  : did not adjust instead of adjusting downward.

## 4.5. Selection or Extensive Margin Effect?

- Given the micro adjustment frequency  $\mathcal{A}$ ,  $\mathcal{E}^{\text{Ss}} > 0$  is necessary and sufficient for

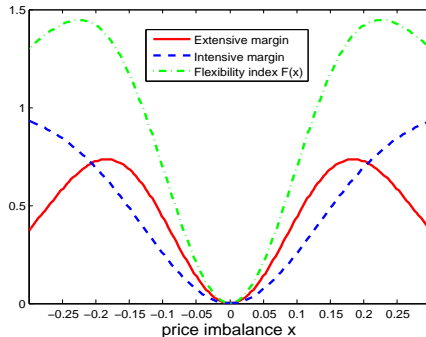
$$\mathcal{F}^{\text{Ss}} > \mathcal{F}^{\text{Calvo}}.$$

- Next we show that the selection effect is neither necessary nor sufficient for the above relation to hold



# Example 1: No Selection Effect and $\mathcal{E} > 0$

$$\Lambda(x) = 1 - e^{-\lambda_2 x^2}.$$



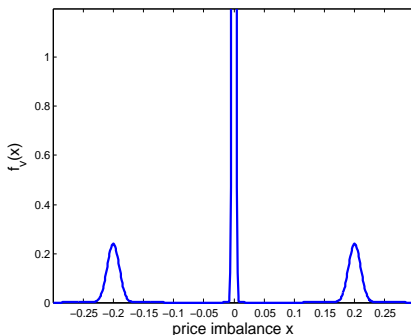
- Intensive margin: increasing in  $|x|$
- Extensive margin: decreasing in  $|x|$  for large  $|x|$

## Property 6

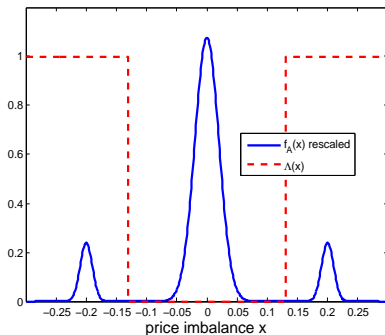
In generalized Ss models, the firms that contribute the most to the aggregate price level are not necessarily those that benefit the most from adjusting. This is due to the fact that the contribution of the extensive margin decreases with the size of the price imbalance when the latter is sufficiently large.

## Example 2: Selection Effect and $\mathcal{E} = 0$

- Aggregate shocks: zero mean, bounded support, small variance
- Leptokurtic idiosyncratic shocks: Midrigan (2006), Gertler and Leahy (2006)



## Example 2: Selection Effect and $\mathcal{E} = 0$



Strong selection effect, yet  $\mathcal{E} = 0$ .

## 4.6. An Application to U.S. Consumer Prices

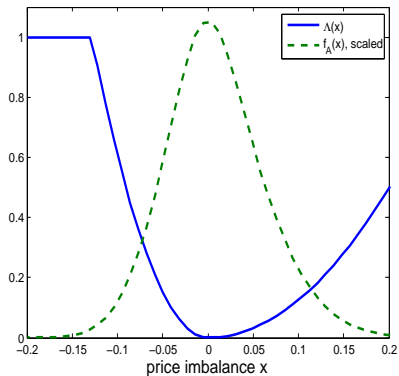
Consider a quadratic (asymmetric) hazard:

$$\Lambda(x) = \begin{cases} \lambda_2^p x^2, & x < 0, \\ \lambda_2^n x^2, & x > 0. \end{cases}$$

Select 4 parameters:  $\sigma_A$ ,  $\sigma_I$ ,  $\lambda_2^p$ , and  $\lambda_2^n$  to match five 5 statistics from Nakamura-Steinsson (2006):

- Frac. pos. adj.: 6.1%
- Frac. neg. adj.: 2.6%
- $E[|\Delta p| | \Delta p > 0]$ : 7.3%
- $E[|\Delta p| | \Delta p < 0]$ : 10.5%
- Volatility (IQR)  $\mathcal{A}_t$ : 0.88%

# Estimated Hazard



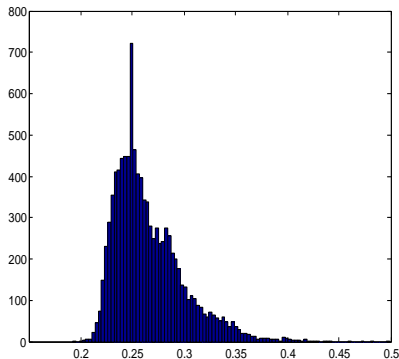
# Estimated Model

- Absolute average relative error: 2.2%
- Without asymmetry: fit much worse
- Matching a moment we did not use:

	NS	Model
IQR yearly fraction adjusters:	0.88%	0.88%
IQR yearly fraction positive adjusters:	1.51%	1.44%
IQR yearly fraction negative adjusters:	0.71%	0.68%

# Skewed Distribution of $\mathcal{F}_t$

Based on 10,000 simulations of the model:

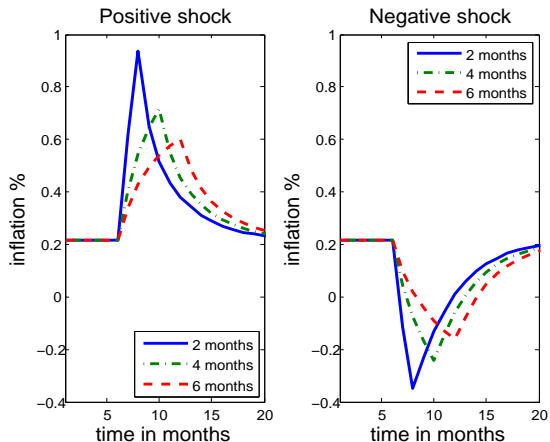




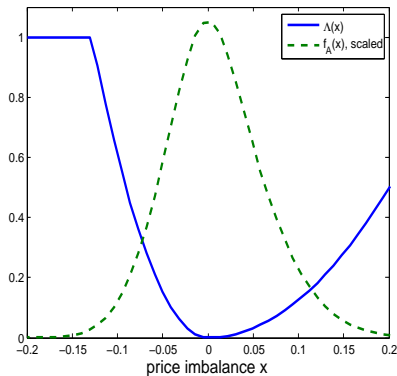
# Why Skewed?

- Cumulative shock (deviation from mean) of  $\pm 4\sigma_A$ ...
- ...distributed over 2, 4 and 6 months
- Compare positive and negative shock
- Look at  $\Delta p_t$  and  $\mathcal{F}_t$

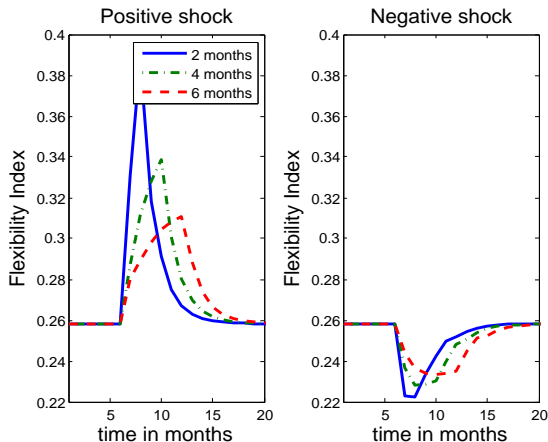
# Evolution of $\Delta p$



# Estimated Hazard



# Evolution of $\mathcal{F}_t$



## 4.7. Conclusion

- To look for Ss-type models that match VAR price inertia:
  - need  $\mathcal{E} < 0$
  - i.e., need a decreasing hazard (in  $x$ -space) in a significant region
  - Kehoe and Midrigan (2007): first of (possibly) many papers
- This paper's framework and insights are valid for:
  - time-varying IRFs
  - other aggregates where lumpy micro adjustment is important