

## Financial markets in development, and the development of financial markets

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### Abstract

What is the relationship between markets and development? It is argued that markets promote growth, and that growth in turn encourages the formation of markets. Two models with endogenous market formation are presented to analyze this issue. The first examines the role that financial markets – banks and stock markets – play in allocating funds to the highest valued use in the economic system. It is shown that intermediation will arise under weak conditions. The second focuses on the role that markets play in supporting specialization in economic activity. The consequences of perfect competition in market formation are highlighted.

*Key words:* Development; Markets; Efficiency

*JEL classification:* E1; G1; O1

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### 1. Introduction

What is the relationship between the development of markets and economic development? It is argued here that markets – especially financial markets – play a central role in economic development and that economic development leads to the formation of new markets. In pursuing this thesis, the analysis will focus on five themes.

(1) Markets enhance growth to the extent that they serve to allocate resources to the place in the economic system where their social return is greatest. Markets, through the price signals and other information they provide, aid in

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this allocation. So does the risk sharing provided by (primarily financial) markets, since this allows agents to reallocate their savings toward more productive investments by eliminating idiosyncratic risk.

(2) Market formation permits increased specialization. As production technologies advance over time, they tend to require increasingly specialized inputs and yield increasingly specialized outputs. The exploitation of these technological advances requires markets so that agents can trade these specialized goods and services.

(3) Market structures affect agents' incentives to accumulate various types of physical and human capital, as well as other kinds of assets. This is true both because changes in market structure affect the perceived returns to various kinds of investments, and the risks associated with them.

(4) Market formation is an endogenous process. Arranging and effecting trades requires resource expenditures. Bankers, stockbrokers, insurance agents, realtors, placement agencies, and agents who enforce the terms of contracts make a living doing precisely this. But poor economies are less well placed to devote substantial resources to the trading process than are wealthier economies. Thus growth should lead to an increase in market activity, and this increase may in turn further stimulate growth.

(5) If there is competition in the provision of market services, this provision is likely to occur in a way that is perceived to be efficient by market participants. This has several implications for what kinds of market structures are likely to be observed.

While it may be taken as a truism that the trading opportunities provided by the marketplace are essential to growth, the empirical evidence is only available for financial markets. Goldsmith (1969), Jung (1986), Antje and Jovanovic (1993), and King and Levine (1993) document a positive correlation between a variety of measures of financial market activity and economic development. Economic history is also replete with examples illustrating the importance of financial markets for growth.<sup>1</sup>

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<sup>1</sup>Particularly striking are the experiences of the least well-known, and most impressive growth successes of the early 19th century – Belgium and Scotland. Both of these countries were distinguished primarily by the efficiency of their financial markets. According to Cameron (1967, pp. 94–7), 'in 1750 the per capita income of Scotland was no more than half that of England, but ... by 1845 it very nearly equalled England's. Given its many disadvantages, and few positive advantages for growth compared with its neighbors, the superiority of its banking system stands out as one of the major determining factors.' The differences in the level of development of financial markets between Scotland and England are illustrated by the fact that, in 1770, bank assets per capita were approximately equal in the two countries. In 1844, bank assets per capita were 2.5 times greater in Scotland than in England (Cameron, 1967). Similarly, Belgium had few obvious advantages for growth other than the developed state of its financial system. Yet Belgium was the great growth success of continental Europe in the first half of the 19th century (Cameron, 1967).

The economic importance of financial markets for growth derives from the fact that they fulfill several of the functions emphasized in the first three themes. Financial markets are the most prominent means, for instance, of channeling investment capital to its highest return uses. These markets also provide liquidity, and permit the efficient pooling of risk. Both of these activities alter the social composition of savings in a way that is (potentially) favorable to enhanced capital accumulation. Finally, financial markets foster specialization in entrepreneurship, entrepreneurial development, and the adoption of new technologies. They do this by making funds available to potential entrepreneurs for activities which – in developed economies – must typically be undertaken on a larger scale than any small number of individuals can readily afford.

The latter role of financial markets receives substantial attention from Hicks (1969) and North (1981). They argue that the distinguishing feature of the industrial revolution – compared with earlier times – was not particularly the development of new technologies. Indeed the steam engine and several other of the technological advances that played a prominent role in the industrial revolution were invented much earlier. Hicks and North argue that the industrial revolution was a revolution because, for the first time, the implementation of technical advances became a highly capital-intensive process. As a result, new technologies could be employed only by ‘tying up’ large-scale investments in illiquid capital for long periods. This implied inflexibility made the provision of liquidity for short-term needs essential. Moreover, again for the first time, the levels of investment required for the adoption of new technologies were large relative to the wherewithal of even the wealthiest individuals. This made the pooling of funds essential. In addition, as argued by North (1981), the provision of liquidity and the sharing of risk associated with financial market development substantially reduced the perceived costs of investing in innovation.

The importance of financial markets in permitting innovation and the implementation of new technologies has, of course, long been recognized. Bagehot (1873, pp. 3–4) argued that English success in development was due to the superiority of their financial markets:

We have entirely lost the idea that any undertaking likely to pay, and seen to be likely, can perish for want of money; yet no idea was more familiar to our ancestors, or is more common in most countries. A citizen of London in Queen Elizabeth’s time ... would have thought that it was no use inventing railways (if he could have understood what a railway meant), for you would have not been able to collect the capital with which to make them. At this moment, in colonies and all rude countries, there is no large sum of transferable money; there is no fund from which you can borrow, and out of which you can make immense works.

Another feature of economic organization that Hicks (1969) and North (1981) identify as being central to the development process is increased specialization.<sup>2</sup> By its very nature, increasing specialization in an economy implies that economic agents produce goods and services which they may not consume, and consume goods and services which they may not produce. In addition, it likely implies that producers will not be well-diversified in the absence of financial markets, and that therefore they will desire the risk-sharing services and access to external funding provided by such markets. Thus increasing specialization will require the support of a variety of trading institutions.

A final point raised both by Hicks and North is that there are important fixed costs associated with the formation of markets. Therefore, growth in the size of a potential market will reduce the costs to each participant of being active in that market. As an implication, a particular market may not become active until the economy has developed to the point where the market can sustain enough activity to make it 'cost-effective'. In other words 'threshold effects' will be observed in market formation.

The connection between financial intermediation and growth has been modeled recently by Greenwood and Jovanovic (1990), Bencivenga and Smith (1991), and Obstfeld (1994). Each of these papers has the feature that financial intermediation promotes growth by permitting a larger fraction of investment to be directed to activities with high (social) returns. In order to study the relationship between intermediation and growth, Section 2 of this paper uses a variant of the Bencivenga–Smith model. There intermediaries alter the social composition of savings in a way that is favorable to more productive, illiquid capital investment through liquidity provision (which in the model is simply a kind of insurance). As argued by Patrick (1966), Cameron (1967), McKinnon (1973), and Shaw (1973), this liquidity provision is growth-promoting.

Relative to the Bencivenga–Smith model, the model of Section 2 contains several innovations. First, by streamlining the structure several results are sharpened. In particular, intermediation is necessarily growth-enhancing (it was not necessarily growth-enhancing in Bencivenga and Smith, 1991). In addition, the formation of equity markets (rather than banks) need not be growth-enhancing. Conditions that are necessary and sufficient for equity markets to promote accelerated growth are derived.

Second, the framework is extended to allow for the endogenous formation of either banking or equity markets. Interestingly, while intermediation will arise endogenously under weak conditions, equity markets will never emerge if the costs of participating in them are no smaller than the costs of trading with banks. (This is true even when the presence of equity markets is growth

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<sup>2</sup>See Romer (1987) for a model of the relationship between increased specialization and growth.

promoting.) Situations which would allow banks and equity markets to coexist are also described.

Increasing specialization, market development, and growth is the subject of Section 3. There a model is presented where the adoption of new production technologies requires the use of specialized intermediate goods. The production of specialized intermediate goods in turn requires the support of trading institutions. Investment in institutional capital, however, is taken to be costly. Specifically, it is assumed that there are important fixed costs associated with opening and operating markets. The analysis allows for some uncertainty about the gains from specialization and the costs of market formation. The result is a ‘threshold effect’: markets open when the economy is wealthy enough to support them. Growth leads to the creation of markets, and the creation of markets increases the equilibrium rate of growth of an economy. In this latter respect the analysis has a flavor similar to Greenwood and Jovanovic (1990). Finally, since the business of making markets is modeled as being competitive, the equilibrium obtained is Pareto optimal. The work, here, builds on Townsend (1978) which stressed the consequences of perfect competition in the business of intermediating trade.

## **2. Financial markets, liquidity provision, and growth**

The first model considered is intended to demonstrate how the provision of liquidity by financial markets can promote growth. The analysis effectively embeds a version of the liquidity provision model of Diamond and Dybvig (1983) into a modified version of Diamond’s (1965) neoclassical growth model.

### *2.1. The environment*

The economy consists of an infinite sequence of two-period-lived overlapping generations. Each generation is comprised of a continuum of agents with unit mass. Agents born at each date are indexed by  $i \in [0, 1]$ , and time is denoted by  $t = 0, 1, \dots$ .

There is a single consumption good at each date. The consumption good is produced using intermediate inputs according to a constant-returns-to-scale production function. Intermediate goods, in turn, are produced using capital and labor as primary inputs.

Each young agent  $i$  produces a quantity of intermediate goods at  $t$  denoted  $x_t(i)$ . Agent  $i$  produces this good using his own labor input,  $l_t(i)$ , and a capital input  $k_t(i)$ . Suppose that labor is a nontraded factor of production, so that each agent utilizes only his own labor. This agent, in turn, is endowed with one unit of labor, which is supplied inelastically (labor generates no disutility). The

technology for producing intermediate goods is given by

$$x_t(i) = Ak_t(i)l_t(i)^{1-\delta}. \quad (1)$$

Finally, assume that only young agents are endowed with labor, and that capital depreciates completely in production. (The latter assumption is without any real loss of generality.)

Time  $t$  final consumption goods, denoted  $c_t$ , and the time  $t + 1$  capital stock  $k_{t+1}$  are produced using intermediate goods according to the technology

$$c_t + (k_{t+1}/R) = \left[ \int_0^1 x_t(i)^\theta di \right]^{1/\theta}, \quad (2)$$

with  $\theta < 1$ . Note that (2) allows one unit of current consumption to be converted into  $R$  units of future capital.

All young agents at  $t$  have identical ex ante preferences. Letting  $c_j$  denote age  $j$  consumption by a representative individual ( $j = 1, 2$ ), these preferences are given by the utility function

$$u(c_{1t}, c_{2t}; \phi) = - [(1 - \phi)c_{1t} + \phi c_{2t}]^{-\gamma/\gamma}, \quad (3)$$

with  $\gamma > -1$ . The variable  $\phi$ , in turn, is an individual specific, iid (across agents) preference shock. Its probability distribution is given by

$$\phi = \begin{cases} 0 & \text{with probability } 1 - \pi, \\ 1 & \text{with probability } \pi. \end{cases}$$

Agents are assumed to make a savings (portfolio) decision before observing their realization of  $\phi$ . There are two assets which can be held. One is capital. One unit of time  $t$  consumption placed into capital investment returns  $R$  units of capital at  $t + 1$ . However, if this capital investment is interrupted at  $t$ , no capital or consumption is received. Alternatively, each young agent has access to a technology for storing consumption goods. One unit of consumption stored at  $t$  returns  $n$  units either later in period  $t$  (that is, if the investment is interrupted at that date) or at  $t + 1$ . This specification of the menu of assets resembles that of Jacklin and Bhattacharya (1988).

## 2.2. Trade in good and factors

Producers of final consumption goods purchase a quantity  $x_t(i)$  of intermediate goods from agent  $i$  at  $t$ ;  $i \in [0, 1]$ . Let  $p_t(i)$  be the price charged for these inputs (in units of current consumption) by agent  $i$ . Since  $i$  is the only producer

of  $x_t(i)$ , he is modeled as being imperfectly competitive – that is, he does not take  $p_t(i)$  as given.

Final goods producers, however, are assumed to take  $p_t(i)$  as given. With current consumption as the numeraire, they therefore choose a schedule of intermediate inputs  $x_t(i)$  to maximize

$$\left[ \int_0^1 x_t(i)^\theta di \right]^{1/\theta} - \int_0^1 p_t(i)x_t(i)di.$$

Letting  $y_t \equiv \left[ \int_0^1 x_t(i)^\theta di \right]^{1/\theta}$ , the first-order condition for this problem has the form

$$p_t(i) = y_t^{1-\theta} x_t(i)^{\theta-1}. \tag{5}$$

Eq. (5) represents an inverse demand function for  $x_t(i)$  by final goods producers.

Young producers of intermediate goods obtain capital inputs in a competitive rental market, paying the rental rate  $\rho_t$  at  $t$ . Thus, young agent  $i$  chooses values for  $x_t(i)$  and  $k_t(i)$  to solve the problem

$$\max [p_t(i)x_t(i) - \rho_t k_t(i)], \tag{P.1}$$

subject to (1), (5), and  $l_t(i) = 1$ . This problem, in turn, can be transformed into

$$\max_{k_t(i)} \{y_t^{1-\theta} [A k_t(i)]^\theta - \rho_t k_t(i)\}, \tag{P.1'}$$

by substituting (1), (5), and  $l_t(i) = 1$  into (P.1). The problem (P.1') has the first-order condition

$$\theta y_t^{1-\theta} A^\theta k_t(i)^{\theta-1} = \rho_t. \tag{6}$$

*Equilibrium*

Since all young agents are symmetric (as goods producers), an equilibrium is sought where  $x_t(i) = x_t$  and  $k_t(i) = k_t, \forall i \in [0, 1]$ . Eqs. (1) and (2) then imply that

$$y_t = x_t = A k_t. \tag{7}$$

Substituting (7) into (6) yields

$$\rho_t = \theta A. \tag{8}$$

It remains to describe the (maximized) income of young agents, measured in current consumption. Let  $w_t(i)$  denote this income for agent  $i$ . Substituting (6) into the expression (P.1'), one obtains

$$w_t(i) = (1 - \theta)y_t^{1-\theta}[Ak_t(i)]^\theta. \quad (9)$$

Using  $k_t(i) \equiv k_t$  and (7) in (9) gives

$$w_t(i) = (1 - \theta)Ak_t \equiv w_t, \quad (10)$$

which holds for all  $t$ .

### 2.3. Savings behavior

The savings behavior of young agents depends on the kind of financial markets to which they have access. Three financial market structures will be considered: financial autarky, banking, and equity markets. For the present take financial market structure as exogenous; later, the formation of financial markets will be endogenized.

Each market structure assumes the same timing of activity. At the beginning of period  $t$ , young agents undertake the production activity just described. In doing so, they earn an income of  $w_t$ . These agents next decide how to allocate this income among the various assets available to them; of course this availability depends on the structure of financial markets. A savings/portfolio decision must be made by each agent *before*  $\phi$  is realized. This implies that no consumption (by young agents) will take place prior to making a savings decision, since it is not yet known by any agent whether young consumption will generate utility for them.

After savings/portfolio choices are made,  $\phi$  is realized for each young agent. Agents with  $\phi = 1$  wait until old age to liquidate assets and consume. Agents with  $\phi = 0$  value only young consumption, however. Hence they liquidate all their assets at the end of period  $t$  and consume the proceeds. Notice that this timing convention requires such consumption to occur before the next generation appears, and hence all young consumption must be done out of goods storage.<sup>3</sup> The timing structure is depicted in Fig. 1.

<sup>3</sup>In other words, intergenerational transfers are not possible. In this respect the timing conventions differ from Bencivenga and Smith (1991, 1992). The timing conventions employed are essentially drawn from Champ, Smith, and Williamson (1992).

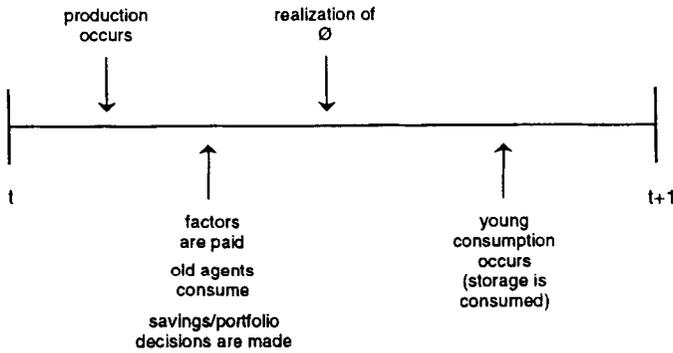


Fig. 1. Timing structure.

### 2.3.1. Financial autarky

When young agents are financially autarkic, they store goods and accumulate capital on their own behalf. If these agents are holding some capital and  $\phi = 0$ , this capital can no longer be rented (factor markets have closed – see Fig. 1) or sold, since there are no equity markets for transferring claims to ownership of capital. Moreover, if  $\phi = 0$  old-age consumption has no value, so it will be assumed that autarkic agents with  $\phi = 0$  simply lose their capital investment.<sup>4</sup> Thus, all young consumption must be financed by storage.

Let  $s_t^a$  be goods storage by an autarkic young agent at  $t$ , and let  $K_{t+1}^a$  be the value, in current consumption, of capital accumulation by this same agent. The return on goods storage is  $n$ , independent of when consumption occurs. The return on capital is zero if  $\phi = 0$ . If  $\phi = 1$ , for each unit of current consumption invested,  $R$  units of  $t + 1$  capital is received. This can be rented for  $\rho_{t+1}$  per unit, so the return on capital invested between  $t$  and  $t + 1$  is

$$R\rho_{t+1} = RA\theta. \tag{11}$$

The resource constraints for an autarkic young agent, then, are

$$s_t^a + K_{t+1}^a \leq w_t, \tag{12}$$

$$c_{1t} \leq ns_t^a, \tag{13}$$

$$c_{2t} \leq ns_t^a + (RA\theta)K_{t+1}^a. \tag{14}$$

<sup>4</sup>If capital had any 'scrap value' (say  $x$  per unit), young agents with  $\phi = 0$  would obviously scrap their capital before it could be rented. The formulation in the text is simply the limiting case of this situation as  $x \rightarrow 0$ .

The problem of a young agent is to solve

$$\max_{c_{1t}, c_{2t}, s_t^a, K_{t+1}^a} - [(1 - \pi)c_{1t}^{-\gamma} + \pi c_{2t}^{-\gamma}] / \gamma, \tag{P.2}$$

subject to (12)–(14).

This problem can be conveniently transformed as follows. Define  $q_t^a = K_{t+1}^a / w_t$  to be the fraction of an autarkic agent’s portfolio held in the form of capital. Then the problem (P.2) can be rewritten as

$$\max_{0 \leq q_t^a \leq 1} - w_t^{-\gamma} \{ (1 - \pi)[n(1 - q_t^a)]^{-\gamma} + \pi[n(1 - q_t^a) + RA\theta q_t^a]^{-\gamma} \} / \gamma. \tag{P.2'}$$

If there is an interior optimum, it satisfies

$$q_t^a = Q^a(RA\theta) \equiv [\lambda(RA\theta) - 1] / \{ [\lambda(RA\theta) - 1] + (RA\theta/n) \}, \tag{15}$$

where

$$\lambda(RA\theta) \equiv [\pi(RA\theta - n) / (1 - \pi)n]^{1/(1 + \gamma)}. \tag{16}$$

Apparently, there is an interior optimum iff  $\lambda(RA\theta) \geq 1$ . This condition is equivalent to

$$\pi RA\theta \geq n. \tag{17}$$

Henceforth, (17) is assumed to hold.<sup>5</sup> In this case, the savings/portfolio behavior of young agents is completely summarized by the function  $Q^a(-)$ .

**2.3.2. Banking**

A discussion of savings behavior in the presence of banks necessitates a description of what banks do. Banks are assumed to take deposits (from young agents), to invest in capital, and to hold goods in storage. As noted previously, young consumption must be financed out of storage: from a bank’s perspective, assets stored constitute reserves against ‘early’ withdrawals.

Having accepted a deposit, a bank promises to pay a time  $t$  depositor who withdraws at  $t$  (one who has  $\phi = 0$ )  $r_{1t}$  per unit withdrawn. If the same agent withdraws at  $t + 1$  (has  $\phi = 1$ ), he receives  $r_{2t}$  per unit deposited. Suppose that

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<sup>5</sup>If (17) fails, the optimal  $q_t^a = 0$ . Then after one period (the autarkic version of) this economy has no capital, and a steady state equilibrium with no real activity is reached.

at the time withdrawals occur, it is too late to undertake further goods storage. This implies that only agents with  $\phi = 0$  withdraw ‘early’.

Banks, then, can be viewed as announcing  $(r_{1t}, r_{2t})$  pairs at  $t$ . It is assumed that banks are Nash competitors, so that these announcements are made taking the interest rates offered by other banks at  $t$  as given.

Banks are identified with generations,<sup>6</sup> so that the resource constraints faced by a bank are as follows. Anticipating the result that all young-period savings (which here equal  $w_t$ ) are deposited,<sup>7</sup> an active bank receives per person deposits of  $w_t$ . Letting  $s_t^b$  denote (per depositor) goods storage by the bank and  $K_{t+1}^b$  denote (per depositor) capital investment by the bank, the bank faces the constraints

$$s_t^b + K_{t+1}^b \leq w_t, \tag{18}$$

$$(1 - \pi)r_{1t}w_t \leq ns_t^b, \tag{19}$$

$$\pi r_{2t}w_t \leq R\rho_{t+1}K_{t+1}^b = (RA\theta)K_{t+1}^b. \tag{20}$$

Eqs. (19) and (20) reflect the fact that a bank serving a large number of depositors has a fraction  $1 - \pi$  ( $\pi$ ) of their depositors withdrawing at  $t$  ( $t + 1$ ). (19) and (20) also assume that the bank liquidates all its reserves (goods in storage) at  $t$ . This will be optimal for them if  $R\rho_{t+1} = RA\theta > n$ . This condition, of course, is implied by (17).

Banks compete against each other for depositors. This competition implies that bank choices  $(r_{1t}, r_{2t}, s_t^b, K_{t+1}^b)$  must be selected to maximize the expected utility of a representative depositor; that is, to solve the problem

$$\max - w_t^{-\gamma} [(1 - \pi)r_{1t}^{-\gamma} + \pi r_{2t}^{-\gamma}] / \gamma, \tag{P.3}$$

subject to (18)–(20) and the obvious nonnegativity constraints.

The problem (P.3) can be transformed as follows. Define  $q_t^b \equiv K_{t+1}^b/w_t$  to be the fraction of the bank’s portfolio invested in capital. Then (19) and (20) can be rewritten as

$$r_{1t} \leq n(1 - q_t^b)/(1 - \pi), \tag{19'}$$

$$r_{2t} \leq (RA\theta)q_t^b/\pi. \tag{20'}$$

<sup>6</sup>See Bencivenga and Smith (1991) for further discussion.

<sup>7</sup>As in Diamond and Dybvig (1983) and all related models, this will be an equilibrium outcome if banks are not regulated.

The bank’s problem can be written as maximizing the expression in (P.3), subject to (19’) and (20’).

The solution to this problem sets

$$q_t^b = Q^b(R\rho_{t+1}) = Q^b(RA\theta), \tag{21}$$

where

$$Q^b(RA\theta) \equiv \eta(RA\theta)/[1 + \eta(RA\theta)], \tag{22}$$

$$\eta(RA\theta) = \pi(RA\theta/n)^{-\gamma/(1+\gamma)}/(1 - \pi). \tag{23}$$

The function  $Q^b(-)$  completely describes savings behavior when banks operate.

Observe that the function  $Q^b(-)$  is decreasing in  $\gamma$ ; that is, the more risk-averse agents are the less is saved in the form of illiquid capital. Note further that as  $\gamma \rightarrow -1$  (as agents become nearly risk-neutral),  $Q^b(-) \rightarrow 1$ , so that all assets are invested in long-term capital. When  $\gamma = 0$  (preferences are logarithmic),  $Q^b(-) \equiv \pi$ . Since in general Eqs. (19’) and (20’) will hold with equality, these observations make it apparent that  $r_{1t} \geq (<) n$  iff  $\gamma \geq (<) 0$ . If agents are more (less) risk-averse than the logarithmic preference case, then they desire a return in excess of (less than)  $n$  in the event of early withdrawal. This corresponds to a willingness to accept a yield less than (in excess of)  $RA\theta$  if assets are held ‘to maturity’. Of course, in either case, banks are exploiting the law of large numbers in order to provide insurance against adverse realizations of  $\phi$ .

An important question concerns how banks affect the fraction of young savings that are placed in the form of capital. This question is answered by Proposition 1.

*Proposition 1.* (a)  $Q^b(RA\theta) > Q^a(RA\theta)$  holds iff

$$\pi/(1 - \pi) > [\pi/(1 - \pi)]^{1/(1+\gamma)} [(RA\theta - n)/RA\theta]^{1/(1+\gamma)} - (n/RA\theta)^{1/(1+\gamma)}. \tag{24}$$

A sufficient condition for (24) is that

$$\gamma(\pi - 0.5) \geq 0.$$

(b)  $Q^b(RA\theta) > \pi Q^a(RA\theta)$  always holds.

The proof of Proposition 1 appears in the Appendix. Part (a) of the proposition states when the improvement in risk sharing attained via intermediation results in a larger fraction of the ‘risky asset’ (capital) being held in the consolidated portfolio of banks and young savers. Part (b) of the proposition states

that the proportion of saving *maturing* in the form of long-term capital must be unambiguously greater in the presence than in the absence of intermediation. (Recall that in autarky the fraction  $\pi$  of long-term investment projects will be lost.)

### 2.3.3. *Equity markets*

Again, in order to describe savings behavior in the presence of equity markets, it is necessary to provide a description of how equity markets operate. To this end, assume that, after each agent's value of  $\phi$  is known at  $t$ , an equity market opens in which agents with  $\phi = 0$  sell claims to capital in process to agents with  $\phi = 1$  in exchange for claims to their storage (this is the description of equity markets in the Diamond–Dybvig model originally given by Jacklin, 1987). Let  $z_t$  be the number of units of storage that must be exchanged for a unit of capital (that is,  $z_t$  is the relative price of capital at  $t$  in the equity market).

Agents who know equity markets will operate at  $t$  choose a storage level,  $s_t^e$ , and a capital investment  $K_{t+1}^e$  at  $t$  in order to solve the problem

$$\max - [(1 - \pi)c_{1t}^{-\gamma} + \pi c_{2t}^{-\gamma}]/\gamma, \tag{P.4}$$

subject to

$$s_t^e + K_{t+1}^e \leq w_t, \tag{25}$$

$$c_{1t} \leq ns_t^e + nz_t K_{t+1}^e, \tag{26}$$

$$c_{2t} \leq (R\rho_{t+1})[K_{t+1}^e + (s_t^e/z_t)] = (RA\theta)[K_{t+1}^e + (s_t^e/z_t)]. \tag{27}$$

As before, let  $q_t^e \equiv K_{t+1}^e/w_t$  be the fraction of a young agent's portfolio held in capital. It is easy to show that the optimal choice of  $q_t^e$  satisfies

$$q_t^e = \begin{cases} 1 & \text{if } z_t > 1, \\ 0 & \text{if } z_t < 1, \end{cases} \tag{28}$$

and that  $q_t^e \in [0, 1]$  if  $z_t = 1$ .

### *Equity market equilibrium*

Young agents who must liquidate their long-term capital investment will supply this capital inelastically in the equity market. The supply of capital at  $t$  is therefore given by  $(1 - \pi)q_t^e w_t$ . The demand for capital in this market is  $\pi(1 - q_t^e)w_t/z_t$  if  $RA\theta/n > z_t$ , and is zero otherwise. From this observation and (28) it is apparent that an equilibrium in the equity market requires that  $z_t = 1$ ,

and that

$$(1 - \pi)q_t^e = \pi(1 - q_t^e)/z_t = \pi(1 - q_t^e). \tag{29}$$

Thus,  $q_t^e = \pi$  if equity markets are active at  $t$ .

### 2.4. General equilibrium

A general equilibrium for each of these three financial market structures will now be characterized.

#### 2.4.1. Financial autarky

Under financial autarky, young agents invest  $Q^a(RA\theta)w_t$  in capital. A fraction  $(1 - \pi)$  of this investment is liquidated before  $t + 1$  by agents who have  $\phi = 0$ . Hence only the fraction  $\pi$  of this investment translates into the time  $t + 1$  capital stock,  $k_{t+1}$ . Therefore

$$k_{t+1} = \pi RQ^a(RA\theta)w_t, \tag{30}$$

since  $R$  units of date  $t + 1$  capital are received per unit of unliquidated capital investment at  $t$ .

Substituting (10) into (30) yields the equilibrium law of motion for the (productive) capital stock:

$$k_{t+1} = (1 - \theta)\pi RAQ^a(RA\theta)k_t. \tag{31}$$

Thus, the growth rate of the capital stock and output is  $(1 - \theta)RA\pi Q^a \times (RA\theta) \equiv \sigma^a$ .

#### 2.4.2. Banking

When banks operate, no capital is liquidated prior to becoming productive. Hence all time  $t$  capital investment –  $Q^b(RA\theta)w_t$  – translates into time  $t + 1$  capital and therefore

$$k_{t+1} = RQ^b(RA\theta)w_t = (1 - \theta)RAQ^b(RA\theta)k_t. \tag{32}$$

The growth rate of the economy in the presence of banks is  $(1 - \theta)RAQ^b(RA\theta) \equiv \sigma^b$ .

*Proposition 2. The growth rate of an economy with banks exceeds that of a financially autarkic economy.*

*Proof.* The statement follows from  $Q^b(RA\theta) > \pi Q^a(RA\theta)$ , as shown in Proposition 1.

Thus banks necessarily raise the rate of growth. This occurs for one or both of the following reasons: either banks shift savings into capital [if  $Q^b(RA\theta) > Q^a(RA\theta)$ ], or at a minimum, they prevent ‘premature’ liquidation of capital [ $Q^b(RA\theta) > \pi Q^a(RA\theta)$ ].

### 2.4.3. Equity markets

In the presence of (active) equity markets, a fraction  $\pi$  of savings is placed in capital investments, and none of these are liquidated ‘prematurely’. Therefore

$$k_{t+1} = \pi R w_t = (1 - \theta) R A \pi k_t. \quad (33)$$

The growth rate of an economy with equity markets is given by  $\pi(1 - \theta) R A \equiv \sigma^e$ .

*Proposition 3.* (a)  $\sigma^e > \sigma^a$ . (b)  $\sigma^e > \sigma^b$  holds if and only if  $\gamma > 0$ .

Proposition 3 is proved in the Appendix. It asserts that equity markets increase the growth rate of an economy relative to autarky. Equity markets increase the growth rate of an economy relative to banks if and only if agents are relatively risk-averse. In particular, in the presence of banks, the more risk-averse agents are the less of their savings is allocated to the capital investment. This effect, which is growth-reducing, is absent in the presence of equity markets. Thus growth is more rapid in the presence of equity markets if (and only if) agents are sufficiently risk-averse – and specifically, more risk-averse than the case of logarithmic utility.

### 2.5. Endogenous formation of financial institutions

The preceding discussion took the structure of financial markets as exogenous. It will now be illustrated how the formation of financial markets can be endogenized.

In order to prevent financial markets from forming immediately, it is clearly necessary either that (a) there be costs to forming and operating in them or (b) regulations or the legal environment inhibit their development. Certainly in practice (b) is quite important.<sup>8</sup> Here it will be shown how costs of operating in financial markets can lead to their endogenous formation at some finite date.

Suppose that agents suffer a utility loss  $e$  from the effort expended to contact banks, and a utility loss  $e'$  from the effort expended to be active in equity

<sup>8</sup>See, for instance, the discussions in Cameron (1967), McKinnon (1973), and Shaw (1973).

markets.<sup>9</sup> This effort will be expended, then, if and only if the utility gain from financial market activity justifies its expenditure.

Let  $v^a(RA\theta)w_t^{-\gamma}$  denote a financially autarkic agent's indirect utility; that is, an autarkic agent's maximized (expected) utility is given by  $v^a(RA\theta)w_t^{-\gamma}$  at  $t$ . This indirect utility function is given by

$$v^a(RA\theta) \equiv - [(1 - \pi)\{n[1 - Q^a(RA\theta)]\}]^{-\gamma} + \pi\{n[1 - Q^a(RA\theta)] + RA\theta Q^a(RA\theta)\}^{-\gamma}/\gamma. \tag{34}$$

Similarly, let  $v^b(RA\theta)w_t^{-\gamma}$  [ $v^c(RA\theta)w_t^{-\gamma}$ ] denote the maximized expected utility of an agent using banks [equity markets] at  $t$ , exclusive of the utility cost of making a transaction. Then

$$v^b(RA\theta) \equiv - [(1 - \pi)\{n[1 - Q^b(RA\theta)]/(1 - \pi)\}]^{-\gamma} + \pi\{(RA\theta)Q^b(RA\theta)/\pi\}^{-\gamma}/\gamma, \tag{35}$$

$$v^c(RA\theta) \equiv - [(1 - \pi)n^{-\gamma} + \pi(RA\theta)^{-\gamma}]/\gamma. \tag{36}$$

Since setting  $q_t^b = \pi$  is always a feasible choice for agents who use banks (but is chosen iff  $\gamma = 0$ ), clearly  $v^b(RA\theta) \geq v^c(RA\theta) \forall (RA\theta)$ . Strict inequality holds unless  $\gamma = 0$ . Similarly,  $v^b(RA\theta) > v^a(RA\theta) \forall (RA\theta) \geq n$ .

Agents who have a choice, then, will make use of banks at  $t$  if and only if

$$v^b(RA\theta)w_t^{-\gamma} - e \geq v^a(RA\theta)w_t^{-\gamma}, \tag{37a}$$

$$v^b(RA\theta)w_t^{-\gamma} - e \geq v^c(RA\theta)w_t^{-\gamma} - e'. \tag{37b}$$

If  $e' \geq e$ , as seems natural to assume, (37b) holds  $\forall w_t$ . Thus equity markets can only be observed as an endogenous outcome if  $e > e'$ . While this result may seem surprising, it is consistent with the casual observation that extensive government intervention is typically required to get equity markets to operate in developing countries (Fry, 1988, pp. 258–259).

Assuming  $e' \geq e$ , only a banking system will ever operate in this economy. Define  $t^*$  to be the first date at which banks operate. If  $t^* > 0$ , the date  $t^*$  must

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<sup>9</sup>Having the costs of financial market activity be utility costs (rather than direct resource losses) simplifies the analysis. It is also consistent with how the costs of 'trips to the bank' are usually treated when Baumol–Tobin type money demand analyses are integrated into more general equilibrium models. In particular, these costs typically do not show up as resource losses anywhere else in the system (introspection on how IS-LM/Aggregate-Supply models are usually treated will indicate this).

satisfy the inequality

$$w_{t^*} \geq \{e/[v^b(RA\theta) - v^a(RA\theta)]\}^\gamma > w_{t^*-1}. \quad (38)$$

Substitution of (10) into (38), along with the observation that the capital stock grows at the rate  $\sigma^a$  in a financially autarkic economy gives

$$(1 - \theta)A(\sigma^a)^{t^*} \geq \{e/[v^b(RA\theta) - v^a(RA\theta)]\}^\gamma/k_0 \geq (1 - \theta)A(\sigma^a)^{t^*-1} \quad (39)$$

as the condition that determines  $t^*$ . Evidently, there are four possibilities.

*Case 1.* Suppose  $k_0$  satisfies

$$(1 - \theta)Ak_0 < \{e/[v^b(RA\theta) - v^a(RA\theta)]\}^\gamma, \quad (40)$$

and that  $\sigma^a > 1$ . Then there is a unique, finite date  $t^* > 0$  when banks open. Agents are financially autarkic for  $t = 0, 1, \dots, t^* - 1$ . At  $t^*$  the growth rate of the economy increases by the factor  $\sigma^b/\sigma^a > 1$ .

*Case 2.* Suppose  $k_0$  violates (40), and that  $\sigma^b \geq 1$ . Then financial intermediaries operate at all dates, and the growth rate of the economy is always  $\sigma^b$ .

*Case 3.* Suppose  $k_0$  violates (40), and that  $\sigma^b < 1$ . Then  $t^* = 0$ , and there will be a finite date,  $\bar{t}$ , at which banks close. This date is determined by the inequality

$$k_0(1 - \theta)A(\sigma^b)^{\bar{t}-1} \geq \{e/[v^b(RA\theta) - v^a(RA\theta)]\}^\gamma > (1 - \theta)A(\sigma^b)^{\bar{t}}k_0. \quad (41)$$

*Case 4.* If  $k_0$  satisfies (40) and  $\sigma^a \leq 1$ ,  $t^* = \infty$ . Banks never open, and the growth rate of the economy is always  $\sigma^a$ .

If  $\gamma > 0$  holds, this economy would grow faster in the presence of equity markets than of banks, even though equity markets never form. This is because, as agents become more risk-averse, banks will hold increasingly larger fractions of their portfolios in the form of ‘reserves’ to better service the liquidity needs of their depositors. While this operates to improve depositor welfare (period-by-period), it also works to reduce the growth rate of the economy. When  $\gamma > 0$  holds, this effect is large enough so that banking reduces the rate of growth of the economy relative to what would occur with equity markets. Such a result indicates that a government which attached sufficient ‘weight’ to the utility of future generations might choose to subsidize the formation of equity markets. This kind of subsidization is often observed in developing countries (Fry, 1988). Also, the economy would attain a high growth rate sooner (assuming  $0 < t^* < \infty$ ) if banks formed before  $t^*$ . This also would require some government subsidization of the banking system (in its early stages). Government

subsidization of banking systems is also commonly observed.<sup>10</sup> This kind of financial market subsidization is effectively advocated by McKinnon (1973) and Shaw (1973). Here we note that such subsidization effectively represents a transfer from current to future generations.

## 2.6. Discussion

As mentioned in the introduction, financial markets promote economic growth by directing resources to their highest return uses. In the framework developed here, savings earn their highest return in illiquid capital investments. The provision of liquidity by financial markets limits the exposure of savers to idiosyncratic risks, and prevents the costly premature liquidation of long-term capital investment. The result is that a higher fraction of savings is channeled into such investments that actually mature.

It is the latter effect on saving, rather than liquidity provision per se, that is growth-promoting. In order to see this, it suffices to consider the model described with only a single asset. In particular, suppose that a unit invested at  $t$  returns  $R$  units of capital if held until  $t + 1$ , and has a scrap value of  $n$  (with  $0 < n < RA\theta$ ) if liquidated in period  $t$ . Otherwise the model is unaltered. It is straightforward to show that the presence of intermediaries in this environment increases growth if and only if

$$(n/RA\theta)^{\gamma/(1+\gamma)} > 1.$$

This condition holds only when  $\gamma < 0$ . But when  $\gamma < 0$ , agents desire a return on early withdrawals of less than  $n$ . In other words, intermediation increases growth if and only if intermediaries reduce the return (relative to autarky) to agents with  $\phi = 0$ . Such intermediaries are *not* insuring against this event, and do not provide liquidity in the sense of Diamond and Dybvig (1983).

Intermediation may also affect growth by changing savings behavior. This effect is absent in the current model where the supply of savings from young agents is inelastic. Intermediation may affect the supply of savings for a variety of reasons: increases in the rate of return, reductions in risk, etc. The impact such considerations have on savings is ambiguous. Devereux and Smith (1994) point out that reductions in risk may reduce savings rates under certain specifications of preferences (Taub, 1989, also partially addresses this point). For example, in

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<sup>10</sup>See Patrick (1966) for an explicit discussion of this point in the context of Japan. Parenthetically, another potential motivation for governments to subsidize early bank formation, or even equity market formation, could arise whenever a government wishes to tax capital income. In this situation early bank – or ultimate equity market formation – can enhance the tax base, and hence might be viewed as desirable by the government.

Bencivenga and Smith (1991), improved liquidity provision can lead to a fall in aggregate savings. As demonstrated there, growth can still increase provided that the effects intermediation has in reallocating savings toward long-term capital investments dominate the ones it has in reducing savings rates.

It is also possible to use models in the class at hand to investigate how various kinds of government interventions in financial markets affect capital accumulation. Bencivenga and Smith (1992) consider the situation of a government monetizing a deficit in an economy like the one here, but in which money plays the role of bank reserves. They consider when a government will want to impose reserve requirements. Such requirements, of course, can raise the inflation tax base, but are also detrimental to capital accumulation. As Bencivenga and Smith show, it will typically be optimal for the government to interfere in the financial system in some way, and the optimal degree of interference increases with the size of the government deficit. Such a result mirrors heuristic arguments made by McKinnon (1982).

Finally, the model investigated here has no role for equity markets (if  $e' \geq e$ ), and certainly will not permit banks and equity markets to (permanently) coexist. However, such a coexistence would be possible if several features of the model in Bencivenga, Smith, and Starr (1996) were integrated into the model of this section. Bencivenga, Smith, and Starr present an economy in which there are several technologies for converting current output into future capital. These technologies differ by gestation period (incorporating a 'time-to-build' aspect) and the amount of capital ultimately received. Long-gestation capital investments must be 'rolled over' in secondary capital (equity) markets. Bencivenga, Smith, and Starr also allow for transactions in these markets to be costly, and investigate how changes in secondary market transactions costs (changes in the liquidity of these markets) affect capital accumulation. Introducing long-gestation capital into the model presented here would allow banks and equity markets to operate simultaneously, if the lifespan of a bank and a generation coincide. In particular, as is the case in the model just presented, banks could be expected to arise in order to provide insurance against the necessity of early asset liquidation. In addition, there would be a role for equity markets where claims to long-term capital investments could be traded. Such a synthesis would permit an investigation of the simultaneous interaction between banks and equity markets, and of the impact this interaction has on real growth.

### **3. Markets and resource allocation**

As economies develop, economic activity has tended to become increasingly specialized. Highly specialized activity requires the support of trading structures. Organizing such structures is a costly process, however. Consequently, an economy will be better disposed to undertake market building activities when its

income level is high rather than low. This observation suggests a nexus between market formation and economic development: economic development promotes the formation of markets, which in turn spurs further economic development.

A stylized model of such a process is presented in this section. Here the operation of more advanced production technologies requires the input of specialized intermediate goods. The production of these goods requires the formation of a supporting market. As suggested by Hicks (1969) and North (1981), there are fixed costs associated with establishing this market, and it is possible that these costs are not known with certainty in advance of setting it up. The date at which the market opens depends on the gains from specialization (which again are allowed to be random), the probability distribution of the costs of market formation, and the initial wealth of the economy.

### 3.1. *The environment*

Consider a continuous time economy populated by a set of infinitely-lived agents. These agents are divided into  $J$  types ( $J < \infty$ ), with type indexed by  $j = 1, \dots, J$ . The measure of type  $j$  agents is denoted  $\mu_j$ ; for simplicity let  $\sum \mu_j = 1$ .

An individual of type  $j$  can produce a single final good by using either of two technologies. Let  $x_j(t)$  be the amount of an intermediate good of type  $j$  (that is, produced by type  $j$  agents) used in final goods production, and let  $y_j(t)$  be the final goods production of type  $j$  agents. The first technology lets agents produce autarkically, in which case

$$y_j(t) \leq ix_j(t). \tag{42}$$

Alternatively, the second technology lets type  $j$  agents make use of intermediate inputs produced by other agents. In this case

$$y_j(t) \leq rJ^{(\theta-1)/\theta} \left[ \sum_{l=1}^J x_l(t)^\theta \right]^{1/\theta},$$

with  $0 < \theta < 1$ . This latter technology obviously requires trade in intermediate inputs, or in other words, its use requires market activity.

The return on the second technology is determined by the parameter  $r$ . This parameter is assumed to be unknown prior to the opening of market activity. More specifically,  $r$  is a random variable drawn from the set  $R = \{r_1, \dots, r_H\}$  at the date the market first opens. The elements of  $R$  are indexed so that  $r_H > r_{H-1} > \dots > r_1$ , and let  $\phi_h \equiv \text{prob}(r = r_h)$ . It is assumed that  $r_1 > i$ , so that the second technology is unambiguously ‘more advanced’ than the first.

Intermediate goods are produced using capital, which is assumed not to depreciate. Letting  $\kappa_j(t)$  denote the quantity of capital input employed by type  $j$  agents at  $t$ , intermediate goods are produced according to

$$x_j(t) \leq \kappa_j(t). \tag{44}$$

Further, let  $c_j(t)$  denote the time  $t$  consumption of a type  $j$  agent. Then this agent obtains a lifetime expected utility level of

$$E \int_0^\infty [\ln c_j(t)] e^{-\beta t} dt,$$

where  $E$  denotes the expectations operator. It is assumed that  $i > \beta$ . This condition implies that the marginal product of capital always lies above the rate of time preference, and so guarantees that sustained growth will occur as in Jones and Manuelli (1990) or Rebelo (1991). Also, let individual  $j$  begin life with  $k_j(0) > 0$  units of capital.

Finally, following Townsend (1978) and Greenwood and Jovanovic (1990), it will be assumed that trading arrangements are costly to establish. Suppose that there is some once-and-for-all fixed cost associated with setting up a market (for instance, the costs of building a trading center, or establishing accounting and communication systems, or drawing up legal contracts, etc.). In the absence of diminishing returns to market size, the most efficient form of trade is to have a single agent intermediate for others – or to establish a market. Denote the fixed cost by  $\gamma$  and let it be incurred in terms of capital. This cost is unknown prior to opening the market. In particular, let  $\gamma$  be a random variable which is drawn at the time the market first opens from the set  $\Gamma = \{\gamma_1, \dots, \gamma_N\}$  according to the probability distribution  $\text{prob}(\gamma = \gamma_n) = \pi_n$  for  $n = 1, \dots, N$ .

### 3.2. Market equilibrium

The following questions will now be addressed: (i) Will a market ever operate and, if it does, when will its operation begin? (ii) How are resources allocated before and after the market opens? In particular, how will the costs of market formation be allocated across agents in the economy? (iii) Is it possible for the economy to operate efficiently, even though markets are initially absent? In order to consider these issues, it is convenient to begin by analyzing what will happen after the market opens in this economy.

#### 3.2.1. The post-market economy

Suppose that a market opens at  $t^*$ . Immediately after the market opens, let type  $j$  agents have  $k_j(t_+^*)$  units of capital. The existence of a market allows agents to trade goods (or factors) freely at this stage.

It is easy to characterize an equilibrium in the spot markets for intermediate and final goods at any date  $t \geq t^*$ . Suppose that capital can be rented for  $\rho(t)$  units of final output at  $t$ , and that intermediate goods of variety  $l$  can be bought or sold for  $p_l(t)$  units of final output at  $t$ . Then an intermediate goods producer of type  $j$  will choose a capital input,  $\kappa_j(t)$ , and an output level  $x_j(t)$  to maximize  $p_j(t)x_j(t) - \rho(t)\kappa_j(t)$ , subject to (44). Clearly, then,  $p_j(t) = \rho(t)$  must hold  $\forall j, \forall t \geq t^*$ . Similarly, a final goods producer will choose a schedule  $\{x_l(t)\}$  of intermediate inputs to solve the problem

$$\max rJ^{(\theta-1)/\theta} \left[ \sum x_l(t)^\theta \right]^{1/\theta} - \sum p_l(t)x_l(t).$$

Evidently,  $x_1(t) = x_2(t) = \dots = x_j(t) \forall t \geq t^*$  must hold, as must  $\rho(t) \equiv r$ .

Now consider the intertemporal consumption–saving problem faced by agent  $j$ . Since production activity generates zero profits, all income for this agent derives from his holdings of capital, which earn the rate of return  $r$  at each date. Thus at  $t^*$  agent  $j$ 's problem is

$$\max_{c_j(t), k_j(t)} \int_{t^*}^{\infty} [\ln c_j(t)] e^{-\beta(t-t^*)} dt, \tag{P.5}$$

subject to

$$\dot{k}_j(t) = rk_j(t) - c_j(t), \tag{45}$$

with  $k_j(t_+^*)$  taken as given. Let  $V[k_j(t_+^*); r]$  denote the maximized objective function for this problem.

It is straightforward to show that the problem (P.5) is solved by setting

$$c_j(t) = \beta k_j(t_+^*) e^{(r-\beta)(t-t^*)}, \tag{46}$$

$$k_j(t) = k_j(t_+^*) e^{(r-\beta)(t-t^*)}, \tag{47}$$

for  $t \geq t^*$ . In addition, it is easy to demonstrate that

$$\beta V[k_j(t_+^*); r] = \ln \beta + \ln k_j(t_+^*) + (r - \beta)/\beta. \tag{48}$$

### 3.2.2. The pre-market economy

Let some individual in this economy assume the task of establishing the market. Clearly whoever does so will want to charge market participants some sort of fee for this service. Moreover, since any agent can undertake to establish a market, this activity can generate no surplus in equilibrium.

The marginal cost to the market-maker of allowing an extra agent to participate in the market is zero. There is also an advantage to having all types of agents participate in the market from its inception, since each producer would like the largest possible variety of intermediate goods to be available on the market.<sup>11</sup> Thus all agents will be active in the market at each date that it operates.

In addition, since the cost of operating the market is not known in advance, the market-maker may wish to condition fees for market participation on the realized cost,  $\gamma_n$ . Let  $f_{jn}$  be the fee that is charged to type  $j$  agents should the event  $\gamma = \gamma_n$  transpire.

A potential market-maker announces a date,  $t^*$ , at which the market will begin operation, and a fee schedule  $\{f_{jn}\}$  (with fees charged when market operation begins). These announcements are constrained by two factors. First, the announcements must induce all agents to enter the market voluntarily at  $t^*$  in the face of the fee schedule  $\{f_{jn}\}$ . That is, the market-maker is constrained by the utility maximization of the potential market participants. Second, since there is free entry into market formation, an equilibrium announcement must leave no other potential market-maker an incentive to announce an alternative plan that can draw away some market participants. In other words, the announcements  $t^*$  and  $\{f_{jn}\}$  must constitute Nash equilibrium announcements. An immediate consequence is that an equilibrium fee schedule must generate zero profits, so that<sup>12</sup>

$$\sum_{j=1}^J \mu_j f_{jn} = \gamma_n, \quad n = 1, \dots, N. \tag{49}$$

Since equilibrium announcements are constrained by how market participants respond to them, it is necessary to begin by analyzing the optimal behavior of these participants. In particular, it is necessary to know the date at which agent  $j$  will wish to enter the market when faced with the fee schedule  $\{f_{jn}\}_{n=1}^N$ . This issue is now considered.

Over the time interval  $[0, t^*)$  agent  $j$ 's capital stock will accumulate according to

$$\dot{k}_j(t) = ik_j(t) - c_j(t). \tag{50}$$

<sup>11</sup>Suppose that at  $t$ , there are  $s$  ( $0 < s < J$ ) types of agents who are excluded from the market. Then it is easy to show that  $\rho(t) = r[(J - s)/J]^{(1-\theta)/\theta} < r$  holds at  $t$ . This is clearly undesirable to market participants. Moreover, for a zero entry fee, the excluded agent types cannot be made worse off by entering the market economy. Thus there is no gain to excluding any agents from the market at any date at which it is open.

<sup>12</sup>Eq. (49) must hold in each state to prevent a potential market-maker from ever becoming bankrupt.

Just prior to  $t^*$  the agent will have  $k_j(t^*)$  units of capital. After paying the fee  $f_{jn}$  in state  $n$ , agent  $j$  will of course be left with  $k_j(t^*_+) = k_j(t^*) - f_{jn}$  units of capital. Then agent  $j$  will choose a consumption schedule  $c_j(t)$ , a capital accumulation program  $k_j(t)$ , and an optimal date of market entry  $t^*$  (which will be the same for all agents in equilibrium), in order to solve the problem

$$\max \int_0^{t^*} [\ln c_j(t)] e^{-\beta t} dt + e^{-\beta t^*} \sum_h \sum_n \phi_h \pi_n V[k_j(t^*) - f_{jn}; r_h], \tag{P.6}$$

subject to (50). It is easy to show (see, for instance, Kamien and Schwartz, 1981, Section 11) that the solution to this maximization problem satisfies

$$c_j(t) = \beta e^{(i-\beta)t} [k_j(0) - e^{-it^*} k_j(t^*)] / [1 - e^{-\beta t^*}], \tag{51}$$

$$(1 - e^{-\beta t^*}) e^{(\beta-i)t^*} = [k_j(0) - e^{-it^*} k_j(t^*)] E[k_j(t^*) - f_j]^{-1}, \tag{52}$$

and

$$\begin{aligned} & (i/\beta) E\{k_j(t^*)/[k_j(t^*) - f_j]\} - E \ln [k_j(t^*) - f_j] - \ln \{E[k_j(t^*) - f_j]^{-1}\} \\ & = E(r/\beta). \end{aligned} \tag{53}$$

For future reference, define

$$\begin{aligned} & W[k_j(t^*), t^*, f_{j1}, \dots, f_{jN}; k_j(0)] \\ & \equiv \int_0^{t^*} \ln \{ \beta e^{(i-\beta)t} / [1 - e^{-\beta t^*}] \} e^{-\beta t} dt \\ & \quad + \int_0^{t^*} \ln [k_j(0) - e^{-it^*} k_j(t^*)] e^{-\beta t} dt + e^{-\beta t^*} \sum_h \sum_n \phi_h \pi_n V[k_j(t^*) - f_{jn}; r_h]. \end{aligned} \tag{54}$$

This equation gives agent  $j$ 's lifetime expected utility as a function of  $k_j(t^*)$ ,  $t^*$ ,  $f_{j1}, \dots, f_{jN}$ ,  $k_j(0)$ , and the probability distributions of  $r$  and  $\gamma$ , given that the agent follows an optimal plan in the pre-market economy, enters the market economy at date  $t^*$  with  $k_j(t^*)$  units of capital, and behaves optimally thereafter in the market economy. Notice that Eqs. (52) and (53) determine  $k_j(t^*)$  and  $t^*$  jointly as a function of the values  $\{f_{j1}, \dots, f_{jN}\}$ . It is, of course, necessary to choose these values so that all agents have the same optimal value  $t^*$ . Also note that (52) and (53) are nothing more than simple rearrangements of the conditions

$$W_1[k_j(t^*), t^*, f_{j1}, \dots, f_{jN}; k_j(0)] = 0, \tag{55}$$

$$W_2[k_j(t^*), t^*, f_{j1}, \dots, f_{jN}; k_j(0)] = 0. \tag{56}$$

**3.2.3. Market establishment**

Nash equilibrium announcements  $t^*$  and  $\{f_{jn}\}$  and equilibrium values  $\{k_j(t^*)\}$  must satisfy (49), (52), and (53). Moreover, in the presence of such announcements, it must be impossible for any alternative potential market-maker to attract some subset of agents to his market in a manner consistent with earning nonnegative profits. Of course any such ‘deviating’ agents are also constrained by the optimizing behavior of potential market participants – that is, by (55) and (56). Thus, to be viable, equilibrium announcements must have the property that there is no alternative set of values  $\hat{t}$ ,  $\{\hat{k}_j(\hat{t})\}$ , and  $\{\hat{f}_{jn}\}$ , with  $j$  belonging to some set  $\hat{J} \subseteq J$ , that result in all agents with  $j \in \hat{J}$  being no worse off than under the announcements  $t^*$ ,  $\{k_j(t^*)\}$ , and  $\{f_{jn}\}$ , and that makes some agent  $j \in \hat{J}$  better off or that earns nonnegative profits for the market-maker. Then  $t^*$ ,  $\{k_j(t^*)\}$ , and  $\{f_{jn}\}$  constitutes an equilibrium if there is no set  $\hat{J}$ , and no alternative set of values  $\hat{t}$ ,  $\{\hat{k}_j(\hat{t})\}$ , and  $\{\hat{f}_{jn}\}$ ,  $j \in \hat{J} \subseteq J$ , satisfying

$$\sum_{j \in \hat{J}} \mu_j \hat{f}_{jn} \geq \gamma_n, \quad n = 1, \dots, N, \tag{57}$$

$$W_1[\hat{k}_j(\hat{t}), \hat{t}, \hat{f}_{j1}, \dots, \hat{f}_{jN}; k_j(0)] = 0, \tag{58}$$

$$W_2[\hat{k}_j(\hat{t}), \hat{t}, \hat{f}_{j1}, \dots, \hat{f}_{jN}; k_j(0)] = 0, \tag{59}$$

$$W[\hat{k}_j(\hat{t}), \hat{t}, \hat{f}_{j1}, \dots, \hat{f}_{jN}; k_j(0)] \geq W[k_j(t^*), t^*, f_{j1}, \dots, f_{jN}; k_j(0)], \tag{60}$$

for all  $j \in \hat{J}$ , and with at least one strict inequality in (57) or (60).<sup>13</sup>

In order to demonstrate the existence of a Nash equilibrium for the market formation game, it is useful to begin by considering the following ‘constrained

<sup>13</sup>If  $\hat{J}$  is a proper subset of  $J$  and  $j \in \hat{J}$ , then agent  $j$  obtains some expected utility level

$$\hat{W}[k_j(\hat{t}), \hat{t}, \hat{f}_{j1}, \dots, \hat{f}_{jN}; k_j(0)] \leq W[k_j(\hat{t}), \hat{t}, \hat{f}_{j1}, \dots, \hat{f}_{jN}; k_j(0)].$$

This is true since the ‘ $\hat{\cdot}$ ’ economy has a growth rate (strictly) less than  $r$  if  $\hat{J}$  is a proper subset of  $J$  for the reasons stated in Footnote 11.  $t^*$ ,  $\{f_{jn}\}$ , and the associated values  $\{k_j(t^*)\}$  do not constitute a Nash equilibrium if there exists a set  $\hat{J}$ , and values  $\hat{t}$ ,  $\{\hat{k}_j(\hat{t})\}$ , and  $\{\hat{f}_{jn}\}$  such that

$$\hat{W}_1[\hat{k}_j(\hat{t}), \hat{t}, \hat{f}_{j1}, \dots, \hat{f}_{jN}; k_j(0)] = 0, \quad \hat{W}_2[\hat{k}_j(\hat{t}), \hat{t}, \hat{f}_{j1}, \dots, \hat{f}_{jN}; k_j(0)] = 0,$$

$$\hat{W}[\hat{k}_j(\hat{t}), \hat{t}, \hat{f}_{j1}, \dots, \hat{f}_{jN}; k_j(0)] \geq W[k_j(t^*), t^*, f_{j1}, \dots, f_{jN}; k_j(0)].$$

The first two relations are analogous to (58) and (59), since the form of the partial derivatives of  $W$  are independent of the rate of growth of the post-market economy. And clearly the last relation is an analogue to (60). Thus the failure of  $t^*$ ,  $\{k_j(t^*)\}$ , and  $\{f_{jn}\}$  to constitute a Nash equilibrium implies the existence of a set of values  $\hat{t}$ ,  $\{\hat{k}_j(\hat{t})\}$ , and  $\{\hat{f}_{jn}\}$  satisfying an analogue to (57)–(60)  $\forall j \in \hat{J} \subset J$ . The absence of a set of such values satisfying an analogue to (57)–(60) also clearly implies that  $t^*$ ,  $\{k_j(t^*)\}$ ,  $\{f_{jn}\}$  does constitute a Nash equilibrium.

Pareto problem':

$$\max_{\{k_j(t^*), t^*, \{f_{jn}\}} \sum \omega_j W [k_j(t^*), t^*, f_{j1}, \dots, f_{jN}; k_j(0)], \tag{P.7}$$

subject to (49), (55), and (56). Here, of course,  $\omega_j$  is a weight attached to agent  $j$ 's utility by a potential market maker. Without loss of generality let  $\sum \omega_j = 1$ .

Let  $\lambda_n$ ,  $\eta_j$ , and  $\psi_j$  denote the multipliers associated with the constraints (49), (55), and (56), respectively. Then the first-order conditions for the problem (P.7) are

$$\omega_j W_1(-) - \eta_j W_{11}(-) - \psi_j W_{21}(-) = 0, \quad \text{all } j, \tag{61}$$

$$\sum_j [\omega_j W_2(-) - \eta_j W_{12}(-) - \psi_j W_{22}(-)] = 0, \tag{62}$$

$$\omega_j W_{2+n}(-) - \eta_j W_{1,2+n}(-) - \psi_j W_{2,2+n}(-) = \mu_j \lambda_n, \quad \text{all } j, n, \tag{63}$$

where it is straightforward to calculate that

$$W_{2+n}(-) = \pi_n e^{-\beta t^*} [k_j(t^*) - f_{jn}]^{-1}, \quad n = 1, \dots, N. \tag{64}$$

Notice that (49), (55), (56), and (61)–(63) constitute a system of  $JN + 3J + N + 1$  equations in the  $JN + 3J + N + 1$  unknowns  $\{f_{jn}\}$ ,  $\{k_j(t^*)\}$ ,  $\{\eta_j\}$ ,  $\{\psi_j\}$ ,  $\{\lambda_n\}$ , and  $t^*$ .

*3.2.4. Existence and optimality of equilibrium*

It is both interesting and instructive at this point to pose two questions. First, is there a Pareto optimal Nash equilibrium of the market formation game? If so, the Nash equilibrium announcements  $t^*$  and  $\{f_{jn}\}$  – and the associated values  $\{k_j(t^*)\}$  – must solve the Pareto problem

$$\max_{\{k_j(t^*), t^*, \{f_{jn}\}} \sum_{j=1}^J \hat{\omega}_j W [k_j(t^*), t^*, f_{j1}, \dots, f_{jN}; k_j(0)], \tag{P.8}$$

subject to (49) alone, for some set of welfare weights  $\{\hat{\omega}_j\}$  with  $\sum \hat{\omega}_j = 1$ . Second, can the Nash equilibrium of the market formation game mimic the outcome that would be observed in an Arrow–Debreu–McKenzie complete markets economy where agents can trade contingent claims against the events  $\gamma = \gamma_n$  and  $r = r_n$  at  $t = 0$ ?

If there is a Pareto optimal competitive equilibrium, Negishi (1960) shows that its allocation solves the problem (P.8) when the welfare weights are chosen to be proportional to the inverse of the marginal utility of initial wealth for each

agent type. One such set of welfare weights is given by

$$\hat{\omega}_j = \mu_j k_j(0)/k(0), \quad \text{all } j, \tag{65}$$

where  $k(0) \equiv \sum \mu_j k_j(0)$ .<sup>14</sup>

The answer to both questions posed is an affirmative one. To begin, it is useful to state the following lemma.

*Lemma.* *Let  $\omega_j = \mu_j k_j(0)/k(0)$  for all  $j$ . Then the following values satisfy (49), (55), (56), and (61)–(63):*

$$k_j(t^*) = \alpha k_j(0), \quad \text{all } j, \tag{66}$$

$$f_{jn} = \gamma_n k_j(0)/k(0), \quad \text{all } j, n, \tag{67}$$

$$\eta_j = \psi_j = 0, \quad \text{all } j, \tag{68}$$

$$\lambda_n = \pi_n e^{-\beta t^*} / [\alpha k(0) - \gamma_n], \quad \text{all } n, \tag{69}$$

and where  $\alpha$  and  $t^*$  are implicitly defined by

$$(1 - e^{-\beta t^*}) e^{(\beta - i)t^*} = (1 - \alpha e^{-it^*}) E[\alpha - \gamma/k(0)]^{-1} \tag{70}$$

and

$$\begin{aligned} & (i/\beta) E\{\alpha/[\alpha - \gamma/k(0)]\} - E\{\ln[\alpha - \gamma/k(0)]\} - \ln \{E[\alpha - \gamma/k(0)]^{-1}\} \\ & = E(r/\beta). \end{aligned} \tag{71}$$

The proof is given in the Appendix. Moreover, since  $\eta_j = \psi_j \equiv 0$  for all  $j$ , it is immediate that:

*Corollary.* *The values  $\{k_j(t^*)\}$ ,  $t^*$ , and  $\{f_{jn}\}$  described in the lemma solve (P.8) when  $\hat{\omega}_j$  is given by (65).*

Thus the allocation exhibited in the lemma is Pareto optimal.

It is now possible to state the main result of this section.

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<sup>14</sup>Given the logarithmic form of the momentary utility function, it is reasonable to conjecture that agent  $j$ 's marginal utility of wealth is proportional to  $1/k_j(0)$ . This conjecture will be born out.

*Proposition 4. There exists a Pareto optimal Nash equilibrium, which is described by the lemma.*

Proposition 4 is proved in the Appendix. It has the immediate implication that trade can be efficiently organized in this economy. In addition, and perhaps surprisingly, trade can be organized in a way that mimics the outcome that would obtain if agents (optimally) coordinated on the date  $t^*$  at which the more advanced technology would be adopted, and shared any associated risk in a set of complete contingent claims markets. In particular, the Nash equilibrium allocation displayed in the lemma can be supported as an Arrow–Debreu–McKenzie equilibrium where the price of a unit of income contingent on the event  $\gamma_n$  occurring at  $t^*$ , denoted by  $p(\gamma_n)$ , is given by

$$\frac{\pi_n V_1[\alpha k_j(0) - \gamma_n k_j(0)/k(0)]}{\pi_1 V_1[\alpha k_j(0) - \gamma_1 k_j(0)/k(0)]} = \frac{\pi_n[\alpha - \gamma_1/k(0)]}{\pi_1[\alpha - \gamma_n/k(0)]} = \frac{p(\gamma_n)}{p(\gamma_1)}, \quad n = 2, \dots, N.$$

Thus, in the equilibrium considered, risk is shared optimally in the sense that the marginal rate of substitution between any two states of nature is equalized across all agents.

### 3.3. Comparative statics

What factors influence the date at which the market opens? As is evident from (70) and (71),  $t^*$  depends on  $E[r]$  (and on no other aspects of the distribution of  $r$ ), on  $i$  and  $\beta$ , and on aggregate initial wealth  $k(0) \equiv \sum_i \mu_i k_i(0)$ . Inspection of (70) and (71) indicates that  $d\alpha/dE[r] < 0$  and  $dt^*/dE[r] < 0$ . Thus an increase in the expected return to specialization will cause the market to open earlier, and the economy will have less capital when it opens.

As far as the cost of market participation is concerned, the entire distribution of  $\gamma$  affects  $\alpha$  and  $t^*$ . In order to investigate the consequences of raising the costs of market operations, consider what happens to  $\alpha$  and  $t^*$  as a result of the following rightward displacement in the probability distribution governing  $\gamma$ :

$$d\gamma_1 = d\gamma_2 = \dots = d\gamma_N = dE[\gamma].$$

As is apparent from (71), this has the result of increasing  $\alpha$ . From (70), it is therefore necessary that  $dt^*/dE[\gamma] > 0$ . In short, if the distribution of costs of opening the market shifts to the right society will wait longer to open the market and accumulate more capital before doing so. In a similar fashion, it is easy to deduce that  $d\alpha/dk(0) < 0$  and  $dt^*/dk(0) < 0$ . Thus, the wealthier society is initially, the sooner the market will open.

Table 1

$\delta/\bar{\gamma}$	$t^*$	$\alpha = k(t^*)/k(0)$
0.0	92.13	3.000
0.5	94.94	3.097
1.0	102.08	3.353
1.5	111.29	3.711

It is also natural to ask how an increase in the degree of uncertainty about  $\gamma$  affects  $t^*$ . While analytical results on this point seem difficult to establish, intuition suggests that – in the presence of risk aversion – this would delay the opening of the market. This intuition is bolstered by the numerical example shown below.

*Example.* Let  $\beta = 0.04$ ,  $i = 0.05$ ,  $r = 0.06$ , and let  $k(0) = 20$ . Assume that  $\gamma \in \Gamma = (\bar{\gamma} - \delta, \bar{\gamma} + \delta) \equiv \{\gamma_1, \gamma_2\}$ , and let  $\pi_1 = \pi_2 = 0.5$ . Also, set  $\bar{\gamma} = 10$ . Clearly,  $E[\gamma] = \bar{\gamma}$ , and the standard deviation of  $\gamma$  is given by  $[E(\gamma - \bar{\gamma})^2]^{0.5} = \delta$ . The results displayed in Table 1 show how  $t^*$  and  $\alpha$  vary with the coefficient of deviation,  $\delta/\bar{\gamma}$ .

### 3.4. Discussion

There are two features of the previous model worth highlighting: (i) the endogenous nature and (ii) the competitive nature of market formation. The development of the economy induced the development of a market, which in turn enhanced subsequent economic growth. Furthermore, the possibility of free entry into the activity of arranging and effecting trades allowed Pareto optimal allocations to be observed. In particular, the equilibrium exhibited had the feature that competition among potential market-makers ensured that market participants received efficiently provided market services.<sup>15</sup> This feature is shared by the models of Townsend (1978) and Greenwood and Jovanovic (1990). It contrasts, however, with the market participation models of Chatterjee (1988) and Pagano (1989), or with the search models of Diamond (1982). In those models trade is largely unorganized, with the consequence that opportunities for

<sup>15</sup>In the current model only one market-maker operates in equilibrium. If there were diminishing returns to market size, however, there could be many market-makers providing similar, and perhaps complementary services simultaneously. It is an interesting issue whether, under these conditions, the possibility of coordination or competition between market makers would tend to augment or reduce the equilibrium rate of growth.

trade go unexploited. Agents who can organize trading activity therefore have an opportunity to enrich themselves. For example, in Chatterjee (1988) each agent chooses to enter an insurance market without taking account of the external benefit that is conferred by the resulting ‘thickening’ of the market. As a result, not enough agents enter the market. Any agent could take it upon himself, however, to become a broker and provide insurance at prices that elicit an ideal level of market participation. The issue, of course, is whether such services will be competitively provided or not.

As indicated previously, Hicks (1969) and North (1981) identify increasing specialization as being central to economic development. Markets are obviously essential in fostering specialization – a feature which the model just described has attempted to capture. McKinnon (1973) has also argued that specialization promotes learning-by-doing, and is therefore growth-enhancing. This argument is formalized by Cooley and Smith (1992).<sup>16</sup> They produce a model in which – in the absence of financial markets – agents delay entry into entrepreneurial activity. This is necessitated by the need to accumulate income prior to entering into entrepreneurship when borrowing is not possible. In the Cooley–Smith model this delay precludes specialization, and therefore interferes with learning-by-doing. When financial markets are present specialization in entrepreneurial activity is possible. The result is more learning-by-doing, and a higher rate of growth of the economy.

Cooley and Smith also demonstrate that financial markets may fail to form for endogenous reasons, even when their formation is costless. This occurs exactly when the real interest rate is too low (lower than the growth rate) in the absence of financial markets. If interest rates are too low, insufficient incentives are provided for agents to specialize. This lack of incentive arises because nonspecialization tends to result in income being earned relatively late in life – an outcome which becomes less desirable as interest rates rise. Thus, when interest rates are low, agents can fail to specialize, with the consequence that specialized entrepreneurs (who need to borrow) do not exist. Similarly, low interest rates make potential lenders content to invest autarkically. The result is an internally consistent situation in which there is no demand for or supply of financial market services.

#### **4. Conclusions**

The previous sections presented two models that illustrate the following points. First, market formation is endogenous. The costs of market formation

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<sup>16</sup>Freeman and Polasky (1992) have a model that bears a considerable resemblance to the Cooley–Smith model. However, they do not consider any role for financial markets.

will typically require that market development follows some period of real development. Second, market formation enhances growth after it occurs. This is because markets promote the allocation of capital to its highest return uses, alter the composition of saving, and foster specialization. Third, competition among potential providers of market services leads markets to form in a way that is perceived to be efficient by market participants.

In order to illustrate these themes, two models of intermediated activity were employed. One is the Diamond and Dybvig (1983) model, which provides a clear description of what liquidity provision entails. This permitted an investigation of how liquidity provision affects capital accumulation, which is the primary focus of Patrick (1966), Cameron (1967), McKinnon (1973), and Shaw (1973). The second is Townsend's (1978) model of intermediated transactions in the presence of a fixed cost associated with transacting. These two models have played an important role in motivating the existing research on the role of financial markets and growth.

There are, of course, other models that could be used to investigate the interactions between financial markets and economic development. One is Williamson's (1986) model, in which banks serve as delegated monitors in a costly-state-verification environment.<sup>17</sup> In such a model banking eliminates socially redundant information acquisition, and thereby enhances efficiency. The Williamson model also has the feature that credit rationing can be observed. Credit rationing is, of course, perceived to be of considerable significance in developing economies. Williamson's model allows an investigation of how banking impacts on credit rationing, and on how this latter factor affects growth. An example of how a costly-state-verification model can be integrated into a growth framework is given by Khan (1992). While the details of the analysis are different, the mechanism through which intermediation affects growth is essentially the same as that modeled here.

Williamson's model considers an environment in which ex post monitoring is relevant. Boyd and Prescott (1986) analyze economies in which banks engage in ex ante information acquisition about borrower 'quality'. While their model has yet to be exploited in an analysis of how financial intermediation and growth are related, such an analysis is likely to have much in common with the current one.

Two extensions to the growing literature on intermediation and growth seem worth pursuing. The first would be to investigate theoretically the relationship between the form of economic institutions and the adoption of new technologies. What type of organizational forms facilitate the introduction of new technologies by innovators, yet provide investors a desirable structure of returns on their savings? Second, the field seems ripe for serious computational general

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<sup>17</sup>The original description of such an environment appears in Townsend (1979). The first model of banks serving a delegated monitoring function is Diamond's (1984).

equilibrium analyses of the relationship between financial and economic growth. Recent work by Parente and Prescott (1992) is instructive in this regard. It suggests that small changes in the incentives to adopt new technologies can have large effects on a country's income.

**Appendix**

*Proof of Proposition 1*

By Eqs. (15) and (22),  $Q^b(RA\theta) > Q^a(RA\theta)$  holds if and only if

$$\eta(RA\theta)/[1 + \eta(RA\theta)] > [\lambda(RA\theta) - 1]/[\lambda(RA\theta) - 1 + (RA\theta/n)]. \tag{A.1}$$

Using the definitions of  $\eta$  and  $\lambda$  in (A.1) and rearranging terms yields the equivalent condition (24) in the text.

Evidently, a sufficient condition for (24) is that

$$1 \geq [\pi/(1 - \pi)]^{-\gamma/(1 + \gamma)}. \tag{A.2}$$

Eq. (A.2) holds if  $\pi \geq \frac{1}{2}$  and  $\gamma \geq 0$  or if  $\pi \leq \frac{1}{2}$  and  $\gamma \leq 0$ . This establishes part (a).

For part (b) note that  $Q^b(RA\theta) > \pi Q^a(RA\theta)$  if and only if

$$\eta(RA\theta)/[1 + \eta(RA\theta)] > \pi[\lambda(RA\theta) - 1]/[\lambda(RA\theta) - 1 + (RA\theta/n)]. \tag{A.3}$$

Rearranging terms in (A.3), one obtains

$$\eta(RA\theta) > [\lambda(RA\theta) - 1]/\{(RA\theta/\pi n) + (1 - \pi)[\lambda(RA\theta) - 1]/\pi\}. \tag{A.4}$$

*Case 1:*  $\gamma \leq 0$ . Obviously, a sufficient condition for (A.4) is

$$\eta(RA\theta) \equiv \pi(RA\theta/n)^{-\gamma/(1 + \gamma)}/(1 - \pi) \geq \pi/(1 - \pi). \tag{A.5}$$

But (17) implies that (A.5) holds  $\forall \gamma \leq 0$ .

*Case 2:*  $\gamma > 0$ . Another sufficient condition for (A.4) is that

$$\begin{aligned} \eta(RA\theta) &\equiv \pi(RA\theta/n)^{-\gamma/(1 + \gamma)}/(1 - \pi) \\ &\geq \pi n \lambda(RA\theta)/RA\theta \\ &= n\pi[\pi(RA\theta - n)/n(1 - \pi)]^{1/(1 + \gamma)}/RA\theta. \end{aligned} \tag{A.6}$$

A sufficient condition for (A.6), in turn, is that

$$\begin{aligned} \pi(RA\theta/n)^{-\gamma/(1+\gamma)}/(1-\pi) &\geq n\pi[\pi RA\theta/n(1-\pi)]^{1/(1+\gamma)}/RA\theta \\ &= \pi[\pi/(1-\pi)]^{1/(1+\gamma)}(RA\theta/n)^{-\gamma/(1+\gamma)}. \end{aligned} \quad (\text{A.7})$$

Eq. (A.7) reduces to

$$1 \geq [\pi(1-\pi)^\gamma]^{1/(1+\gamma)}, \quad (\text{A.7}')$$

which obviously holds  $\forall \gamma > 0$ . This establishes part (b).

*Proof of Proposition 3*

(a) Immediate from  $Q^a(RA\theta) < 1$ .

(b) The growth of an economy with equity markets is more rapid than that of an economy with banks if and only if

$$\pi > Q^b(RA\theta) \equiv \eta(RA\theta)/[1 + \eta(RA\theta)]. \quad (\text{A.8})$$

Using the definition of  $\eta$  in (A.8) and rearranging terms, one obtains

$$1 > (RA\theta/n)^{-\gamma/(1+\gamma)}. \quad (\text{A.9})$$

The restriction (17) implies that (A.9) holds if and only if  $\gamma > 0$ .

*Proof of the Lemma*

The proof proceeds in several steps.

*Step 1.* Eq. (71) has a unique solution  $\alpha \in (\max_n \{\gamma_n/k(0)\}, \infty)$ . The existence of such a solution can be deduced from the intermediate value theorem. The uniqueness of the solution follows from the fact that the left-hand side of (71) is decreasing in  $\alpha$ .

*Step 2.* Eq. (70) has a unique solution  $t^* \in (0, \infty)$ . Again, existence can be established by application of the intermediate value theorem. Uniqueness follows from the fact that  $i > \beta$ .

*Step 3.* When  $\alpha$  and  $t^*$  satisfy (70) and (71), and when  $k_j(t^*)$  and  $f_{j_n}$  satisfy (66) and (67),  $W_1(-) = 0 = W_2(-)$  holds for all  $j$ . This can be verified by direct substitution into (52) and (53). The latter conditions are equivalent to (55) and (56).

*Step 4.* Eqs. (61), (62) and (63) are satisfied. For Eqs. (61) and (62) this is obvious from (68) and step 3. For Eq. (63) this is obvious from (64) and (65)–(69).

*Step 5.* Eq. (49) is satisfied. This is obvious since, when (67) holds,  $\sum \mu_j f_{jn} = \gamma_n \sum \mu_j k_j(0)/k(0) = \gamma_n$ . Thus the values described by (66)–(71) satisfy (49), (55), (56), and (61)–(63), as claimed.

*Proof of Proposition 4*

In view of the corollary, the allocation described in the lemma is Pareto optimal. Thus, if it can be established that it is a Nash equilibrium, this will yield the desired result.

In order to verify that the allocation displayed in the lemma is a Nash equilibrium, it is necessary to show that no potential market maker can profitably attract some subset of agents in the presence of the allocation  $\{k_j(t^*), t^*, \{f_{jn}\}$ . Suppose that this allocation does *not* constitute a Nash equilibrium of the market formation game, then. It follows that there exists a set  $\hat{J} \subseteq J$ , and values  $\hat{t}$ ,  $\{\hat{k}_j(\hat{t})\}$ , and  $\{\hat{f}_{jn}\}$ ,  $j \in \hat{J}$ , satisfying (57)–(60) (with at least one inequality strict). A third allocation  $\tilde{t}$ ,  $\{\tilde{k}_j(\tilde{t})\}$ ,  $\{\tilde{f}_{jn}\}$ ,  $j \in J$ , is now constructed as follows. Set  $\tilde{t} = \hat{t}$ ,  $\tilde{k}_j(\tilde{t}) = \hat{k}_j(\hat{t})$ , and  $\tilde{f}_{jn} = \hat{f}_{jn}$  for all  $j \in \hat{J}$ . For  $j \in J - \hat{J}$ , set

$$\tilde{k}_j(\tilde{t}) = [k_j(0)/k_i(0)]\hat{k}_i(\hat{t}), \tag{A.10}$$

$$\tilde{f}_{jn} = [k_j(0)/k_i(0)]\hat{f}_{in}, \tag{A.11}$$

for some  $i \in \hat{J}$ , and let all agents participate in the market. This is feasible and, by hypothesis,

$$W[\tilde{k}_i(\tilde{t}), \tilde{t}, \tilde{f}_{i1}, \dots, \tilde{f}_{iN}; k_i(0)] \geq W[k_i(t^*), t^*, f_{i1}, \dots, f_{iN}; k_i(0)] \tag{A.12}$$

holds for all  $i \in \hat{J}$ . Moreover, from (54),

$$\begin{aligned} W_j[\tilde{k}_j(\tilde{t}), \tilde{t}, \tilde{f}_{j1}, \dots, \tilde{f}_{jN}; k_j(0)] &= W_i[\tilde{k}_i(\tilde{t}), \tilde{t}, \tilde{f}_{i1}, \dots, \tilde{f}_{iN}; k_i(0)] \\ &\quad + \ln[k_j(0)/k_i(0)]/\beta, \end{aligned} \tag{A.13}$$

for  $j \in J - \hat{J}$ . Finally, it is straightforward to verify that

$$\begin{aligned} W[k_j(t^*), t^*, f_{j1}, \dots, f_{jN}; k_j(0)] &= W[k_i(t^*), t^*, f_{i1}, \dots, f_{iN}; k_i(0)] \\ &\quad + \ln[k_j(0)/k_i(0)]/\beta \end{aligned} \tag{A.14}$$

holds for all  $i, j$ . But then (A.12)–(A.14) imply that

$$W[k_j(t^*), t^*, f_{j1}, \dots, f_{jN}; k_j(0)] \leq W[\tilde{k}_j(\tilde{t}), \tilde{t}, \tilde{f}_{j1}, \dots, \tilde{f}_{jN}; k_j(0)] \quad (\text{A.15})$$

holds  $\forall j \in J$ . In addition, evidently

$$\sum_{j=1}^J \mu_j \tilde{f}_{jn} \geq \gamma_n \quad (\text{A.16})$$

holds, and at least one inequality in (A.15) and (A.16) is strict. But this contradicts the Pareto optimality of the allocation described in the lemma, establishing the desired result.

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