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# STATE-DEPENDENT PRICING AND THE DYNAMICS OF MONEY AND OUTPUT\*

ANDREW CAPLIN AND JOHN LEAHY

Standard macroeconomic models of price stickiness assume that each firm leaves its price unchanged for a fixed amount of time. We present an alternative model in which the pricing decision depends on the state of the economy. We find a method of aggregating individual price changes that allows a simple characterization of macroeconomic variables. The model produces a positive money-output correlation and an empirical Phillips curve. In addition, the impact of monetary shocks depends crucially on the current level of output, which points to a natural connection between state-dependent microeconomics and state-dependent macroeconomics.

## I. INTRODUCTION

There is a long tradition in macroeconomics of attributing the real effects of nominal demand shocks to nominal price stickiness. In this view, if there is no change in prices when nominal demand rises, then quantities must bear the burden of adjustment. Hence nominal price rigidity provides the friction needed for nominal demand shocks to be transmitted to the real economy.

Standard models of this transmission mechanism, such as Fischer [1977] and Taylor [1980], are based on the assumption that each firm leaves its price unchanged for a fixed amount of time. The main reason for considering such time-dependent pricing rules is their analytic tractability. Constraining firms to adjust their prices at prespecified times both simplifies the derivation of equilibrium strategies and allows the use of powerful time series techniques to analyze aggregate dynamics. The main disadvantage of the time-dependent approach is that between price adjustments firms are not allowed to respond even to extreme changes of circumstance. This makes it difficult to know whether the qualitative effects of money in these models are the result of nominal rigidities per se or of the exogenously imposed pattern of price changes.

An alternative approach to modeling price stickiness is to allow the price-setting decision to depend on the actual state of the

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economy, not just the date.<sup>1</sup> Microeconomic models of state-dependent pricing were introduced by Barro [1972] and Sheshinski and Weiss [1977, 1983]. They derived optimal policies for a firm facing a fixed cost of adjusting its nominal price, and found these policies to be of the (s,S) variety: a firm should change its price discretely each time it deviates a certain amount from its optimal value.

As yet, little is known about the macroeconomic implications of state-dependent pricing. The best understood example is due to Caplin and Spulber [1987]. Their model reveals the surprising possibility that price stickiness may disappear altogether at the aggregate level. With a continuously increasing path of the money supply, one-sided (s,S) pricing rules and a uniform initial distribution of prices, shocks to the money supply feed immediately into prices, and nominal shocks have no real effects. Caballero and Engel [1989] show that while the strict neutrality result is lost with arbitrary price distributions, the unconditional correlation between money and output remains zero. Beyond these special cases the macroeconomic implications of state-dependent price rigidity are not well understood.<sup>2</sup>

In this paper we provide the first example of a dynamic economy with state-dependent pricing in which monetary shocks have systematic effects on output. We find a method of aggregating individual price changes that allows a simple characterization of the money-output-price process when nominal shocks are symmetrically distributed. This characterization allows us to examine the statistical properties of the model. We confirm that some of the qualitative results of time-dependent models, such as a positive money-output correlation and an empirical Phillips curve, generalize to our state-dependent framework. Our model also has distinctive features. For example, we show that monetary expansion is more effective in expanding output when output is currently low, while monetary contraction is more effective in reducing output when output is currently high. Overall, the model points to a natural connection between state-dependent microeconomics and state-dependent macroeconomics.

We present the basic model in Section II. In Section III we use

1. Blanchard and Fischer [1989] discuss the distinction between time- and state-dependent pricing rules.

2. Tsiddon [1988] considers the impact of a once-and-for-all change in the rate of growth of the money supply. Blanchard and Fischer [1989] construct a two-period example with symmetric monetary shocks (see Section II).

geometric reasoning to characterize the joint money-output-price process. Section IV addresses the contemporaneous relation between money and output. Section V establishes the existence of an empirical Phillips curve, and studies other features of the price process such as inflationary inertia. In Section VI we extend results of Caballero and Engel [1989] to rationalize an earlier assumption on the initial distribution of prices. Finally, Section VII discusses the sensitivity of the results to various alterations in the assumptions.

## II. THE MODEL

We follow Blanchard and Kiyotaki [1987] in focusing on a monopolistically competitive economy with fixed costs of price adjustment. There is a continuum of price-setting firms indexed by  $i \in (0,1]$ . Each firm treats the current level and future evolution of the price index as independent of its own pricing decisions. At all times  $t \geq 0$ , the log of the price level,  $p(t)$ , is determined as the simple geometric mean of individual nominal prices:

$$(1) \quad p(t) = \int p_i(t) di.$$

The aggregate relationship between money, output, and prices is captured by the quantity equation:

$$(2) \quad m(t) = p(t) + y(t),$$

where  $m(t)$  denotes the log of the money supply and  $y(t)$  the log of output.

The final three components of the model are less standard and are given a fuller introduction below. The first assumption specifies the precise form of the monetary disturbance. The second assumption focuses on the pricing policies. The final assumption concerns the initial conditions.

### *The Money Supply Process*

As in standard menu cost models, monetary disturbances are the only source of uncertainty.<sup>3</sup> Previous theoretical work on the aggregate implications of fixed adjustment costs has focused exclusively on the case in which the state variable changes in only one direction. In this paper we provide a first approach to the case

3. For example, see Rotemberg [1982], Caplin and Spulber [1987], and Blanchard and Fischer [1989].

with two-sided shocks. We make the strong assumption that increases and decreases in the money supply are equally likely.

ASSUMPTION A1. The process  $m(t)$  is a Brownian motion with zero drift.

The consideration of symmetric shocks reflects a natural evolution in the macroeconomic literature on state-dependent pricing and parallels the development of the original microeconomic literature. The (s,S) inventory model of Arrow, Harris, and Marschak [1951] involves a one-sided shock to the inventory. Similarly, the model of monopoly pricing with inflation due to Sheshinski and Weiss [1983] rules out deflation. Early models with nonmonotone shocks include the model of a firm's demand for money due to Miller and Orr [1966] and the model of monopoly pricing with cost shocks due to Barro [1972]. In both cases the underlying shock is modeled as a symmetric random walk.

### *The Strategies*

Following Barro [1972] and Sheshinski and Weiss [1983], the standard approach to state-dependent pricing is to consider a firm that pays an explicit real resource cost each time it changes its nominal price. We adopt this "menu cost" approach, viewing it as a valuable shortcut in deriving sensible state-dependent pricing strategies.

When it is costly to change nominal prices, the optimal pricing policy must balance the loss due to nonadjustment against the cost of changing price. In static menu cost models the cost of nonadjustment is often captured by a profit function that depends on a linear combination of real balances and the relative price:<sup>4</sup>

$$(3) \quad (m-p) - b(p_i - p).$$

Here the level of real balances influences profits through its effect on the level of aggregate demand, while the relative price influences the division of aggregate demand among firms. Changes in the money supply affect profits directly through the level of real money balances and indirectly by inducing changes in relative prices.

In order to reduce the number of state variables, we consider the special case in which the effect of a change in the money supply on the firm's profitability is independent of the aggregate price

4. For example, see Blanchard and Kiyotaki [1987].

level. In the static models this assumption corresponds to setting  $b = 1$  in equation (3). In this case the effect of money is the same whether it is transmitted through a change in real balances or through a change in the price level, thus removing the firm's need to keep track of the price level as an independent state variable. The firm's profits and pricing strategy depend only on its price relative to the money supply. It is convenient to define firm  $i$ 's state as

$$(4) \quad \alpha_i(t) \equiv m(t) - p_i(t),$$

so that in the absence of price adjustment, increases in the money supply cause the state variable to increase.

Given that the firm's profitability depends only on  $\alpha_i$ , we may impose enough symmetry and regularity on the profit function that the firm finds it optimal to pursue a symmetric two-sided (s,S) policy.

ASSUMPTION A2. Each firm adopts a symmetric two-sided (s,S) strategy in the state variable  $\alpha_i(t)$ , adjusting it to zero each time  $|\alpha_i(t)|$  reaches  $S$ .

We do not pursue the issue of optimality here and instead regard this assumption as a simple state-dependent alternative to time-dependent pricing rules.

While the reduction to a single state variable plays a valuable simplifying role, it can in fact be dispensed with. When real balances and relative prices influence profits separately, the price process will influence the firm's choice of strategies. Equilibrium requires consistency between the pricing strategies and price process to which they give rise. Although this is in general a difficult problem, the single state formulation points the way to an essentially identical model with two state variables. This extension is outlined in Section VII and is given a complete treatment in Caplin and Leahy [1991a].

### *The Initial Conditions*

We close the model with a specific assumption on the initial distribution of prices across firms and the initial level of the money supply.

ASSUMPTION A3. Initial nominal prices satisfy

$$p_i(0) = (\frac{1}{2} - i).$$

The initial money supply  $m(0)$  is a random variable distributed uniformly on  $(-S/2, S/2]$ .

Assumption A3 implies that at time  $t = 0$  the  $\alpha_i$  are distributed uniformly over an interval of length  $S$  which is randomly placed in the range  $(-S, S]$ . The most important feature of the assumption is that the initial distribution of nominal prices across firms is uniform on  $(-S/2, S/2]$ . Starting with the initial money supply uniform on  $(-S/2, S/2]$  merely serves to start output off in its long-run distribution, as shown in Section III.

To understand the value of this assumption, it is instructive to contrast it with the case in which the initial distribution of nominal prices across firms is triangular on  $(-S, S]$ . This cross-sectional distribution is appealing because over the long run, individual prices spend more time near the return point than near the adjustment barriers.<sup>5</sup> The problem with the triangular distribution, however, is that as soon as there is a shock the distribution across firms is no longer triangular. For example, a reduction in the money supply will empty a region of the state space as the high  $\alpha_i$  firm is pulled below  $S$ . Further analysis then requires the consideration of other cross-sectional distributions, so that tracking the evolution of the economy becomes tremendously complicated. In contrast, we show below that our initial distribution has an invariance property which greatly simplifies aggregate dynamics.

There are two arguments that support Assumption A3 in addition to its analytic convenience. In Section VI we show that these initial conditions arise as a natural limit in a series of models with idiosyncratic as well as common shocks. Furthermore, in Section VII we use A3 as a stepping-stone in the study of arbitrary initial conditions.

### III. THE MONEY-OUTPUT-PRICE PROCESS

In this section we provide a complete characterization of the joint money-output-price process. This characterization follows from one fundamental observation: with Assumption A3 the distribution of prices across firms remains forever uniform over an interval of length  $S$ .

To see this, picture the initial distribution of the  $\alpha_i$  variables as

5. This distribution is the long-run state occupancy probability for a single firm following a symmetric two-sided  $(s, S)$  policy, and is employed by Blanchard and Fischer [1989, p. 411] in a two-period example.

an elevator of height  $S$  moving inside an elevator-shaft of height  $2S$ , as in Figure 1a. The elevator starts at a random position in the shaft. Until the elevator has either risen to the top or fallen to the base of the shaft, the nominal prices of all firms remain unchanged. During this time the elevator moves precisely with the money supply. Only when the elevator seeks to go through the top or the base of the shaft is its motion constrained.

It is when the elevator reaches one of the barriers that the invariance property is in evidence. For example, if the money supply decreases when the elevator is at the base, firms are simply rotated around the range  $(-S, 0]$ . The *distribution* of the  $\alpha$  variables is unchanged, as in the one-sided  $(s, S)$  story of Caplin and Spulber. The firms that lower their price simply fill in the space vacated by firms pulled below the origin as in Figure 1b.

Note that the invariance property depends critically on the symmetry of the model. If price increases and decreases were of a different size, then the Brownian shocks would produce a continual and chaotic bunching and shattering of the price distribution.

The formal argument for the maintenance of uniformity in the symmetric case requires only that the path of the monetary process is continuous.

**PROPOSITION 1.** Assume that A2 holds, that the variables  $\alpha_i(0)$  are uniformly distributed over an interval of length  $S$  contained in  $(-S, S]$ , and that the path of the money supply is continuous. Then, at any given time  $t \geq 0$ , the variables  $\alpha_i(t)$  remain uniformly distributed over an interval of length  $S$  contained in  $(-S, S]$ .

*Proof.* First, we show that at any given time  $t$ , the distribution of  $\alpha_i(t)$  taken modulo  $S$  is uniform. The proof is completed by

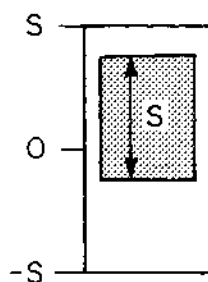


FIGURE 1a

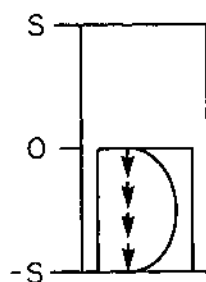


FIGURE 1b

showing that at any time  $t$  all values  $\alpha_i(t)$  lie within a length  $S$  subset of  $(-S, S]$ .

With A2 all nominal price adjustments are of identical magnitude  $S$ . Since multiples of  $S$  are irrelevant to the modulo arithmetic, we arrive at the equation,

$$(5) \quad \begin{aligned} \alpha_i(t) \pmod{S} &= [m(t) - p_i(t)] \pmod{S} \\ &= [m(t) - p_i(0)] \pmod{S}. \end{aligned}$$

The distribution of  $p_i(0) \pmod{S}$  across firms is uniform by assumption. Equation (5) guarantees that this uniformity property is inherited by the distribution of  $\alpha_i(t)$ , since the addition of the constant  $m(t)$  does not disturb uniformity modulo  $S$ .

To complete the proof, we show that at any given time  $t$ , no two firms' real prices differ by more than  $S$ :

$$(6) \quad |\alpha_i(t) - \alpha_j(t)| \leq S \quad \forall i, j \in (0, 1].$$

This holds by assumption when  $t = 0$ . The only time that the difference between two firms' prices alters is when one of them changes its price. But at these times one firm adjusts to  $\alpha_i(t) = 0$ , so that equation (6) continues to apply.

Q.E.D.

It is now straightforward to study the evolution of prices and output. From the quantity equation and the definitions of  $p(t)$  and  $\alpha_i(t)$ , it follows that output corresponds to the mean of the distribution of the  $\alpha_i$  variables:

$$y(t) = m(t) - \int p_i(t) di = \int \alpha_i(t) di.$$

With Proposition 1 the mean is simply the midpoint. To follow the output process, it is sufficient to keep track of the midpoint of the "price-elevator," as in Figure II. Conversely, output is a sufficient statistic for the cross-sectional distribution of the state variables  $\alpha_i(t)$ , and hence for the overall state of the economy.

While all prices are in the interior of the range  $(-S, S]$ , changes in the money supply leave all nominal prices unchanged and feed directly into output. When output reaches  $S/2$ , the price elevator is at the top of the elevator shaft. Further increases in the money supply feed directly into prices and leave output unchanged, while decreases feed into output. When output is at  $-S/2$ , decreases in the money supply feed directly into prices, while

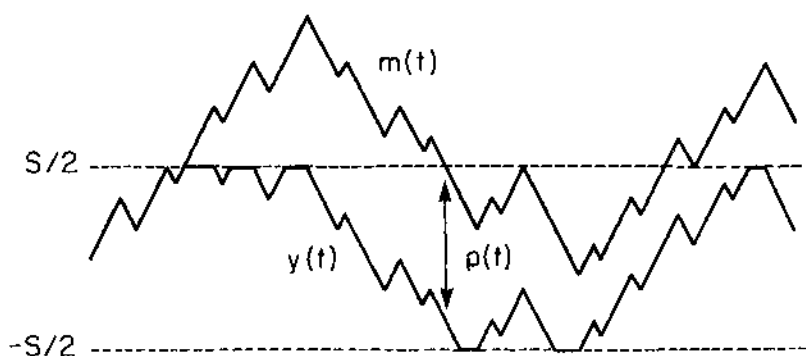


FIGURE II

increases feed into output. Formally, these properties define output as a regulated version of the money process, in the sense of Harrison [1985].<sup>6</sup>

**PROPOSITION 2.** Given the assumptions of Proposition 1, the output process is identical to the money process regulated at  $S/2$  and  $-S/2$ .

*Proof.* It is immediate from Proposition 1 that  $y(t)$  is always in the range  $[-S/2, S/2]$ . We define functions  $u(t)$  and  $l(t)$  as, respectively, the gross cumulative inflation and deflation in the aggregate price index up until time  $t$ . Note that  $u(t)$  and  $l(t)$  are increasing functions, and they also inherit continuity from  $m(t)$ . By the quantity equation,

$$y(t) = m(t) - p(t) = m(t) - u(t) + l(t).$$

Finally, it follows from Proposition 1 and Assumption A2 that increases in  $u(t)$  require  $y(t) = S/2$ . Similarly,  $l(t)$  increases only when  $y(t) = -S/2$ . Hence,  $y(t)$  satisfies the conditions for a regulated process [Harrison, 1985, p. 22].

Q.E.D.

6. There is an interesting analogy between multi-agent menu cost models and a model with a single price-setting agent facing a linear cost of adjusting prices. Just as in Figure II, linear adjustment costs lead to a range of inaction and regulation at the boundaries. Note that the analogy applies equally to the one-sided case. Regulation against an increasing process leads to the state variable being kept at the top of the range: hence the nominal price adjusts precisely in line with money increases, as in Caplin and Spulber [1987].

Proposition 2 places the analysis of money and output in a standard mathematical context: the theory of the regulated Brownian motion. An immediate implication is that the long-run distribution of output is uniform on  $(-S/2, S/2]$ .<sup>7</sup> This confirms that taking  $m(0)$  to be uniform on  $(-S/2, S/2]$  in Assumption A3 indeed starts the model in its long-run distribution. The fact that output is ergodic implies a form of monetary neutrality in the long run. This is, however, not the standard form of long-run neutrality in which the effect of an individual shock, as measured by an impulse response function, falls to zero as time passes. In our model, all shocks are permanent: in the absence of further shocks, the economy would remain forever at rest at the resulting level of output. It is only the cumulative effects of later shocks which ensure that expected output eventually returns to zero.

With Proposition 2 it is clear that output dynamics in this model are far more intricate than in the one-sided model. The model is a hybrid of the static menu cost model of Mankiw [1985] and the one-sided dynamic menu cost model of Caplin and Spulber [1987]. While output is in the interior of the range  $[-S/2, S/2]$ , small changes in the money supply feed directly into output, just as in the static model. The one-sided neutrality result emerges only at extreme levels of output. There is a clean separation between inflationary and noninflationary states of the economy.<sup>8</sup>

#### IV. THE INTERACTION OF MONEY AND OUTPUT

We now turn to the statistical properties of the model. We show that our model with state-dependent pricing strategies produces novel predictions concerning the impact of money on the economy. In contrast to the one-sided model, there is a systematic relationship between monetary shocks and output: the overall correlation of money and output is positive. In contrast to time-dependent models, the effect of money on the economy is closely tied to the state of the economy, as reflected in the level of output. For example, monetary expansion is more effective in expanding output when output is currently low, while monetary contraction is more effective in reducing output when output is currently high.

7. See Harrison [1985, p. 90].

8. Of course, addition of realistic elements such as idiosyncratic shocks and heterogeneity in menu costs and alternative price distributions will soften the boundary between inflationary and noninflationary states. We consider some of these elements in Section VII.

The relationship between output and the effects of monetary shocks follows directly from an analysis of an arbitrary path of the money supply. Figure III illustrates the paths of output associated with two different initial levels of output. The figure shows that for a given path of the money supply a higher level of initial output raises the entire path of output. The paths associated with different levels of initial output may join, but they can never cross. The figure also shows that the expected increment to output is a decreasing function of the current output level, so that money growth is less expansionary when output is already high. A higher initial output both increases the cumulative amount of inflation and reduces the amount of deflation. Geometrically, this corresponds to the declining distance between the output paths as time passes, so that an increase in initial output leads to a less than one-for-one increase in final output. These results are presented in Proposition 3.

It is useful to note that we lose no generality in fixing the initial time at zero in the study of all correlations since we have started the model with output in its long-run distribution.

**PROPOSITION 3.** For a given path of the money supply, consider the paths of output given two different initial output levels,  $\hat{y}(0) \geq y(0)$ . Then at all times  $t \geq 0$ ,  $\hat{y}(t) \geq y(t)$ , and  $\hat{y}(t) - \hat{y}(0) \leq y(t) - y(0)$ .

Another natural issue is the effect of money growth on expected future output. It appears reasonable that a higher rate of money growth will result in a higher level of output. However, this is not universally true. Note that a given change in the money supply over any period,  $m(t) - m(0) \equiv \Delta m(t)$ , is consistent with many different *paths* of the money supply. It is readily confirmed that knowing the initial level of output and the change in the money supply is not enough to pin down the level of final output.

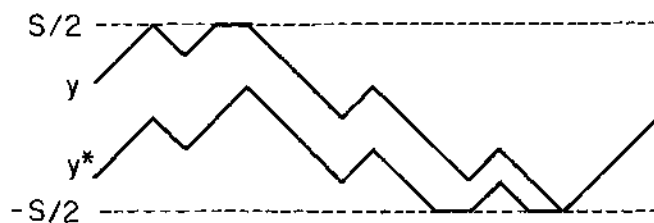


FIGURE III

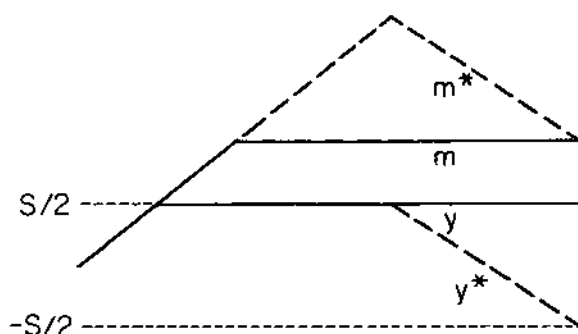


FIGURE IV

Furthermore, Example 1 shows that it is possible to raise the *entire* path of the money supply and yet dramatically lower final output. If there is a relationship between money and output, it is certainly not apparent from the analysis of isolated paths of the money supply.

**EXAMPLE 1.** Given  $y(0)$ , consider the following two alternative paths for the money supply. In the first case the money supply  $m$  increases monotonically by an amount  $S$  between 0 and  $t$ , so that  $y(t) = S/2$ . In the second case the money supply  $m^*$  initially rises more rapidly: the maximal increase exceeds  $2S$ . Having risen monotonically to this maximum,  $m^*$  then decreases monotonically by  $S$ . Final output in this case is at a minimum  $y^*(t) = -S/2$ , despite the fact that this path lies everywhere above the first path, as in Figure IV.<sup>9</sup>

In spite of the possibility of a perverse relationship between money and output on individual paths, there is a simple overall statistical relationship. By averaging across paths, we show that larger increases in the money supply are associated with larger increases in output. This result is stated in Proposition 4. The proof makes heavy use of probabilistic reasoning, and is presented in Caplin and Leahy [1991b].

9. This example illustrates why two-sided  $(s, S)$  policies are so much more complex than one-sided  $(s, S)$  policies. The process of averaging is trivial in the case of one-sided policies with a monotonic money supply, since knowledge of  $\alpha_i(0)$  for all  $i$ ,  $m(0)$ , and  $m(t)$  fully determines all  $\alpha_i(t)$ , and therefore the level of output. Contrary to Example 1, output dynamics are not influenced by the path of the money supply between 0 and  $t$ .

PROPOSITION 4. For all  $t \geq 0$  the conditional expectation of output, given initial output and the change in the money supply,  $E[y(t)|y(0), \Delta m(t)]$  is increasing in  $\Delta m(t)$ .

Proposition 4 allows an easy demonstration that the correlation between money and output is positive.

PROPOSITION 5. The correlation between money and output is positive:

$$\rho(y(t), \Delta m(t)) > 0.$$

*Proof.* Since  $Ey(t) = E\Delta m(t) = 0$ ,

$$\begin{aligned} \text{cov}(y(t), \Delta m(t)) &= Ey(t)\Delta m(t) \\ &= E[\Delta m(t) \cdot E[y(t)|\Delta m(t)]]. \end{aligned}$$

Note that  $E[y(t)|\Delta m(t)]$  is increasing in  $\Delta m(t)$ , since the result in Proposition 4 survives when we remove the conditioning on the initial level of output. In addition, as a direct consequence of the symmetry of the model,  $E[y(t)|\Delta m(t) = 0] = 0$ . It follows that  $E[y(t)|\Delta m(t)]$  has the sign of  $\Delta m(t)$ , establishing the result.

Q.E.D.

While the results in this section are derived for a very special model of state-dependent pricing, there is a general moral. State-dependent policies tend to produce state-dependence in the effect of macroeconomic shocks. Testing such models will require nonlinear estimation techniques in which the estimated parameters are allowed to depend on the state of the economy.

## V. PRICES AND OUTPUT

In continuous time the price level increases only when output is at its maximum value. This is reminiscent of the old-style Keynesian treatment of prices, with inflation occurring only at "full employment." Due to the accumulation of shocks, however, the discrete time data will not reveal such a simple relation. High *net* inflation over a discrete time period does not necessarily imply high output. For example, if money rises monotonically by some multiple of  $S$  and then falls by  $S$ , output will be at a minimum even though only price increases have been observed.

Once again, a probabilistic approach clarifies the issue. Proposition 6 establishes that the sign of the coefficient in a regression of

output on inflation is positive, implying the presence of an empirical Phillips Curve. This result is proved in Caplin and Leahy [1991b].

PROPOSITION 6. The correlation between inflation and output is positive:

$$\rho(y(t), \Delta p(t)) > 0.$$

A second important issue is the presence of inflationary inertia. Even though shocks to the money supply are independent and identically distributed over time, changes in the price level follow a far more complex pattern. In fact, the inflation rate displays positive autocorrelation, since inflation during period  $t - 1$  is associated with above average final output  $y(t - 1)$ , which in turn makes inflation more likely in period  $t$ . Hence there are both inflationary and deflationary spells in the economy.

Finally, the model also has implications for the much investigated topic of the relationship between inflation and relative price variability. Note that the steady state distribution of  $p_i - p$  is uniform over  $(-S/2, S/2]$ . Thus, the relative price formula is identical to that found for the one-sided model of  $(s, S)$  aggregation. This implies, for example, that in widely separated observations the variance of individual inflation rates around the economywide inflation rate approaches  $S^2/6$  [Caplin and Spulber, 1987, pp. 717-18].

## VI. CONVERGENCE

In this section we provide some justification for the assumption that the  $\alpha_i(0)$  are distributed uniformly over an interval of length  $S$ . We show that this distribution arises as a natural limiting case in models with idiosyncratic as well as common shocks.

We introduce the idiosyncratic shock in a way that does not alter the economic environment from the individual firm's perspective. Let  $x_i(t)$  be an idiosyncratic shock to the profits of firm  $i$ , and suppose that firms' profits depend on the new state variable  $z_i(t)$ :

$$z_i(t) \equiv m(t) - p_i(t) + x_i(t).$$

We assume that all idiosyncratic shocks and the money process are independent, mean zero Brownian motions. We further assume that the infinitesimal variance of the idiosyncratic shocks is  $\epsilon^2$ , and that of the money supply is  $\sigma^2 - \epsilon^2$ . Standard results on the

Brownian motion then ensure that the evolution of  $z_i(t)$  does not depend on the variance of the idiosyncratic shock. We therefore assume that the firm's pricing policy is to adjust  $z_i(t)$  to zero when it deviates by  $S$ , regardless of the size of the idiosyncratic shock.

We are interested in the long-run behavior of the distribution of the  $z_i(t)$  across firms. In Proposition 7 we show that for small enough values of  $\epsilon$  the cross-sectional distribution of the  $z_i(t)$  converges over time to a distribution arbitrarily close to a uniform distribution with support of length  $S$ .<sup>10</sup> The proposition is proved in the Appendix.

**PROPOSITION 7.** Assume that all firms pursue symmetric  $(s, S)$  policies in the variables  $z_i(t)$ , and that the path of the money supply is continuous. Then for any given cross-sectional distribution of the  $z_i(0)$  on  $(-S, S]$ , there are small enough values of  $\epsilon$  and large enough values of  $t$  so that the cross-sectional distribution of the  $z_i(t)$  is arbitrarily close to a distribution uniform over a range  $S$  within  $(-S, S]$  with an arbitrarily high probability.

The demonstration of convergence follows from logic similar to that used in the proof of Proposition 1. There, in the absence of idiosyncratic shocks, uniformity of the distribution taken modulo  $S$  and a support of the distribution of length  $S$  were sufficient to prove the invariance property. To prove Proposition 7, we show that each of these observations has an analog in models with idiosyncratic shocks.

While it is no longer true that the distribution of the  $z_i(t)$  taken modulo  $S$  is always uniform, the distribution of the  $z_i(t) \pmod{S}$  does converge over time to the uniform distribution irrespective of the size of the idiosyncratic shock. This result follows directly from an adaptation of Theorem 1 of Caballero and Engel [1989, p. 14]. They show that in a one-sided  $(s, S)$  model the cross-sectional distribution approaches uniformity over  $(0, S]$  in the long run for all values of  $\epsilon$ . Our two-sided model taken modulo  $S$  is equivalent to their one-sided model. In both models all changes in price are of size  $S$  and are therefore irrelevant to the distribution taken modulo  $S$ .

In the absence of idiosyncratic shocks, the only force effecting

10. We use the variation norm to measure the distance between densities  $f$  and  $g$ ,  $\frac{1}{2} \int_{-S}^S |f(x) - g(x)| dx$ .

the distance between nominal prices was price adjustment, which itself placed all firms within  $S$  of one another. While price adjustment still pulls firms together in the present case, the idiosyncratic shocks tend to pull them apart. We can no longer guarantee that all firms lie within a range  $S$ . Lemma 2 in the Appendix, however, shows that with a small enough idiosyncratic shock we can ensure that most of the time most of the firms lie within a range close to  $S$ . Lemma 2 points to an important source of nonneutrality in two-sided  $(s, S)$  models. Since adjustment is to some point in the interior of the range of inaction, common shocks tend to group firms together. This suggests that the more important the common shock, the greater is the bunching and the greater is the nonneutrality of money.<sup>11</sup>

## VII. RELAXING THE ASSUMPTIONS

We now consider the consequences of relaxing the assumptions of Section II. We show that many of the characteristics of the basic model survive in richer settings.

### *The Initial Conditions*

When we allow for nonuniform price distributions, output is no longer a regulated Brownian motion, but is instead the sum of a regulated Brownian motion and an independent error term. Thus, alternative initial conditions simply add noise to the output dynamics.

To see this, first note that the assumption that the support of the initial distribution is of length  $S$  is innocuous. Without idiosyncratic shocks there is no force other than price adjustment that affects the difference between two firms' prices, and this always works to bring these prices within  $S$  of one another. After all firms have adjusted their price once, all the  $\alpha_i$  will always lie in an interval of length  $S$ .

We may now confine our attention to initial distributions on  $(0, S]$ .<sup>12</sup> We wish to compare the output dynamics associated with

11. Our result is a limit result as the idiosyncratic shock is removed. The dynamic implications of two-sided  $(s, S)$  policies in the presence of both idiosyncratic and common shocks are studied in Bertola and Caballero [1990].

12. Assuming that this distribution has all of its mass at a single point corresponds to a single firm following a two-sided  $(s, S)$  policy, so that the following results naturally apply to a representative agent model.

an arbitrary initial distribution to the output dynamics under the uniform distribution.

Once again, a geometric approach is illuminating. Since both distributions have a support of length  $S$ , we may superimpose them in the elevator shaft of Figure I and analyze their evolution under a specific path of the money supply. Let  $y^*(t)$  denote output with the new initial distribution, and let  $y(t)$  denote output under the uniform initial distribution. In each case output is equal to the mean value of the respective  $\alpha$  distribution. While the distributions are in the interior of the shaft, money supply shocks affect  $y(t)$  and  $y^*(t)$  equally. At the top and the bottom of the shaft price adjustment occurs. Price adjustment leaves  $y(t)$  constant, but changes  $y^*(t)$  by rotating the distribution of prices.

Figure V illustrates output dynamics in the nonuniform case. The density of firms at prices inside the lift is represented by the amount of shading. At both times  $t$  and  $t'$  the price-lift is at the top of the shaft. The only difference is that the distribution has been rotated by an amount  $S/2$  between  $t$  and  $t'$ . As a result,  $y^*(t)$  has risen, while  $y(t)$  has remained at  $S/2$ .

In general, the difference between  $y(t)$  and  $y^*(t)$  is a function of the amount by which the initial density has been rotated:

$$y^*(t) - y(t) = f(r(t)).$$

Here the rotation is captured by the relative position of the firm initially at the base of the distribution, which we denote by  $r(t)$ .

The next result formalizes the sense in which arbitrary initial distributions add noise to  $y(t)$ .

PROPOSITION 8. In the long run,  $E[f(r(t))|y(t)] = 0$ .

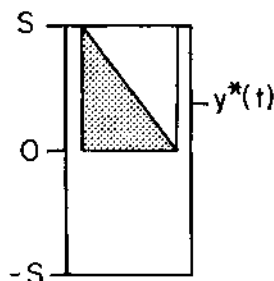


FIGURE Va

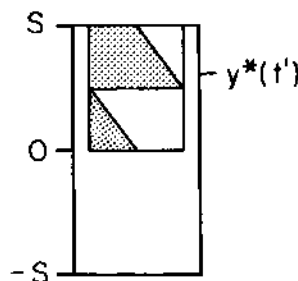


FIGURE Vb

*Proof.* Let  $G(a)$  denote the initial distribution on  $a \in [0, S]$ . Then in the long run,

$$\begin{aligned} E\{f(r(t))|y(t)\} &= E\{y^*(t)|y(t)\} - y(t) \\ &= E\left\{\int_0^{S-r(t)} (a + r(t)) dG(a) \right. \\ &\quad \left. + \int_{S-r(t)}^S (a + r(t) - S) dG(a)\right\} - \frac{S}{2} \\ &= Ea + Er(t) - ES(1 - G(S - r(t))) - \frac{S}{2}. \end{aligned}$$

Since the long-run distribution of  $r(t)$  is independent of  $y(t)$  and is uniform on  $(0, S]$ , the second term in the final expression is  $S/2$ . A straightforward change of variable shows that the third term equals the mean of  $a$ .

Q.E.D.

Proposition 8 shows that in the long run  $y^*(t)$  is a mean-preserving spread of  $y(t)$ . Not only does this result imply that  $y(t)$  may provide a good approximation for  $y^*(t)$ , but it also allows us to apply many of our earlier results directly to arbitrary distributions. For example, altering the initial distribution does not alter the correlation of money and output, since in the long run the added noise in output is independent of the money supply.<sup>13</sup>

### *The Money Supply Process*

Recent developments in the microeconomic literature on fixed adjustment costs point to possible future developments in the literature on aggregation. There is an emerging literature on optimal control against asymmetric two-sided processes.<sup>14</sup> Frequently, the optimal strategy is to adjust the state variable to an intermediate level from asymmetrically placed upper and lower boundaries. While this strategy is closely related to the symmetric strategy, the loss of symmetry makes distributional dynamics prohibitively complex. This undermines our ability to track macro-

13. A related result is shown by Caballero and Engel [1989] for the one-sided (s,S) model, p. 27.

14. For example, Dixit [1989], Grossman and Laroque [1990], and Harrison, Selke, and Taylor [1983] consider the geometric Brownian motion. Tsiddon [1987] derives the stationary density for an individual firm's prices with an asymmetric two-sided (s,S) policy.

economic aggregates analytically. Fortunately, there are important topics that can be understood without explicit reference to the distribution of prices.

For example, consider the covariance between changes in the money supply and output:

$$E\{y(t)\Delta m(t)\} = E\{(m(t) - p(t))\Delta m(t)\}.$$

Note that since  $m(t) - p(t) = \int_0^t \alpha_i(t) di$ , it follows by Fubini's theorem that,

$$(7) \quad E\{y(t)\Delta m(t)\} = E\{\alpha_i(t)\Delta m(t)\}.$$

Hence the covariance between money and  $\alpha_i$  is the same as the covariance between money and output. We may therefore be able to calculate the covariance between money and output from firm data even though we are unable to characterize the output process.

Note that this approach can only work in the case with a single state variable. With two state variables one cannot escape the need to follow the entire distribution of prices over time, since this determines the evolution of the price level and hence influences the choice of strategies.

### *The Single State Variable*

So far, we have avoided the potentially separate influence that real balances and relative prices may exert on the firm's pricing decision. Allowing  $m - p$  and  $p_i - p$  to play distinct roles appears to require a fundamental change of perspective. We must now face head-on issues such as the determination of complex strategies in two state variables and the consistency between these strategies and the resulting price processes. Our single state model, however, provides a shortcut.

We first examine why the model as it stands is not well suited to the presence of two state variables. When a firm is only concerned with the future evolution of the money supply, the economy always looks the same at all points of price adjustment. The firm therefore chooses the same value of  $\alpha_i$  regardless of whether it is increasing or decreasing its price. But, with the firm interested in both the money supply and the price level, it no longer makes sense for the firm to choose the same value of  $\alpha_i$  when increasing and decreasing its price, since in the former case the firm is expecting inflation, while in the latter deflation.

The simplest alteration in the basic model that incorporates these considerations is to assume a constant size of price adjust-

ment, which we shall denote by  $D$ , and an initial distribution of prices that is uniform over an interval of length  $D$ . These amendments preserve the invariance property of the original model, and hence the simple characterization of the joint money-output-price process. The only difference from the earlier analysis is that the price-lift need not be half the length of the shaft, as illustrated in Figure VI.

These simple amendments are indeed consistent with a separate role for real balances and the relative price. This is confirmed in Caplin and Leahy [1991a]. Here we provide a sketch of the argument. Under general conditions the firm with the highest relative price,  $D/2$  will be the first firm to lower its price. For any given beliefs concerning the probability law governing the future evolution of money and prices, there is a critical value of  $(m - p)^*$  that will trigger this firm to adjust its price. Equilibrium requires only that expectations are rational and that the size of the price adjustment precisely equals  $D$ . The invariance property then ensures that all firms will act in an identical fashion when theirs is the highest relative price, while symmetry ensures that the same arguments apply for the firm with the lowest real price.

Thus, there is a substantively identical model consistent with real money balances and relative prices having separate influences on profits. In equilibrium, output is a regulated Brownian motion with range  $2(m - p)^*$ , and the price level is the difference between  $m$  and  $y$ . All the Propositions in Sections III–VI of the paper apply without alteration. Allowing for two state variables complicates the microeconomics, but leaves the macroeconomics intact.

### VIII. CONCLUDING COMMENTS

We construct a simple dynamic menu cost model in which monetary disturbances have real effects. In our example money is a

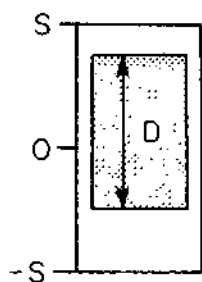


FIGURE VIa

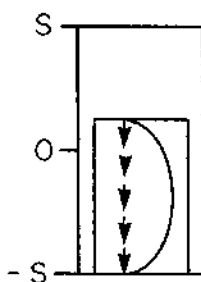


FIGURE VIb

Brownian motion, and output is a regulated Brownian motion. This characterization allows us to fully analyze the interaction between money, prices, and output. As in static menu cost models, the proximate cause of nonneutrality is the bunching of firms' real prices. When all agents' real prices are close together, there will be long periods in which the price level does not change in response to monetary disturbances.

There are now two macroeconomic models with state-dependent pricing with radically different implications for aggregate price inertia. In contrast to the symmetric two-sided (s,S) model considered here, money and output are unrelated in the one-sided model of Caplin and Spulber [1987]. It is remarkable that the presence or absence of neutrality hinges on such an apparently orthogonal issue as the one-sided or two-sided nature of the shocks. The basic difference is that in the two-sided model a prolonged fall in the money supply ensures that all firms will be in the lower half of the state space. In the one-sided story there is no pattern of monetary disturbances that coordinates prices in this way.

The model also shows that state-dependent pricing models imply aggregate dynamics very different from those encountered in time-dependent models. In time-dependent models, the evolution of output is often captured by an ARMA process in which the coefficients on the shocks are constant over time. With state-dependent pricing the effect of money on output will depend on the state of the economy. In our model increases and decreases in the money supply have different effects depending on whether output is high or low. At higher output levels the expansionary effects of increases in the money supply are diminished, while the contractionary effects of decreases in money are enhanced.

The techniques and results of this paper can be developed in several directions. For example, the model may be used to analyze such issues as the connection between the variance of monetary growth and the slope of the Phillips curve, investigated in Lucas [1973]. In addition the techniques we use to analyze the statistical properties of the regulated Brownian motion can be applied to other areas in which transactions costs play a role. These developments are contained in Caplin and Leahy [1991a, 1991b].

At a more general level our approach to dynamic macroeconomics discards the fiction of a representative agent. Instead, we view the economy as a collection of heterogeneous agents who allow their control variables to drift away from their optimal values. The important object of analysis is then the cross-sectional distribution of these control variables. In this paper we find conditions under

which the dynamic strategies of the individual agents aggregate to yield simple and interesting macroeconomic conclusions. It is clear that further study of distributional dynamics is vital in the many areas of macroeconomics in which transactions costs play a role.

## IX. APPENDIX

### *Formal Treatment of Convergence*

We now provide a formal proof of Proposition 7, which shows how the distribution in A3 arises as a limiting case in models with idiosyncratic shocks. To measure the distance between two distributions  $F$  and  $G$ , we use the variation distance,

$$d(F, G) = \frac{1}{2} \int_{-S}^S |f(x) - g(x)| dx,$$

where  $f$  and  $g$  denote the densities corresponding to  $F$  and  $G$ , respectively.

As in the proof of Proposition 1, two observations combine to give the overall result. The first, contained in Lemma 1, involves the distribution of  $z_i(t)$  taken modulo  $S$ .

**LEMMA 1.** At time  $t$  consider the distribution of  $z_i(t)$ ,  $i \in (0, 1]$ , taken modulo  $S$ . For all  $\epsilon > 0$ , this distribution converges over time to a distribution uniform on  $(0, S]$ .

*Proof.* Apply Theorem 1 in Caballero and Engel [1989] to the  $z_i$  taken modulo  $S$ .

Q.E.D.

The second observation states that as we consider models with smaller and smaller idiosyncratic shocks, the bulk of the firms tend to gather within an interval of length  $S$ .

**LEMMA 2.** For small enough values of  $\epsilon$  and large enough values of  $t$ , there is an arbitrarily high probability that an arbitrarily large proportion of the firms have values of  $z_i(t)$  within a range arbitrarily close to  $S$ .

*Proof.* Fix  $\alpha$ ,  $\beta$ , and  $\gamma \in (0, 1)$ . We pick  $\bar{\epsilon}$  and  $\bar{t}$  such that  $\epsilon < \bar{\epsilon}$  and  $t > \bar{t}$  imply that the probability that a proportion  $(1 - \gamma)$  of the  $z_i(t)$  lie within  $S(1 + \beta)$  of each other is at least  $(1 - \alpha)$ .

To define  $\bar{t}$ , let  $b(t)$  be a mean zero Brownian motion with infinitesimal variance  $\sigma^2/2$  and  $b(0) = 0$ . We set  $\bar{t}$  such that

$$(A1) \quad \Pr \{ \max |b(\bar{t})| > 2S + S\beta \} > 1 - \alpha.$$

Note that  $\bar{t}$  is finite almost surely.

To define  $\bar{\epsilon}$ , we first choose  $\hat{\epsilon}$  so that

$$(A2) \quad \Pr \left\{ \max_{t \in (0, \bar{t})} |x(t)| < \frac{S\beta}{4} \right\} > 1 - \gamma.$$

We then define  $\bar{\epsilon} \equiv \min \{\hat{\epsilon}, \sigma/\sqrt{2}\}$ .

We now consider an idiosyncratic shock of standard deviation  $\epsilon < \bar{\epsilon}$  and a time  $t > \bar{t}$  as prescribed. Since  $\epsilon < \sigma/\sqrt{2}$ , the infinitesimal variance of the common monetary shock  $m(t)$  is greater than  $\sigma^2/2$ . Our choice of  $\bar{t}$  then implies that at all times  $t > \bar{t}$ ,

$$(A3) \quad \max_{s \in (0, \bar{t})} |m(t) - m(t-s)| > 2S + S\beta,$$

with probability greater than  $1 - \alpha$ .

On the set of paths for which inequality (A3) holds, the money supply has either risen or fallen by  $2S + S\beta$  at some time in the interval  $[t - \bar{t}, t]$ . Thus, in the absence of idiosyncratic shocks all firms would lie within  $S$  of one another. The only forces that may prevent firms from being within  $S$  of one another are the idiosyncratic shocks. We use the fact that  $\epsilon < \bar{\epsilon}$  to bound the amount of divergence caused by the idiosyncratic shocks.

Inequality (A2) shows that for any firm  $i$ ,

$$\Pr \left\{ \max_{s \in (0, \bar{t})} |x_i(t) - x_i(t-s)| < \frac{S\beta}{4} \right\} > 1 - \gamma.$$

Since the idiosyncratic shocks are independent and identically distributed across firms, the Glivenko-Cantelli lemma implies that this inequality also applies to a proportion  $1 - \gamma$  of firms.<sup>15</sup> Let  $\mathcal{A} \subset [0, 1]$  be the set of firms for which

$$(A4) \quad \max_{s \in (0, \bar{t})} |x_i(t) - x_i(t-s)| < S\beta/4.$$

To complete the proof, we show that for all firms  $i$  and  $j$  in  $\mathcal{A}$ ,  $|z_i(t) - z_j(t)| < S(1 + \beta)$  whenever (A3) holds.

In combination, inequalities (A3) and (A4) show that for all firms in  $\mathcal{A}$ , the variable  $z$  has traveled over a range larger than  $2S$  over the period  $[t - \bar{t}, t]$ . Therefore, all firms in  $\mathcal{A}$  have adjusted their price during this period. Given firms  $i$  and  $j$  in  $\mathcal{A}$ , consider the last time  $\hat{t} \in [t - \bar{t}, t]$  at which one of them adjusted their price.

15. We adopt the conventional treatment in which a continuum of independent random variables is treated as an idealized limit of the large finite case.

Note that  $|z_i(\hat{t}) - z_i(\bar{t})| \leq S$ , and that in the period  $(\hat{t}, \bar{t}]$  only the idiosyncratic shocks have separated these firms. But inequality (A4) shows that the most the idiosyncratic shock has moved either of them since  $\hat{t}$  is  $S\beta/2$ . They therefore lie within  $S(1 + \beta)$  of one another at time  $\bar{t}$ , and the proof is complete.

Q.E.D.

We are now in a position to prove the main result.

**PROPOSITION 7.** Assume that all firms pursue symmetric  $(s, S)$  policies in the variables  $z_i(t)$ , and that the path of the money supply is continuous. Then for any given cross-sectional distribution of the  $z_i(0)$  on  $(-S, S]$ , there are small enough values of  $\epsilon$  and large enough values of  $t$  so that the cross-sectional distribution of the  $z_i(t)$  is arbitrarily close to a distribution uniform over a range  $S$  within  $(-S, S]$  with an arbitrarily high probability.<sup>16</sup>

*Proof.* Fix  $\alpha, \delta \in (0, 1)$ . We pick  $\bar{\epsilon}$  and  $\bar{t}$  such that  $\epsilon < \bar{\epsilon}$  and  $t > \bar{t}$  imply that

$$\Pr [d(F_t, G) < \delta] > 1 - \alpha,$$

where  $F_t$  is the distribution of the  $z_i(t)$  and  $G$  is a distribution uniform over a range  $S$  within  $(-S, S]$ .

We now pick positive numbers  $\beta, \gamma$ , and  $\xi$ , such that,

$$\beta/S + \gamma < \xi \delta.$$

With Lemma 1 we find a time  $t_1$  such that for  $t > t_1$ ,

$$\int_0^S \left| f(x) + f(x - S) - \frac{1}{S} \right| dx < \xi.$$

With Lemma 2 we can find  $t_2$  and  $\bar{\epsilon}$  such that for  $t > t_2$  and  $\epsilon < \bar{\epsilon}$  a proportion  $1 - \gamma$  of the  $z_i$  lie in a range  $S + \beta$  with probability  $(1 - \alpha)$ .

16. This is a form of convergence in probability. Let  $\epsilon_n$  and  $t_n$  denote any pair of sequences converging to zero and infinity, respectively. Define the random variable  $X_n$  as the distribution of the  $z_i$  at time  $t_n$  in a model in which the initial distribution is  $F_0$  and the variance of the idiosyncratic shock is  $\epsilon_n$ . We prove

$$\lim_{n \rightarrow \infty} \Pr [d(X_n, \operatorname{argmin}_G d(X_n, G)) > \delta] = 0,$$

where  $G$  is chosen from the class of distributions that are uniform over an interval of length  $S$ .

Let  $\bar{t} = \max(t_1, t_2)$ . We now confirm that for  $t > \bar{t}$  and  $\epsilon < \bar{\epsilon}$ ,

$$\Pr[d(F_t, G) < \delta] > 1 - \alpha.$$

Consider a realization in which at least  $1 - \gamma$  of the  $z_i$  lie within a set of length  $S + \beta$ , which we denote as  $(a, a + S + \beta]$ . We shall compute the distance between  $F_t$  and the uniform distribution on  $(a + \beta, a + S + \beta]$ . Without loss of generality take  $a = -\beta$ .<sup>17</sup> In this case  $G$  is uniform on  $(0, S]$ , and  $d(F_t, G)$  can be decomposed as

$$\begin{aligned} 2d(F_t, G) &= \int_{-S}^{-\beta} f(x) dx + \int_{-\beta}^0 f(x) dx \\ &\quad + \int_0^{S-\beta} \left| f(x) - \frac{1}{S} \right| dx + \int_{S-\beta}^S \left| f(x) - \frac{1}{S} \right| dx. \end{aligned}$$

The first term is bounded above by  $\gamma$ , the maximum proportion of firms in the range  $(-S, -\beta]$ . The third term is bounded using the triangle inequality,

$$\begin{aligned} \int_0^{S-\beta} \left| f(x) - \frac{1}{S} \right| dx &\leq \int_0^{S-\beta} f(x - S) dx \\ &\quad + \int_0^{S-\beta} \left| f(x) + f(x - S) - \frac{1}{S} \right| dx < \gamma + \xi, \end{aligned}$$

where the second inequality follows from the  $\gamma$  population in  $(-S, -\beta]$  and Lemma 1 which bounds the distance between the distribution of  $z_i \pmod{S}$  and the uniform distribution. Finally, note from Lemma 1 that

$$\int_{S-\beta}^S \left| f(x) + f(x - S) - \frac{1}{S} \right| dx < \xi.$$

Hence we can place an upper bound on the total population in the union of the regions  $(S - \beta, S]$  and  $(-\beta, 0]$ ,

$$\int_{S-\beta}^S f(x) + f(x - S) dx < \xi + \frac{\beta}{S}.$$

To maximize  $\int_{-\beta}^0 f(x) dx + \int_{S-\beta}^S |f(x) - 1/S| dx$  subject to this population constraint, we place all the population in the region

17. This choice simplifies the notation. The basic point is that there are three regions to consider:  $(-S, a) \cap (a + S + \beta, S)$ ,  $(a + \beta, a + S)$ , and  $(a, a + \beta) \cap (a + S, a + S + \beta)$ . These regions correspond to the three cases considered below.

$(-\beta, 0]$  to arrive at

$$\int_{-\beta}^0 f(x) dx + \int_{s-\beta}^s \left| f(x) - \frac{1}{S} \right| dx < \xi + \frac{\beta}{S} + \frac{\beta}{S}.$$

This completes the proof.

Q.E.D.

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