IN34A - Optimización
Leonardo López H.
lelopez@ing.uchile.cl

Método del Gradiente: Ejemplo 1

restart;

$f(x, y) := x^2 + y^2$

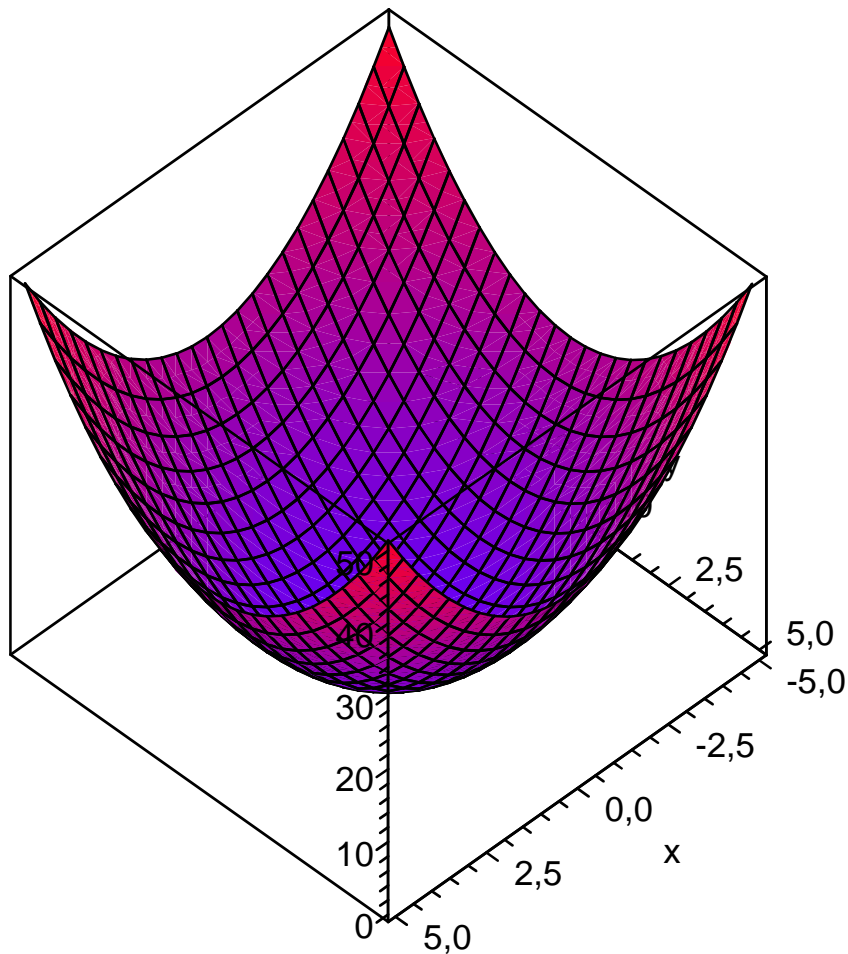$$(x, y) \rightarrow x^2 + y^2 \tag{1}$$

$a := 5$:
with(*plots*) :
surface := *plot3d*($f(x, y)$, $x = -$a ..a, $y = -$a ..a , *shading* $= Z$, *axes* $= box$) :
surface2 := *plot3d*($f(x, y)$, $x = -$a ..a, $y = -$a ..a , *shading* $= Z$, *axes* $= box$, *style* $= contour$) :
contour2d := *contourplot*($f(x, y)$, $x = -$a ..a, $y = -$a ..a) :
gradiente := *gradplot*(f(x, y), $x = -$a ..a, $y = -$a ..a, *grid* $= [15, 15]$) :
display({*surface*});

display({*surface2*});
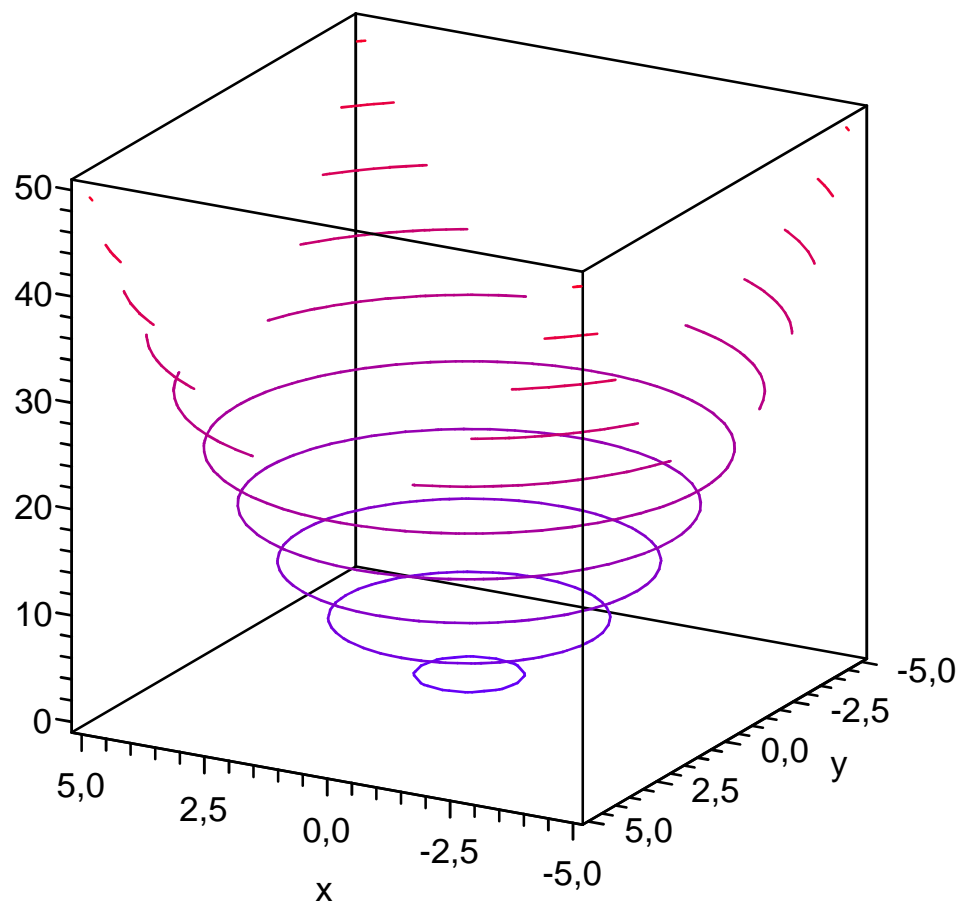
display({*contour2d, gradiente*});
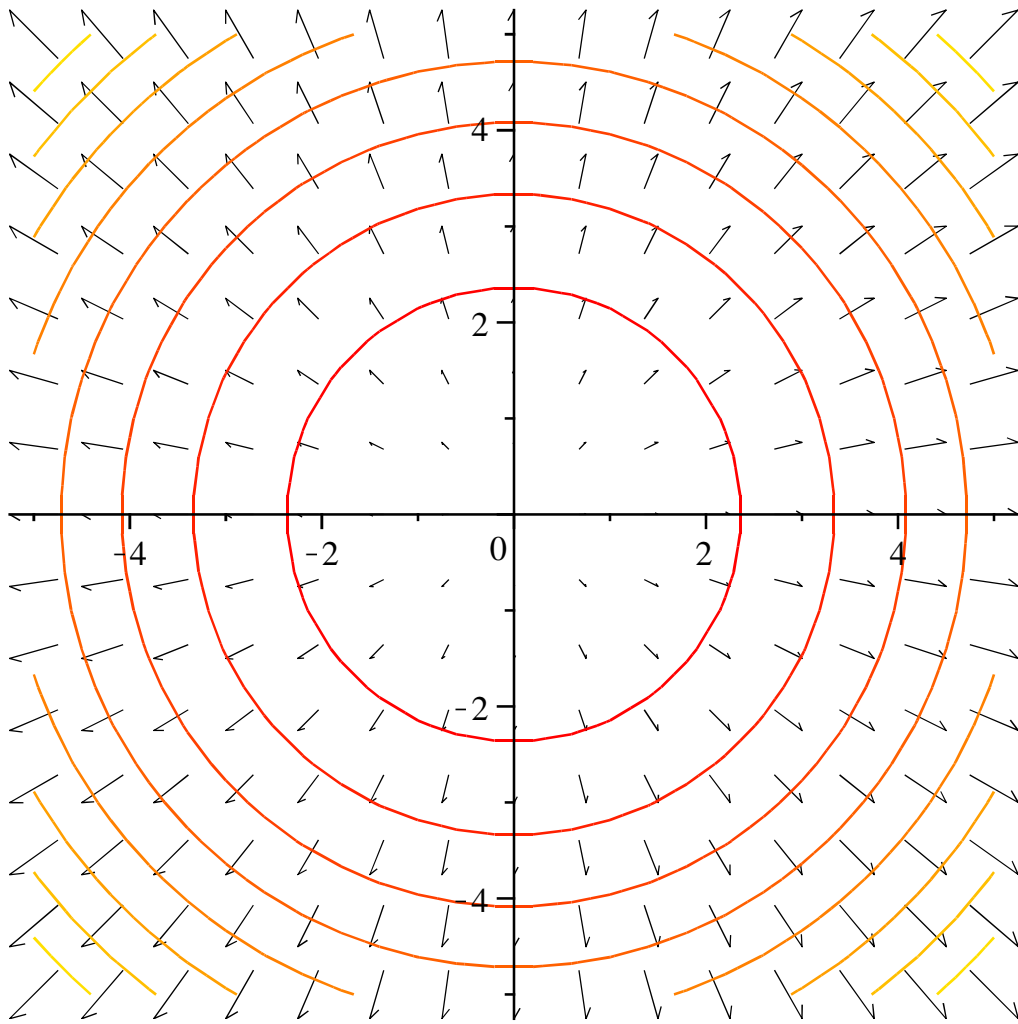
$fx := diff(f(x, y), x);$

$$2x \tag{2}$$

$fy := diff(f(x, y), y);$

$$2y \tag{3}$$

$x0 := 2 : y0 := 4 :$

Iteración 0:

$[subs(x = x0, y = y0, fx), subs(x = x0, y = y0, fy)]$

$$[4, 8] \tag{4}$$

$h0(\lambda) := (f(x0 - subs(x = x0, y = y0, fx) * \lambda, y0 - subs(x = x0, y = y0, fy) * \lambda))$

$$\lambda \rightarrow f(x0 - subs(x = x0, y = y0, fx)\, \lambda, y0 - subs(x = x0, y = y0, fy)\, \lambda) \tag{5}$$

$plot(h0(\lambda), \lambda = -3..3);$
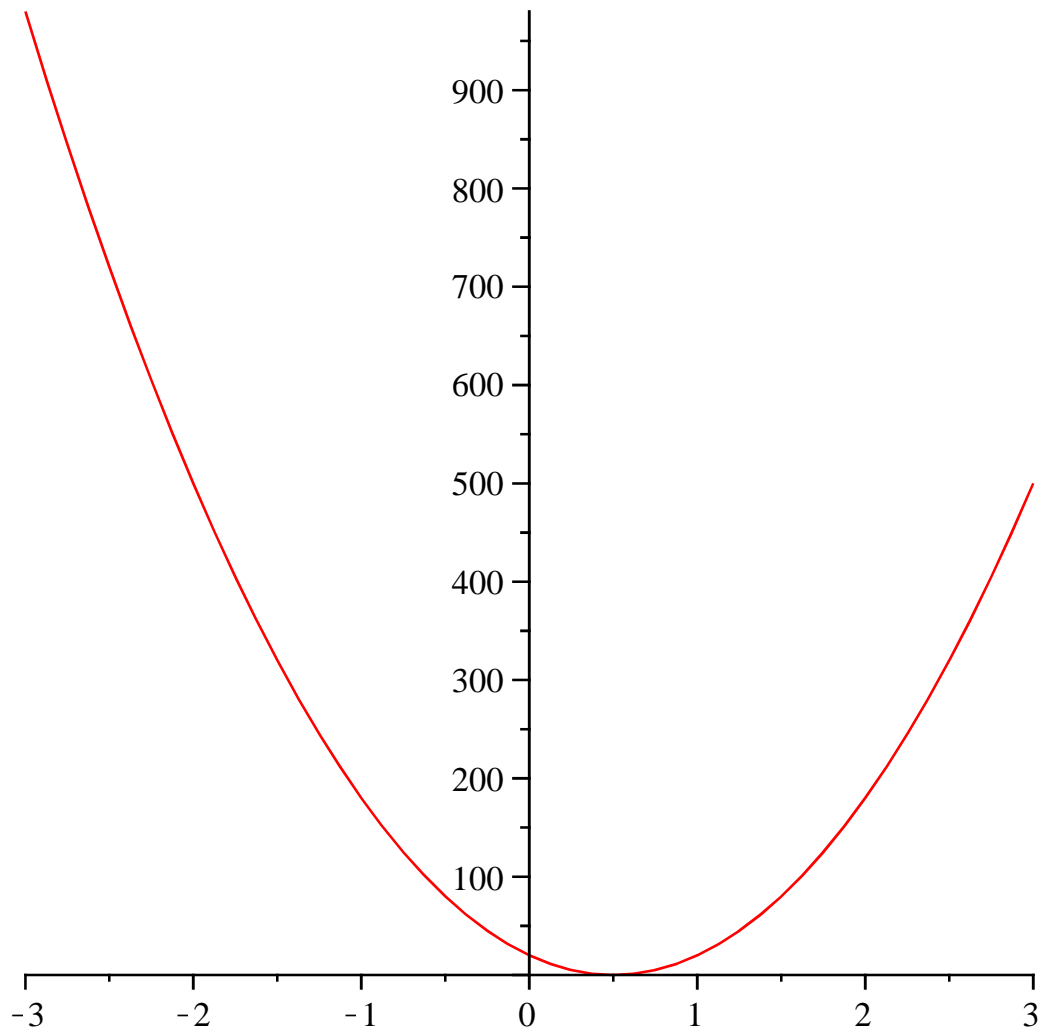
$dh0 := diff(h0(\lambda), \lambda)$

$$-80 + 160\,\lambda \tag{6}$$

$\lambda0 := fsolve(dh0 = 0, \lambda);$

$$0.5000000000 \tag{7}$$

$x1 := evalf(x0 - subs(x = x0, y = y0, fx) * \lambda0);$

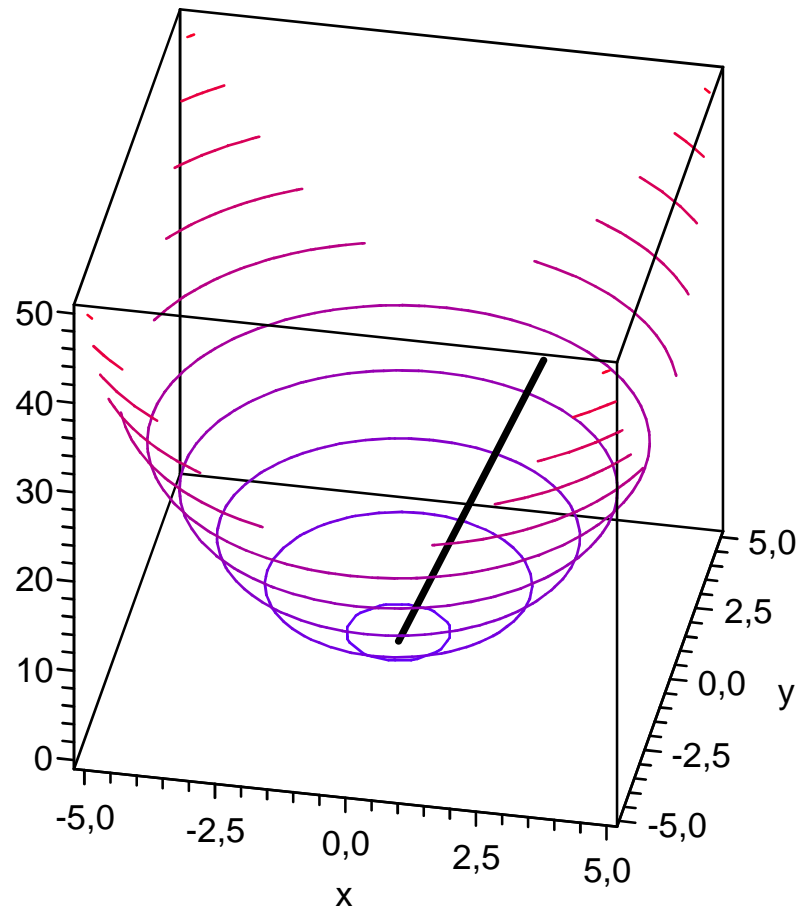$$0. \tag{8}$$

$y1 := evalf(y0 - subs(x = x0, y = y0, fy) * \lambda0);$

$$0. \tag{9}$$

$mejora := evalf(f(x1, y1) - f(x0, y0));$

$$-20. \tag{10}$$

$Linea3dN0 := pointplot3d([[x0, y0, f(x0, y0)], [x1, y1, f(x1, y1)]], color = black, style = line, thickness = 2,):$

$Linea2dN0 := pointplot([[x0, y0], [x1, y1]], color = black, style = line, thickness = 2,):$

$display(\{surface2, Linea3dN0\});$

display({*contour2d, gradiente, Linea2dN0*});
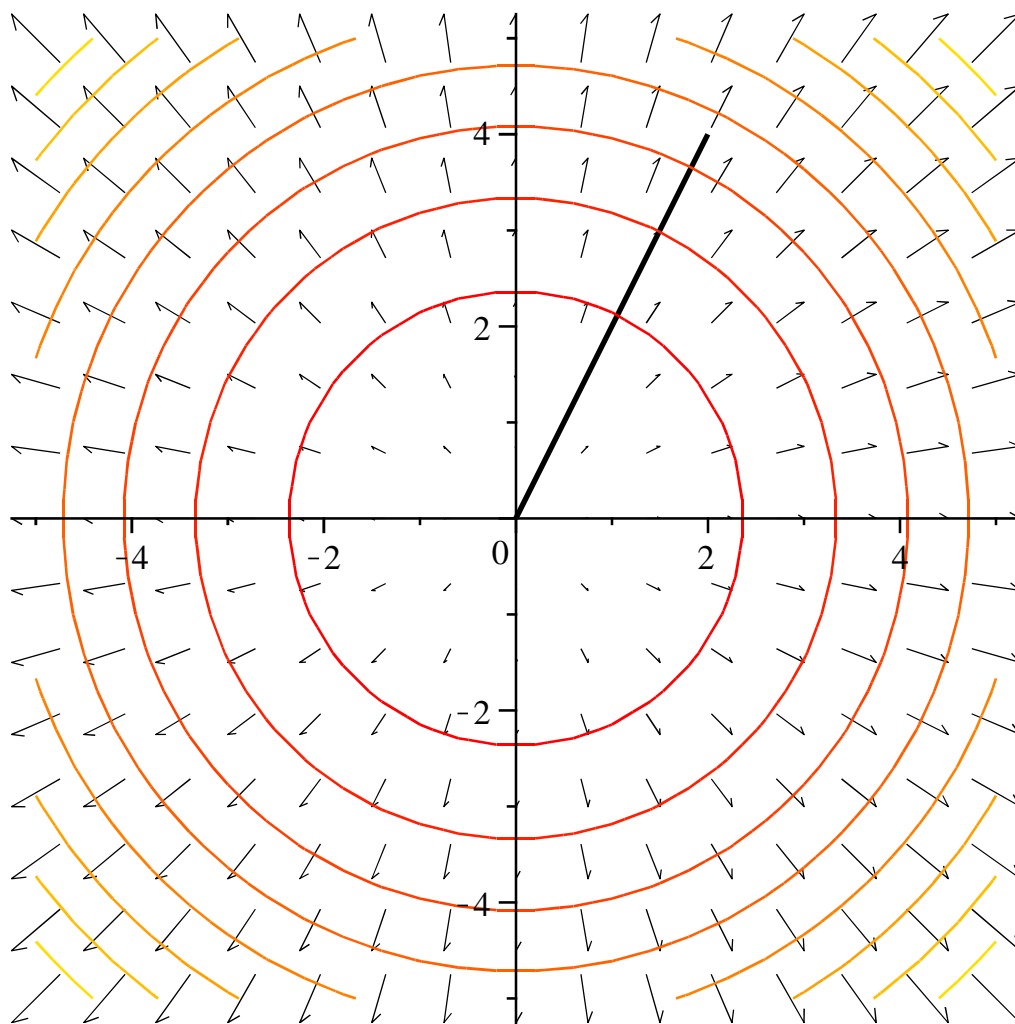
Iteración 1:

$$[\,subs(x = x1, y = y1, fx), subs(x = x1, y = y1, fy)\,]$$
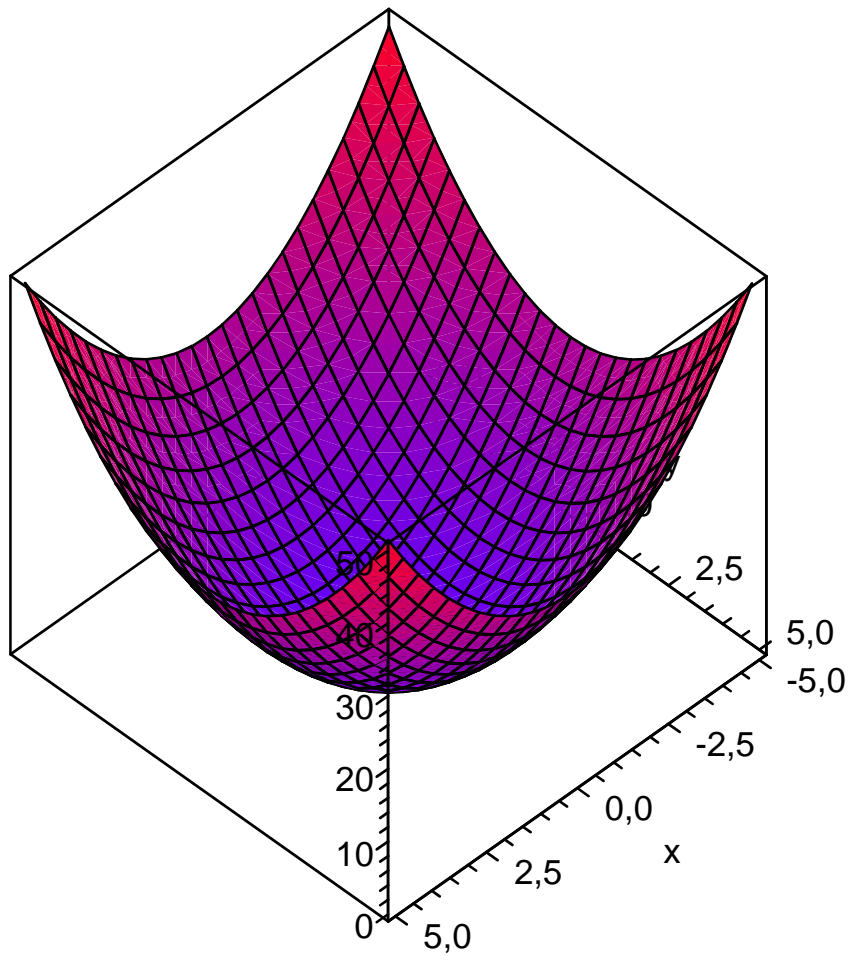$$[0., 0.]$$

(11)

Método de Newton: Ejemplo 1
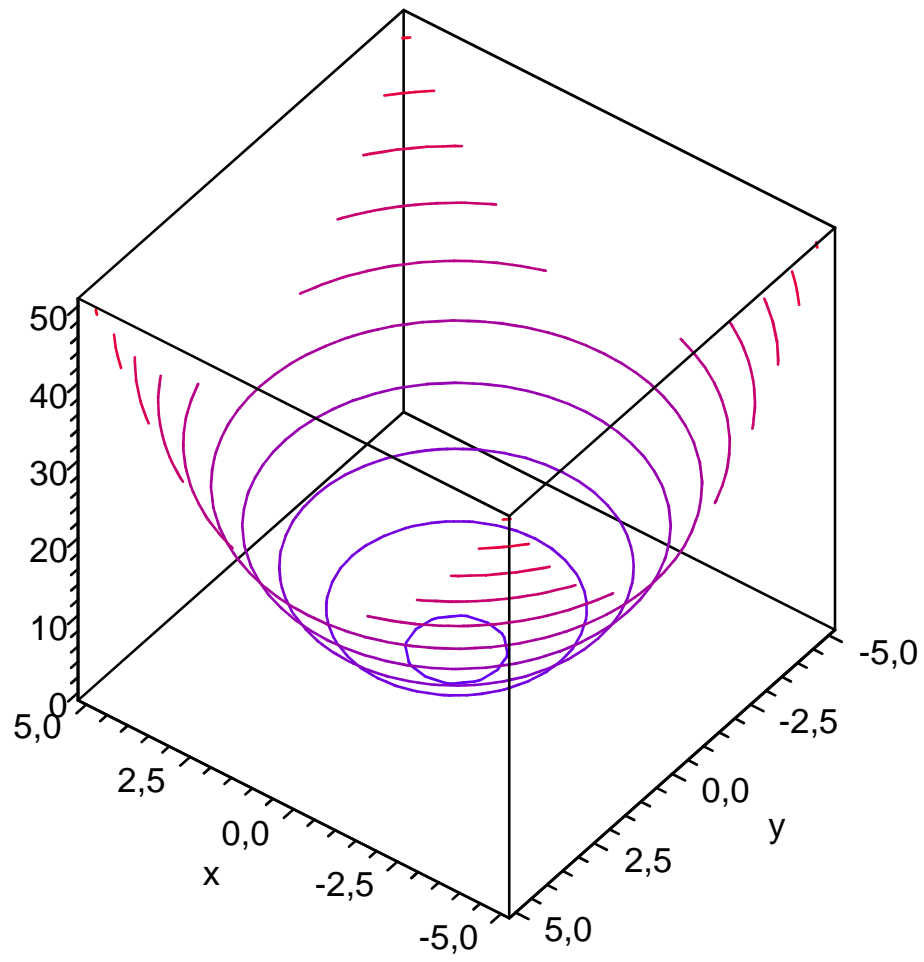
restart;

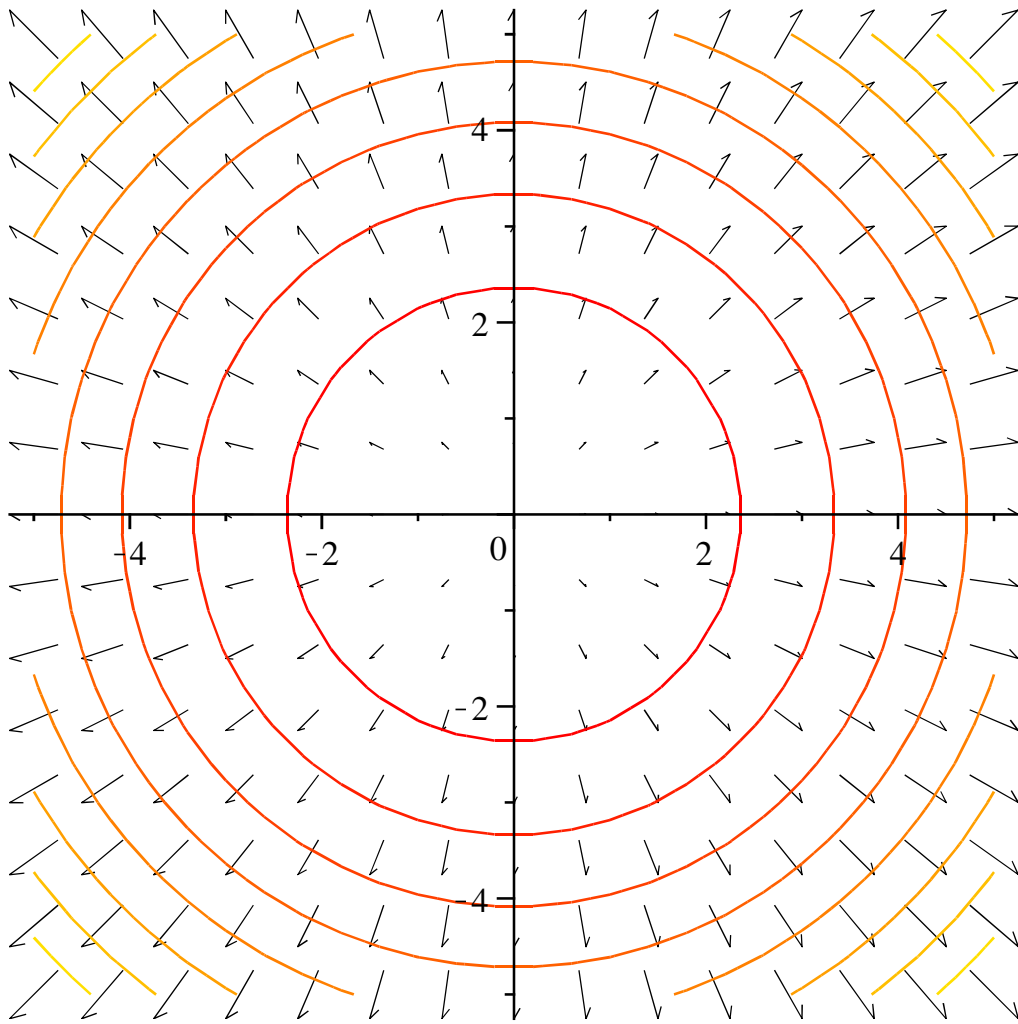$f(x, y) := x^2 + y^2$

$$(x, y) \rightarrow x^2 + y^2 \tag{1}$$

a := 5 :

with(*plots*) :

surface := *plot3d*($f(x, y)$, $x = -a..a$, $y = -a..a$, *shading* = Z, *axes* = *box*) :

surface2 := *plot3d*($f(x, y)$, $x = -a..a$, $y = -a..a$, *shading* = Z, *axes* = *box*, *style* = *contour*) :

contour2d := *contourplot*($f(x, y)$, $x = -a..a$, $y = -a..a$) :

gradiente := *gradplot*(f(x, y), $x = -a..a$, $y = -a..a$, *grid* = [15, 15]) :

display({*surface*});

display({*surface2*});

$display(\{contour2d, gradiente\});$

$fx := diff(f(x, y), x);$

$$2x \tag{2}$$

$fy := diff(f(x, y), y);$

$$2y \tag{3}$$

$fxx := diff(fx, x);$

$$2 \tag{4}$$

$fyy := diff(fy, y);$

$$2 \tag{5}$$

$fxy := diff(fx, y);$

$$0 \tag{6}$$

$fyx := diff(fy, x);$

$$0 \tag{7}$$

$H := matrix([[fxx, fxy], [fyx, fyy]])$

$$\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \tag{8}$$

$x0 := 2 : y0 := 4 :$

<u>Iteración 0:</u>

$[\, subs(x = x0, y = y0, fx), subs(x = x0, y = y0, fy)\,]$

$$[4, 8] \tag{9}$$

$f(x0, y0)$

$$20 \tag{10}$$

$G := subs\left(x = x0, y = y0, evalm\left(H^{-1}\right)\right)$

$$\begin{bmatrix} \dfrac{1}{2} & 0 \\[2mm] 0 & \dfrac{1}{2} \end{bmatrix} \tag{11}$$

$x1 := x0 - (evalm(G[1, 1]) \cdot subs(x = x0, y = y0, fx) + evalm(G[2, 1]) \cdot subs(x = x0, y = y0, fy))$

$$0 \tag{12}$$

$y1 := y0 - (evalm(G[2, 1]) \cdot subs(x = x0, y = y0, fx) + evalm(G[2, 2]) \cdot subs(x = x0, y = y0, fy))$

$$0 \tag{13}$$

$mejora := evalf(f(x1, y1) - f(x0, y0));$

$$-20. \tag{14}$$

$Linea3dN0 := pointplot3d(\,[\,[x0, y0, f(x0, y0)\,], [x1, y1, f(x1, y1)\,]\,], color = black, style = line, thickness$
$\quad = 2,) :$

$Linea2dN0 := pointplot(\,[\,[x0, y0], [x1, y1]\,], color = black, style = line, thickness = 2,) :$

$display(\{surface2, Linea3dN0\});$

display({*contour2d, gradiente, Linea2dN0*})

Iteración 1:

$$[\, subs(x = x1, y = y1, fx), \, subs(x = x1, y = y1, fy) \,]$$
$$[0, 0]$$

(15)

Método del Gradiente: Ejemplo 2

restart;

$$f(x, y) := \frac{3}{2}x^2 + \frac{1}{2} \cdot y^2$$

$$(x, y) \rightarrow \frac{3}{2}\, x^2 + \frac{1}{2}\, y^2 \tag{1}$$
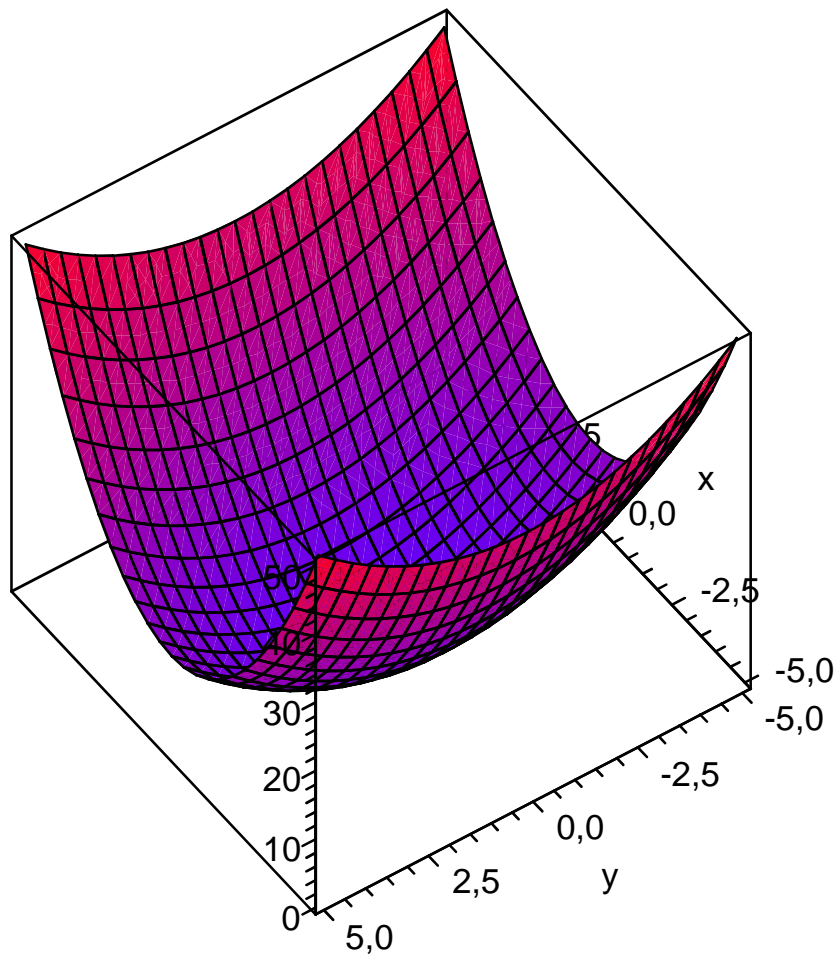
a := 5 :
with(*plots*) :
surface := *plot3d*(*f*(*x*, *y*), *x* = −a ..a, *y* = −a ..a , *shading* = *Z*, *axes* = *box*) :
surface2 := *plot3d*(*f*(*x*, *y*), *x* = −a ..a, *y* = −a ..a , *shading* = *Z*, *axes* = *box*, *style* = *contour*) :
contour2d := *contourplot*(*f*(*x*, *y*), *x* = −a ..a, *y* = −a ..a) :
gradiente := *gradplot*(f(x, y), *x* = −a ..a, *y* = −a ..a, *grid* = [15, 15]) :
display({*surface*});

display({*surface2*});
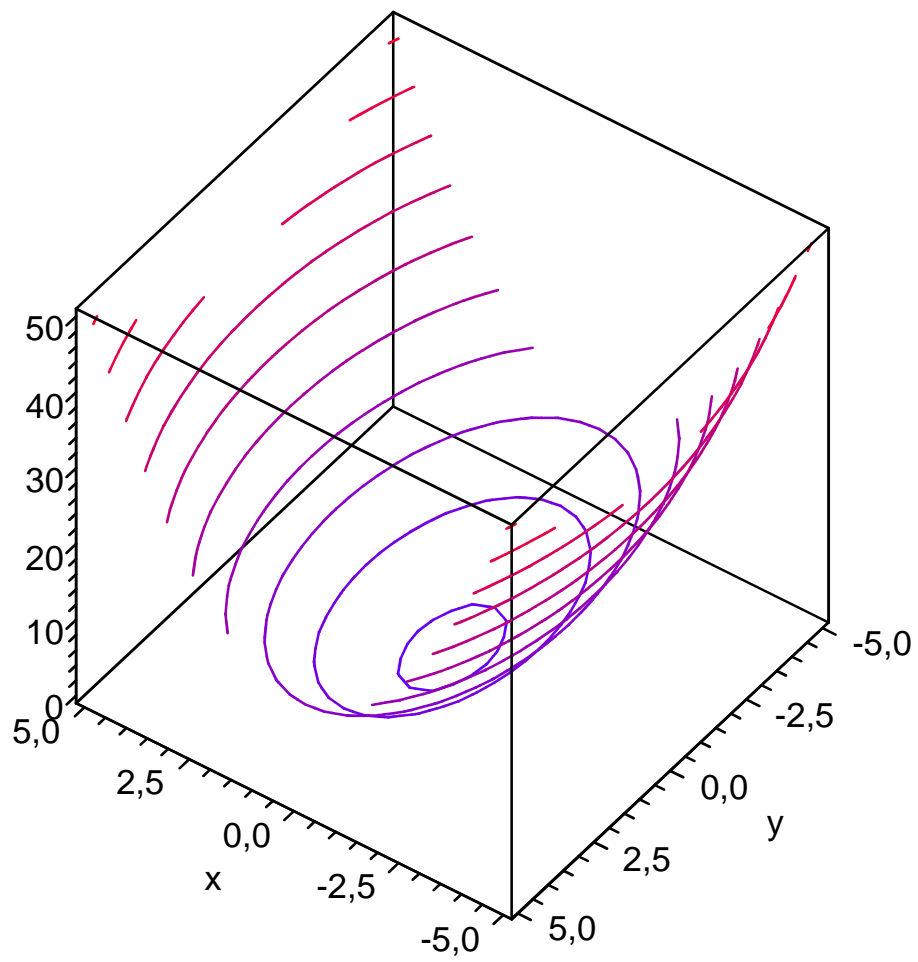
$display(\{contour2d, gradiente\});$
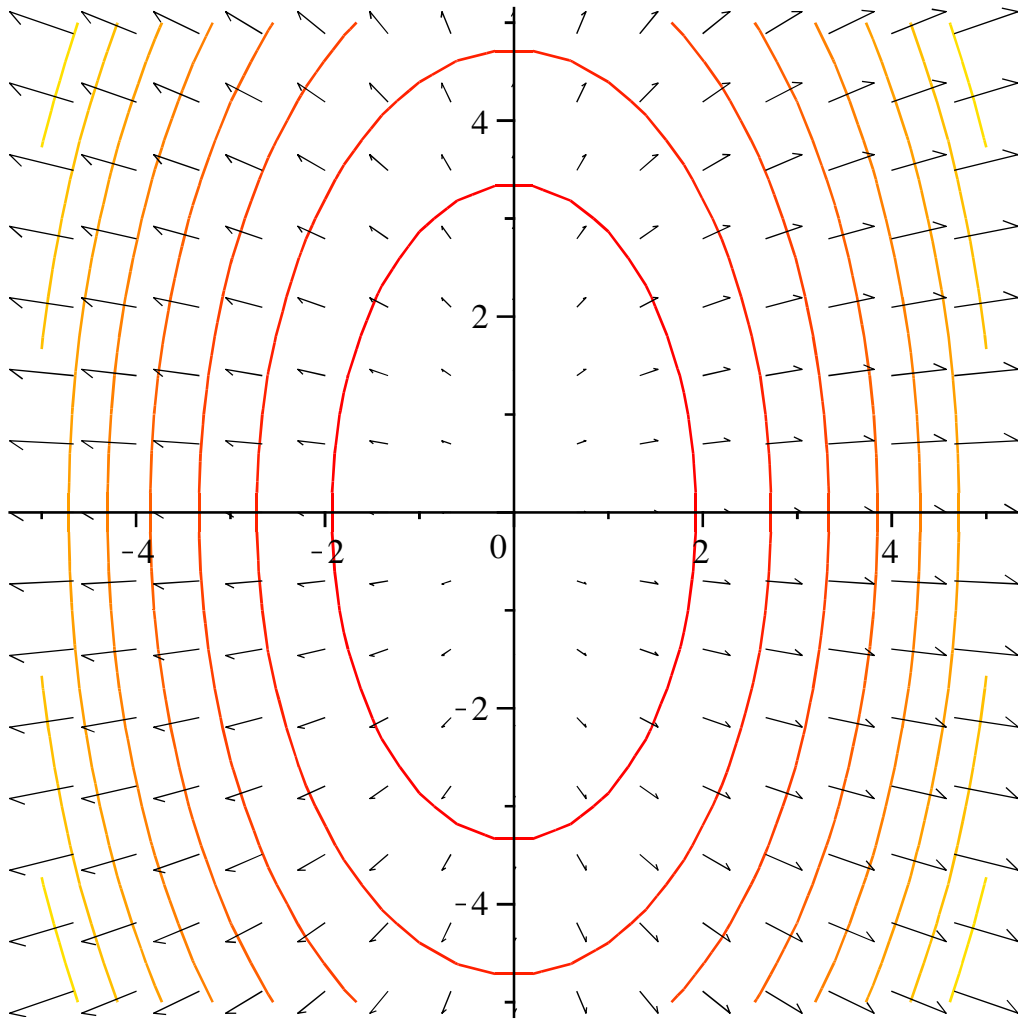
$fx := diff(f(x, y), x);$

$$3\,x \qquad\qquad (2)$$

$fy := diff(f(x, y), y);$

$$y \qquad\qquad (3)$$

$x0 := 2 : y0 := 4 :$

Iteración 0:

$[subs(x = x0, y = y0, fx), subs(x = x0, y = y0, fy)]$

$$[6, 4] \qquad\qquad (4)$$

$f(x0, y0)$

$$14 \qquad\qquad (5)$$

$h0(\lambda) := (f(x0 - subs(x = x0, y = y0, fx) * \lambda, y0 - subs(x = x0, y = y0, fy) * \lambda))$

$$\lambda \rightarrow f(x0 - subs(x = x0, y = y0, fx)\,\lambda, y0 - subs(x = x0, y = y0, fy)\,\lambda) \qquad (6)$$

$plot(h0(\lambda), \lambda = -0.5\,..1);$
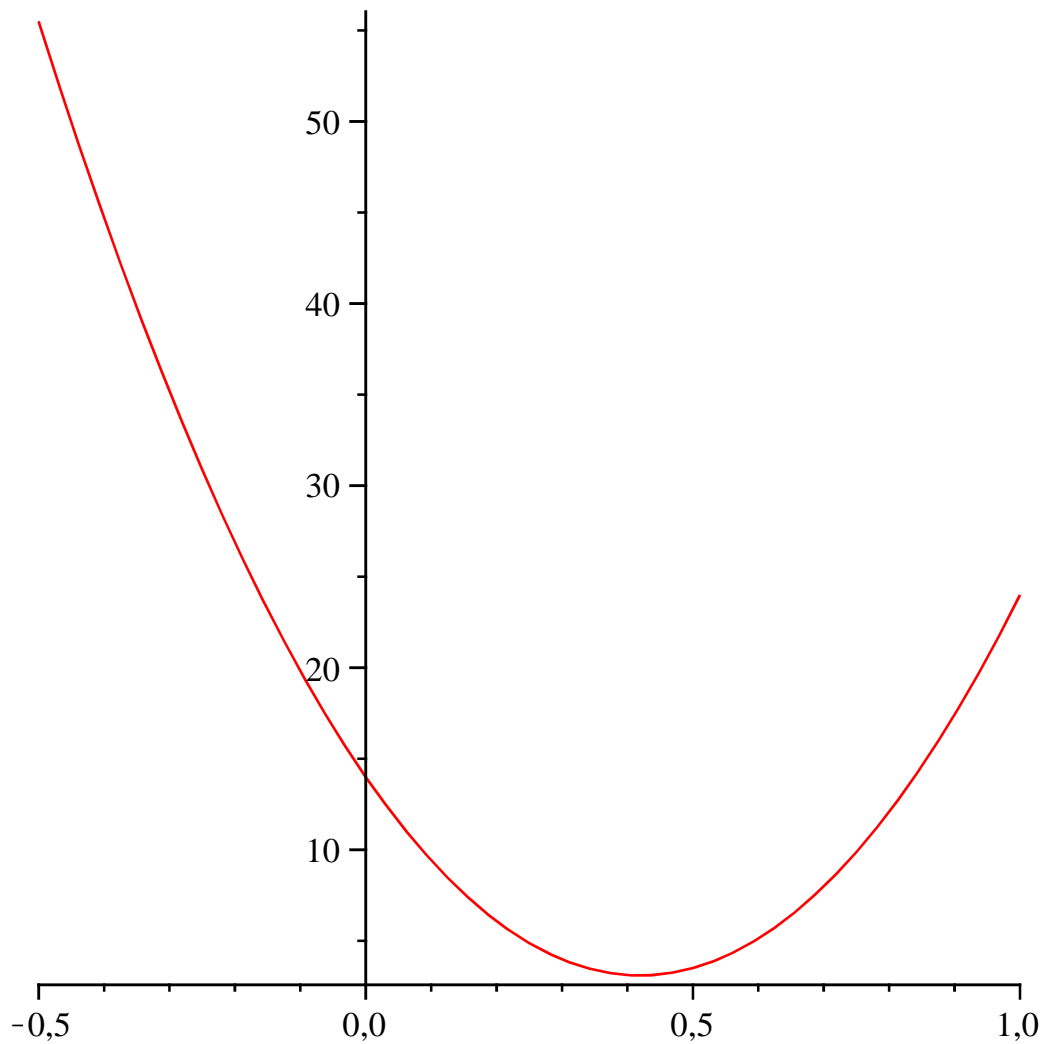
$dh0 := diff\left(h0(\lambda), \lambda\right)$

$$-52 + 124\,\lambda \tag{7}$$

$\lambda0 := fsolve(dh0 = 0, \lambda);$

$$0.4193548387 \tag{8}$$

$h0\left(\lambda0\right)$

$$3.096774193 \tag{9}$$

$x1 := evalf\left(x0 - subs(x = x0, y = y0, fx) * \lambda0\right);$

$$-0.516129032 \tag{10}$$

$y1 := evalf\left(y0 - subs(x = x0, y = y0, fy) * \lambda0\right);$

$$2.322580645 \tag{11}$$

$f(x1, y1)$

$$3.096774193 \tag{12}$$

$mejora := evalf\left(f(x1, y1) - f(x0, y0)\right);$

$$-10.90322581 \tag{13}$$

$Linea3dN0 := pointplot3d\left(\left[\,[x0, y0, f(x0, y0)],\ [x1, y1, f(x1, y1)]\,\right], color = black, style = line, thickness = 2,\ \right):$

$Linea2dN0 := pointplot\left(\left[\,[x0, y0],\ [x1, y1]\,\right], color = black, style = line, thickness = 2,\ \right):$

display({*surface2*, *Linea3dN0*});
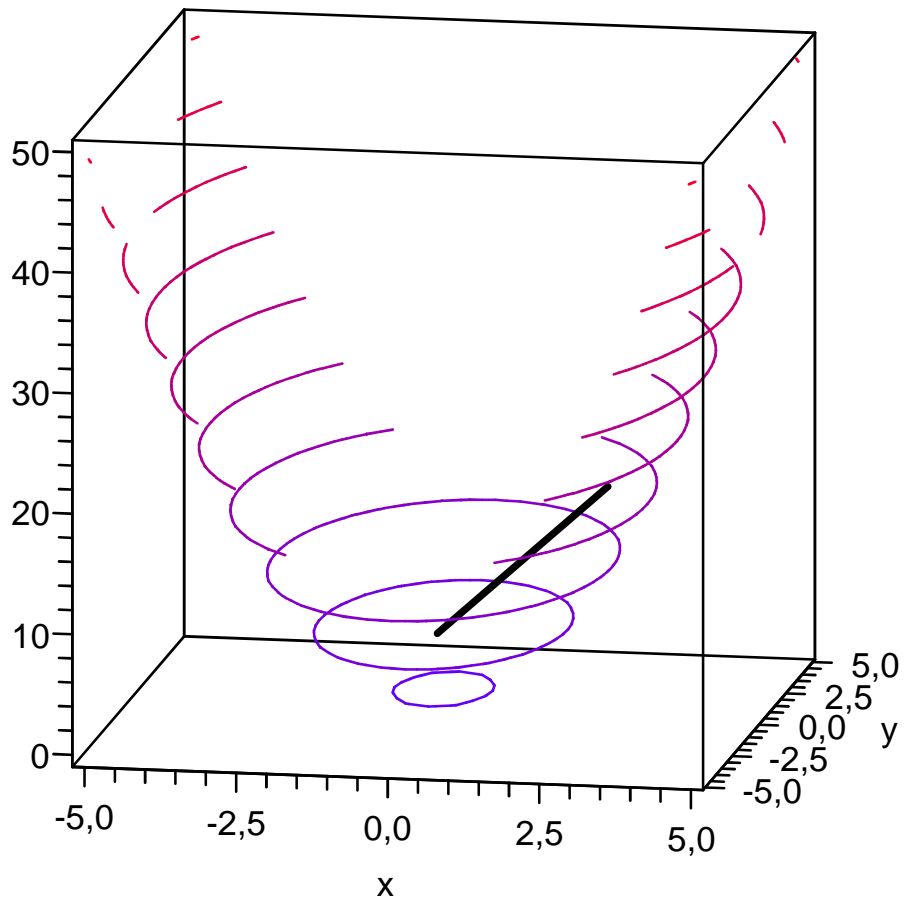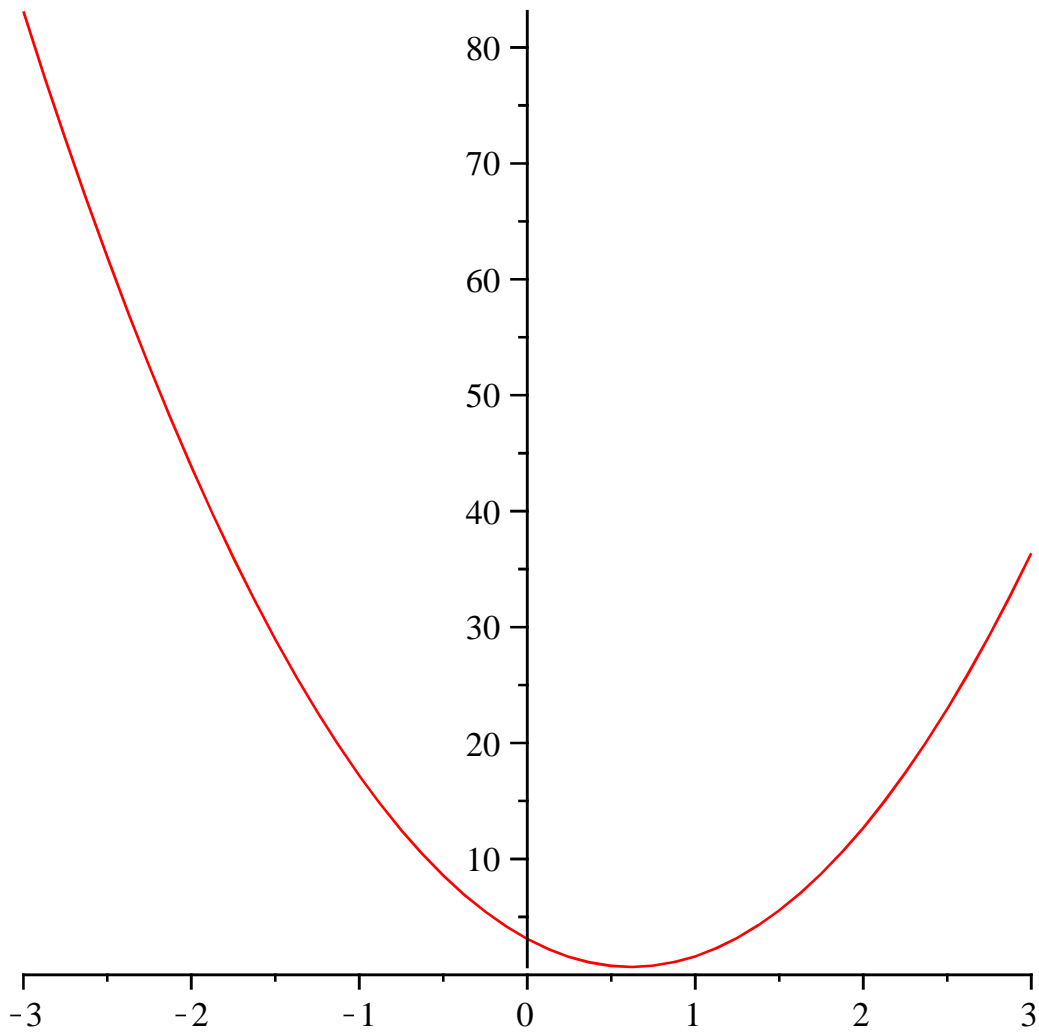

display({*contour2d*, *gradiente*, *Linea2dN0*});
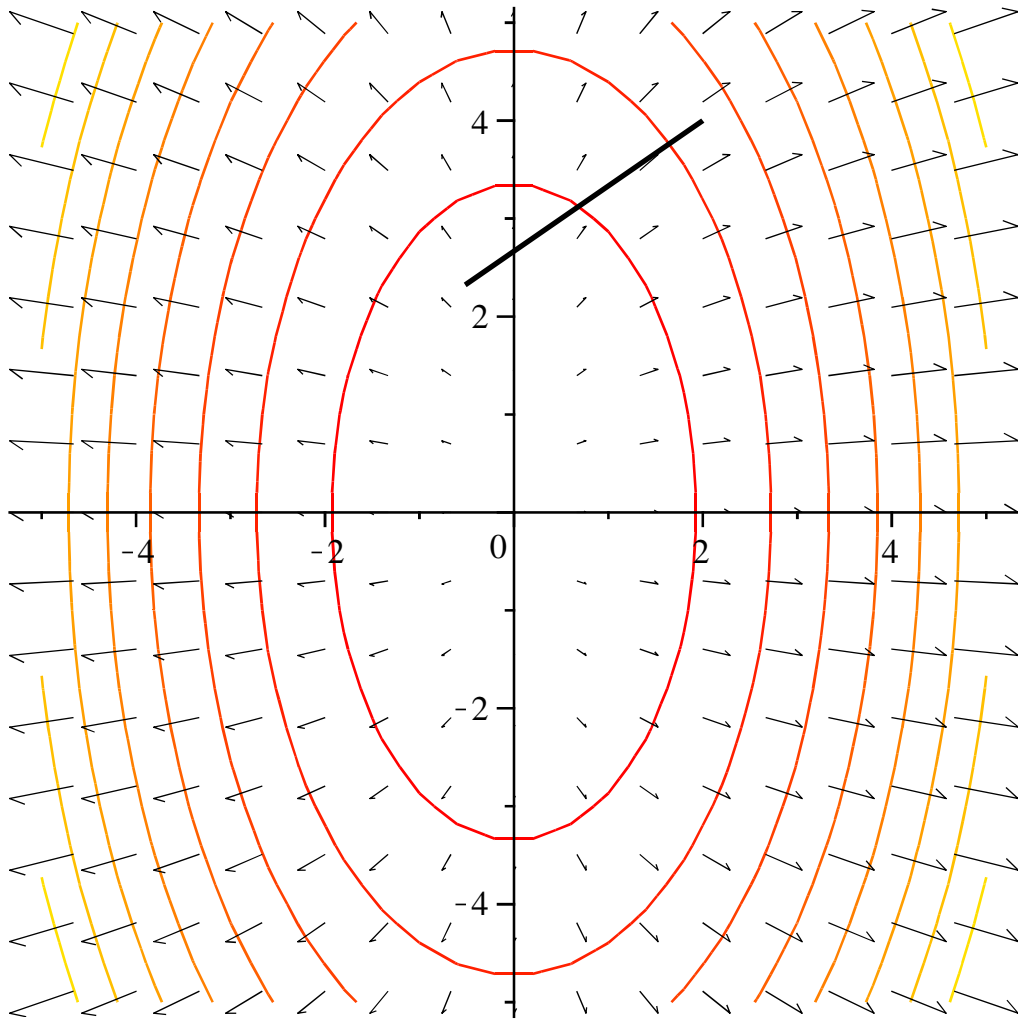
Iteración 1:

$[subs(x = x1, y = y1, fx), subs(x = x1, y = y1, fy)]$

$$[-1.548387096, 2.322580645] \tag{14}$$

$h1(\lambda) := (f(x1 - subs(x = x1, y = y1, fx) * \lambda, y1 - subs(x = x1, y = y1, fy) * \lambda))$

$$\lambda \rightarrow f(x1 - subs(x = x1, y = y1, fx)\ \lambda, y1 - subs(x = x1, y = y1, fy)\ \lambda) \tag{15}$$

$plot(h1(\lambda), \lambda = -3..3);$

$dh1 := diff(h1(\lambda), \lambda)$

$$-7.791883454 + 12.58688865\,\lambda \tag{16}$$

$\lambda1 := fsolve(dh1 = 0, \lambda);$

$$0.6190476194 \tag{17}$$

$x2 := evalf(x1 - subs(x = x1, y = y1, fx) * \lambda1);$

$$0.4423963137 \tag{18}$$

$y2 := evalf(y1 - subs(x = x1, y = y1, fy) * \lambda1);$
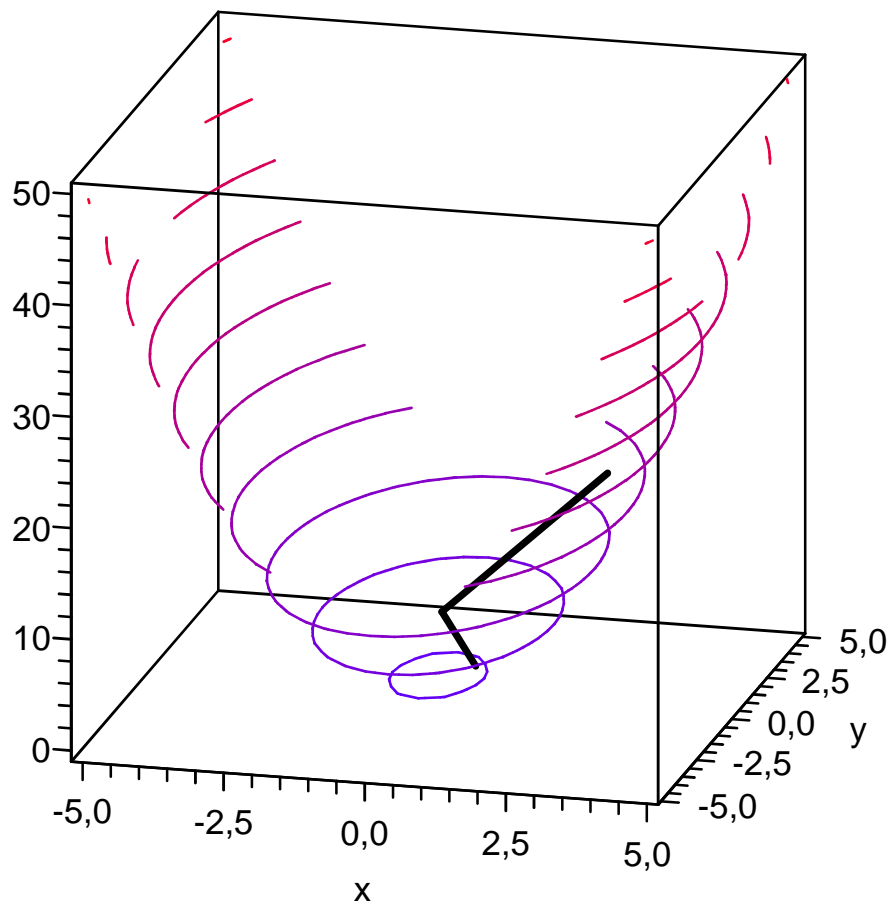
$$0.884792626 \tag{19}$$

$f(x2, y2)$

$$0.6850007431 \tag{20}$$

$mejora := evalf(f(x2, y2) - f(x1, y1));$

$$-2.411773450 \tag{21}$$

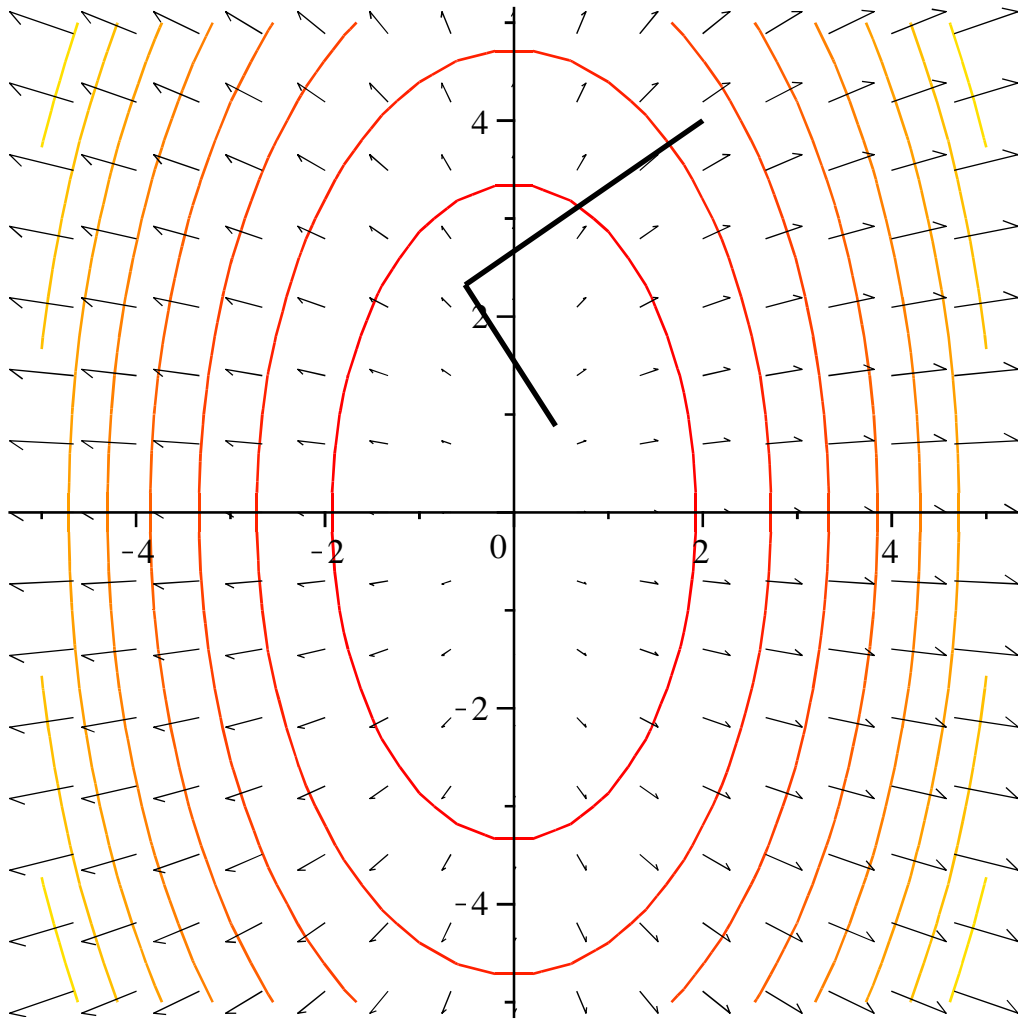$Linea3dN1 := pointplot3d([[x1, y1, f(x1, y1)], [x2, y2, f(x2, y2)]], color = black, style = line, thickness = 2):$

$Linea2dN1 := pointplot([[x1, y1], [x2, y2]], color = black, style = line, thickness = 2):$

$display(\{surface2, Linea3dN0, Linea3dN1\});$

$display(\{contour2d, gradiente, Linea2dN0, Linea2dN1\});$

Iteración 2:

$[subs(x = x2, y = y2, fx), subs(x = x2, y = y2, fy)]$

$$[1.327188941, 0.884792626] \tag{22}$$
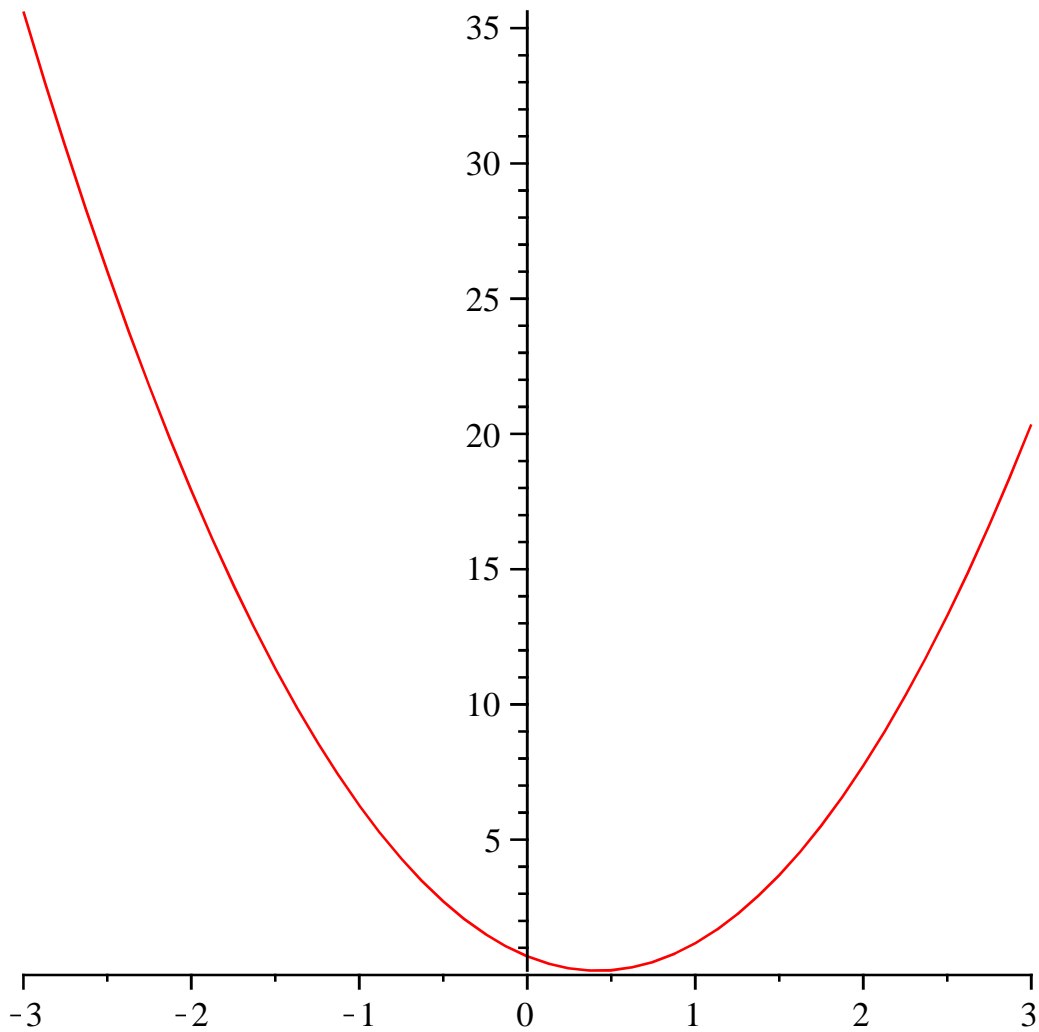
$h2(\lambda) := (f(x2 - subs(x = x2, y = y2, fx) * \lambda, y2 - subs(x = x2, y = y2, fy) * \lambda))$

$$\lambda \rightarrow f(x2 - subs(x = x2, y = y2, fx)\, \lambda, y2 - subs(x = x2, y = y2, fy)\, \lambda) \tag{23}$$

$plot(h2(\lambda), \lambda = -3 .. 3);$

$dh2 := diff\left(\mathrm{h2}(\lambda), \lambda\right)$

$$-2.544288476 + 6.067149446\,\lambda \tag{24}$$

$\lambda 2 := fsolve\left(dh2 = 0, \lambda\right);$

$$0.4193548385 \tag{25}$$

$x3 := evalf\left(x2 - subs(x = x2, y = y2, fx) * \lambda 2\right);$

$$-0.1141667903 \tag{26}$$

$y3 := evalf\left(y2 - subs(x = x2, y = y2, fy) * \lambda 2\right);$

$$0.5137505572 \tag{27}$$

$mejora := evalf\left(f(x3, y3) - f(x2, y2)\right);$

$$-0.5334798416 \tag{28}$$

$Linea3dN2 := pointplot3d\left(\left[\,[x2, y2, f(x2, y2)],\, [x3, y3, f(x3, y3)]\,\right], color = black, style = line, thickness = 2\right):$

$Linea2dN2 := pointplot\left(\left[\,[x2, y2],\, [x3, y3]\,\right], color = black, style = line, thickness = 2\right):$

$display\left(\{Linea3dN0,\, Linea3dN1,\, Linea3dN2,\, surface2\}\right);$
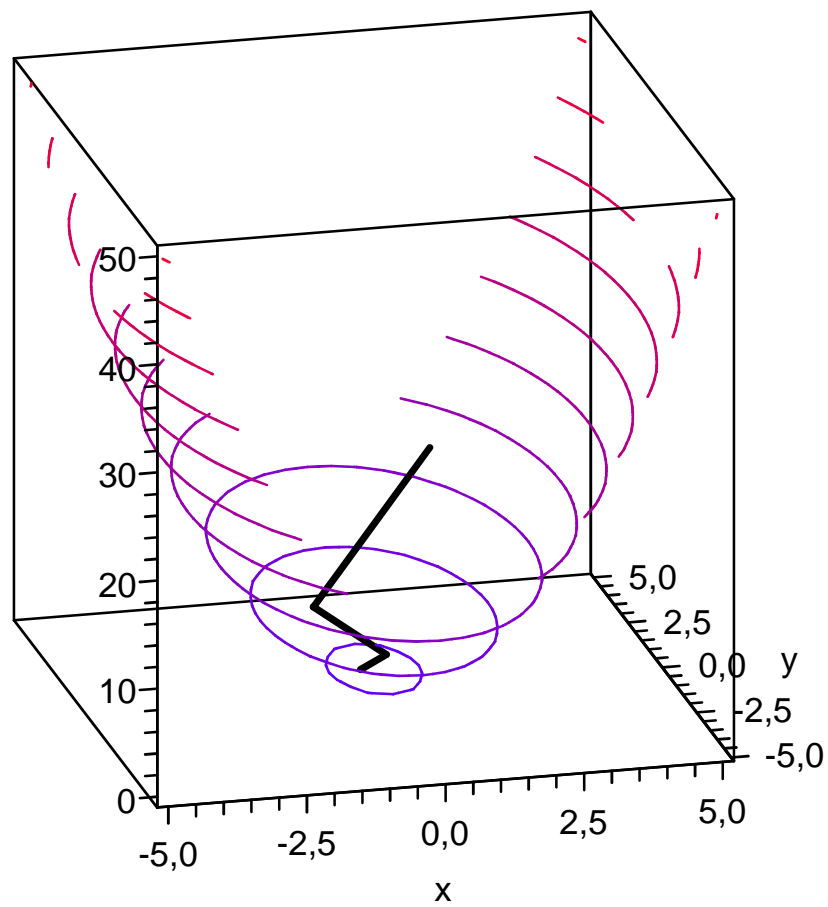
display({*contour2d, gradiente, Linea2dN0, Linea2dN1, Linea2dN2*});
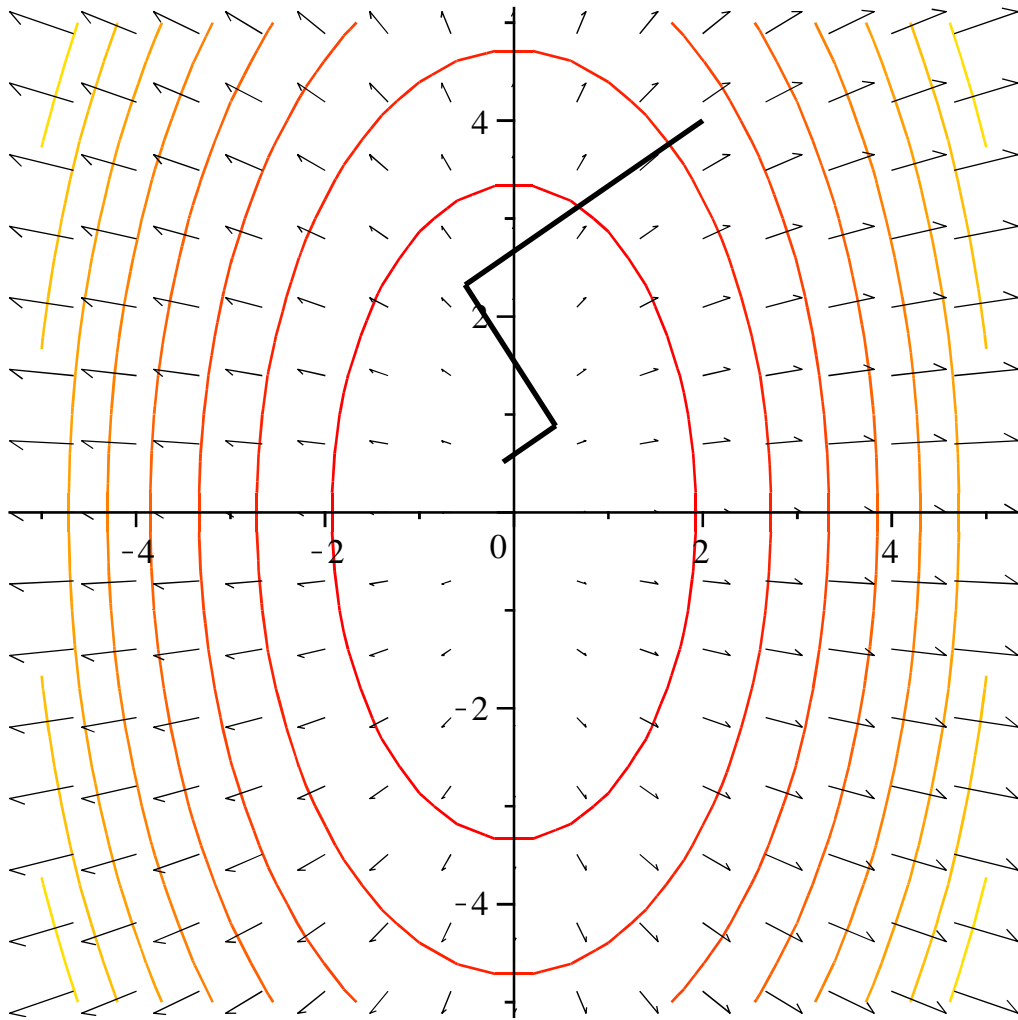
Iteración 3:

$[subs(x = x3, y = y3, fx), subs(x = x3, y = y3, fy)]$
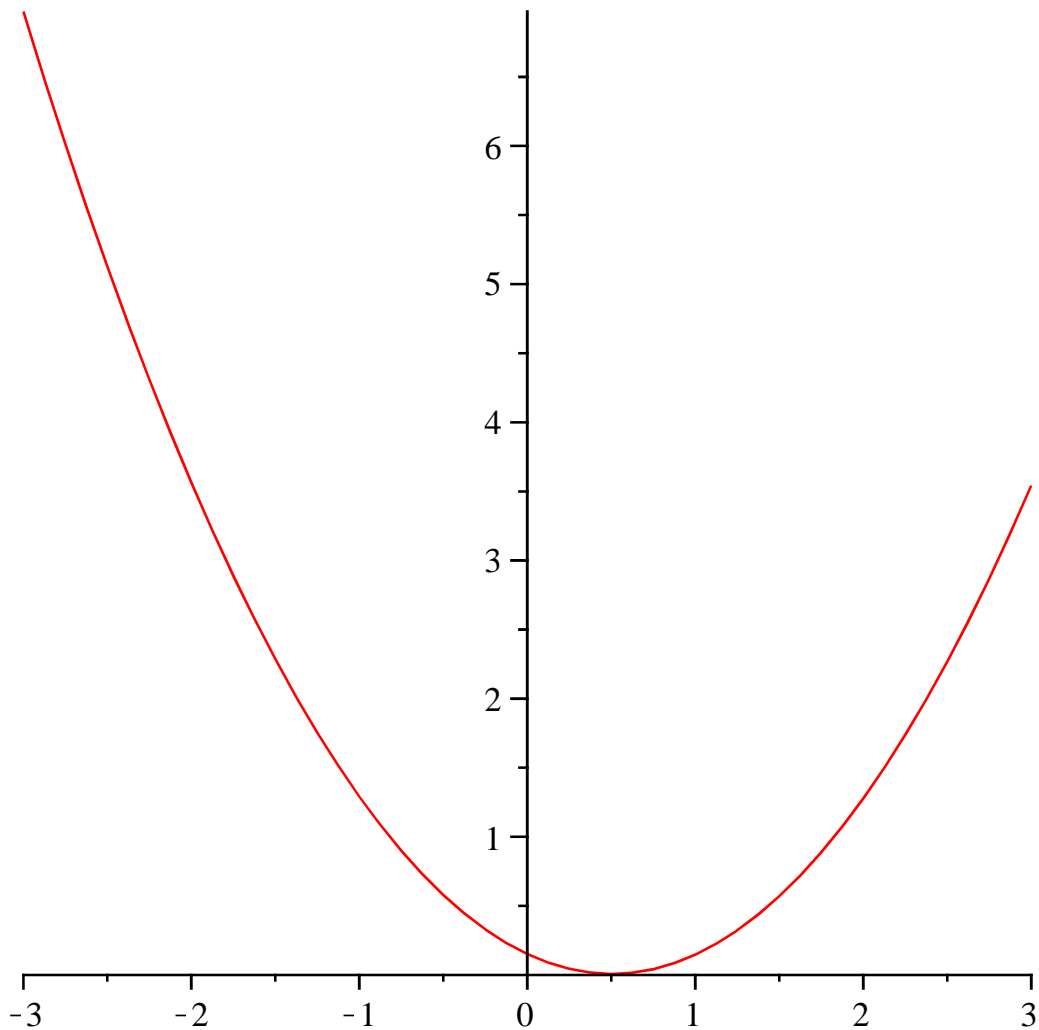
$$[-0.3425003709, 0.5137505572] \qquad (29)$$

$h3(\lambda) := (f(x3 - subs(x = x3, y = y3, fx) * \lambda, y3 - subs(x = x2, y = y2, fy) * \lambda))$

$$\lambda \rightarrow f(x3 - subs(x = x3, y = y3, fx)\,\lambda, y3 - subs(x = x2, y = y2, fy)\,\lambda) \qquad (30)$$

$plot(h3(\lambda), \lambda = -3..3);$

$dh3 := diff(h3(\lambda), \lambda)$

$$-0.5718692087 + 1.134777503\, \lambda \tag{31}$$

$\lambda3 := fsolve(dh3 = 0, \lambda);$

$$0.5039483134 \tag{32}$$

$x4 := evalf(x3 - subs(x = x3, y = y3, fx) * \lambda3);$

$$0.0584356940 \tag{33}$$

$y4 := evalf(y3 - subs(x = x3, y = y3, fy) * \lambda3);$

$$0.2548468304 \tag{34}$$
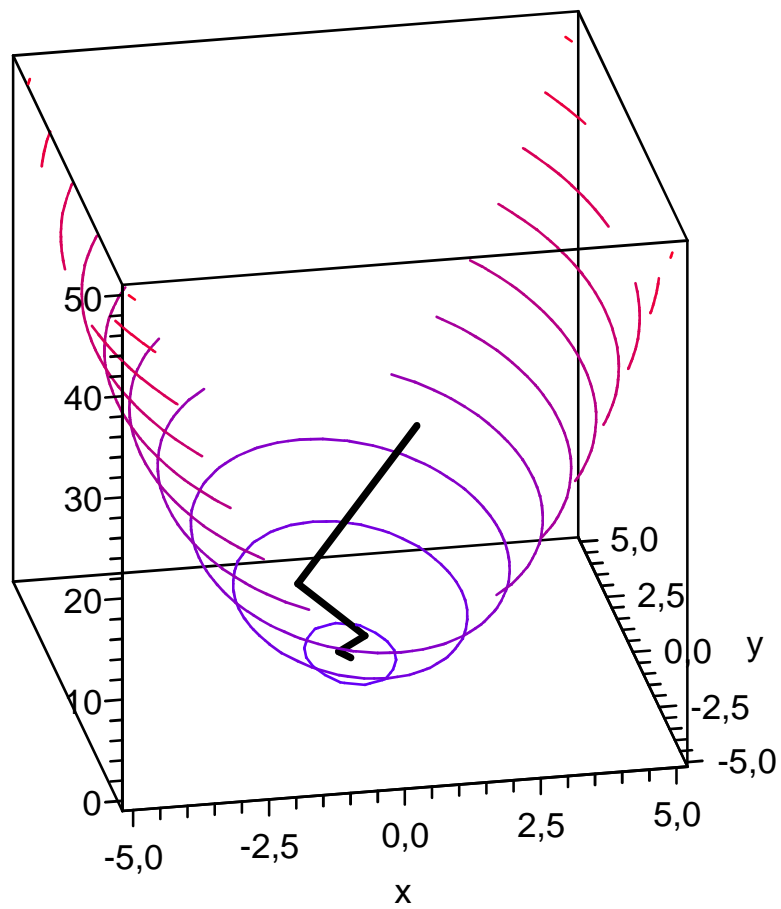
$mejora := evalf(f(x4, y4) - f(x3, y3));$

$$-0.1139253525 \tag{35}$$

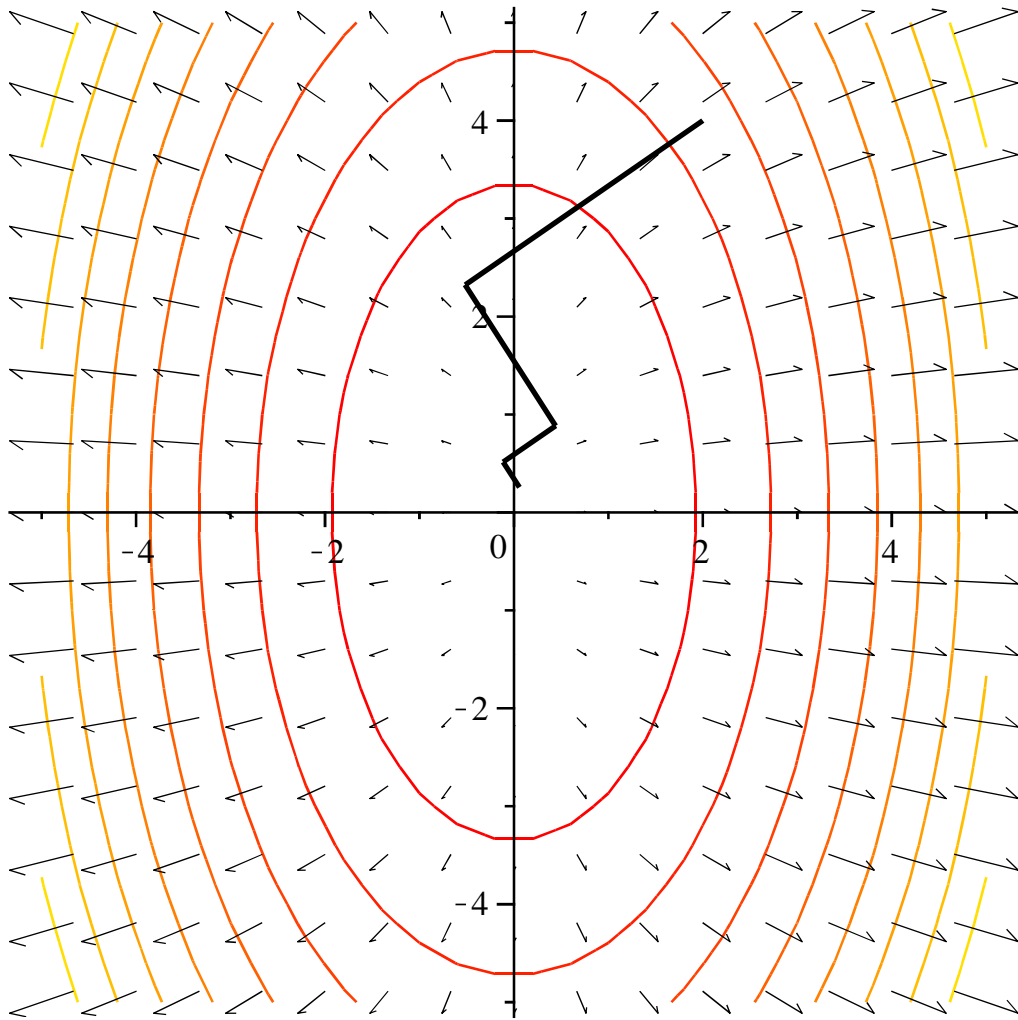$Linea3dN3 := pointplot3d([[x3, y3, f(x3, y3)], [x4, y4, f(x4, y4)]], color = black, style = line, thickness = 2):$

$Linea2dN3 := pointplot([[x3, y3], [x4, y4]], color = black, style = line, thickness = 2):$

$display(\{Linea3dN0, Linea3dN1, Linea3dN2, Linea3dN3, surface2\});$

$display(\{contour2d, gradiente, Linea2dN0, Linea2dN1, Linea2dN2, Linea2dN3\});$

Iteración 4:

$[\,subs(x = x4, y = y4, fx),\ subs(x = x4, y = y4, fy)\,]$

$$[\,0.1753070820,\ 0.2548468304\,] \tag{36}$$

Etc, etc.

Método de Newton: Ejemplo 2

restart;

$f(x, y) := \dfrac{3}{2}x^2 + \dfrac{1}{2} \cdot y^2$

$$(x, y) \rightarrow \dfrac{3}{2}\, x^2 + \dfrac{1}{2}\, y^2 \qquad\qquad (1)$$

$a := 5:$
with(*plots*) :
surface := *plot3d*($f(x, y)$, $x = -$a..a, $y = -$a..a , *shading* $= Z$, *axes* $= box$) :
surface2 := *plot3d*($f(x, y)$, $x = -$a..a, $y = -$a..a , *shading* $= Z$, *axes* $= box$, *style* $= contour$) :
contour2d := *contourplot*($f(x, y)$, $x = -$a..a, $y = -$a..a) :
gradiente := *gradplot*(f(x, y), $x = -$a..a, $y = -$a..a, *grid* $= [15, 15]$) :
display({*surface* });
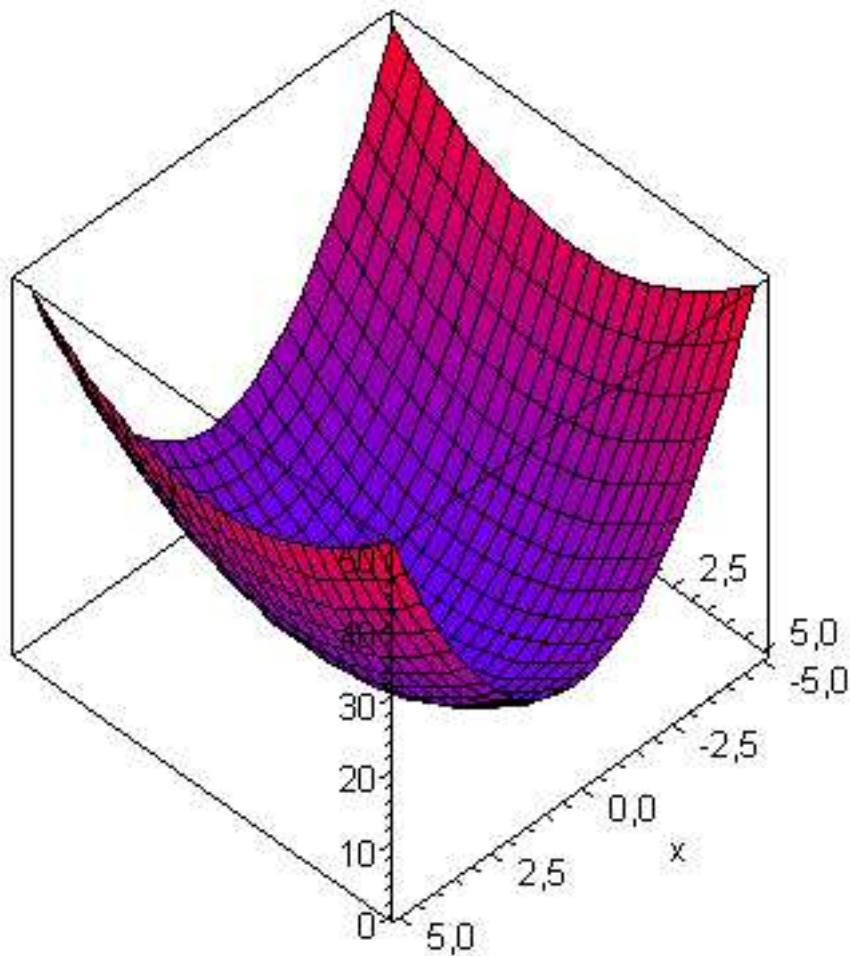
display({*surface2* });
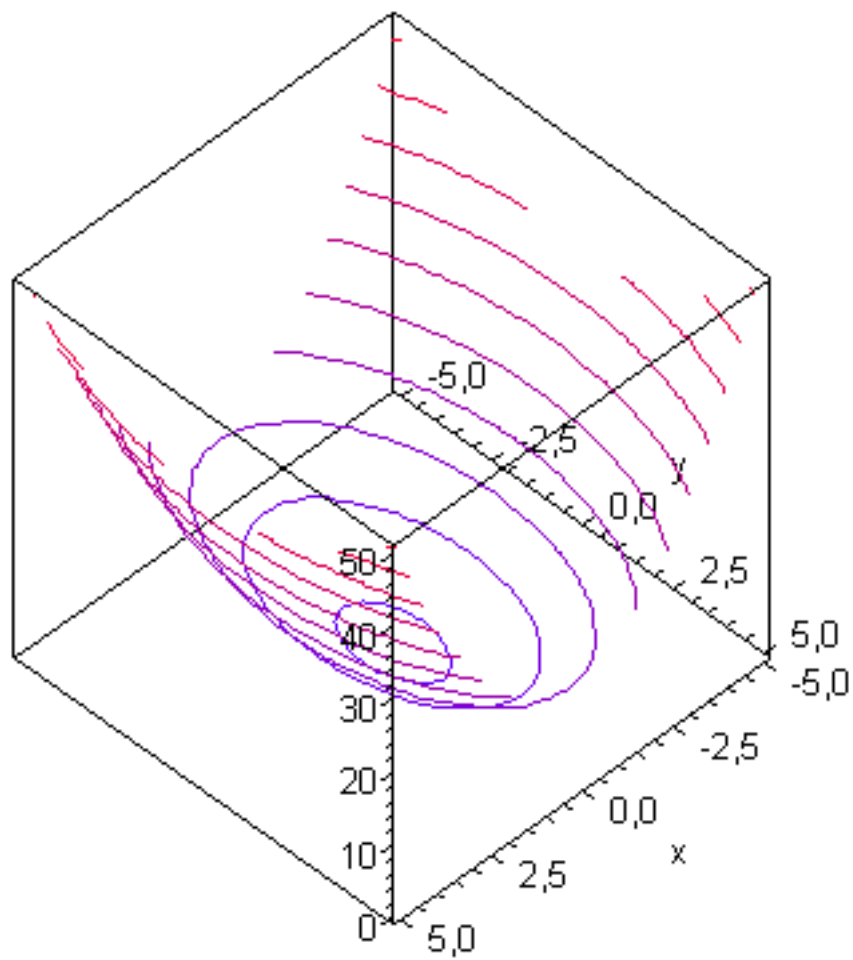
display({*contour2d, gradiente*});
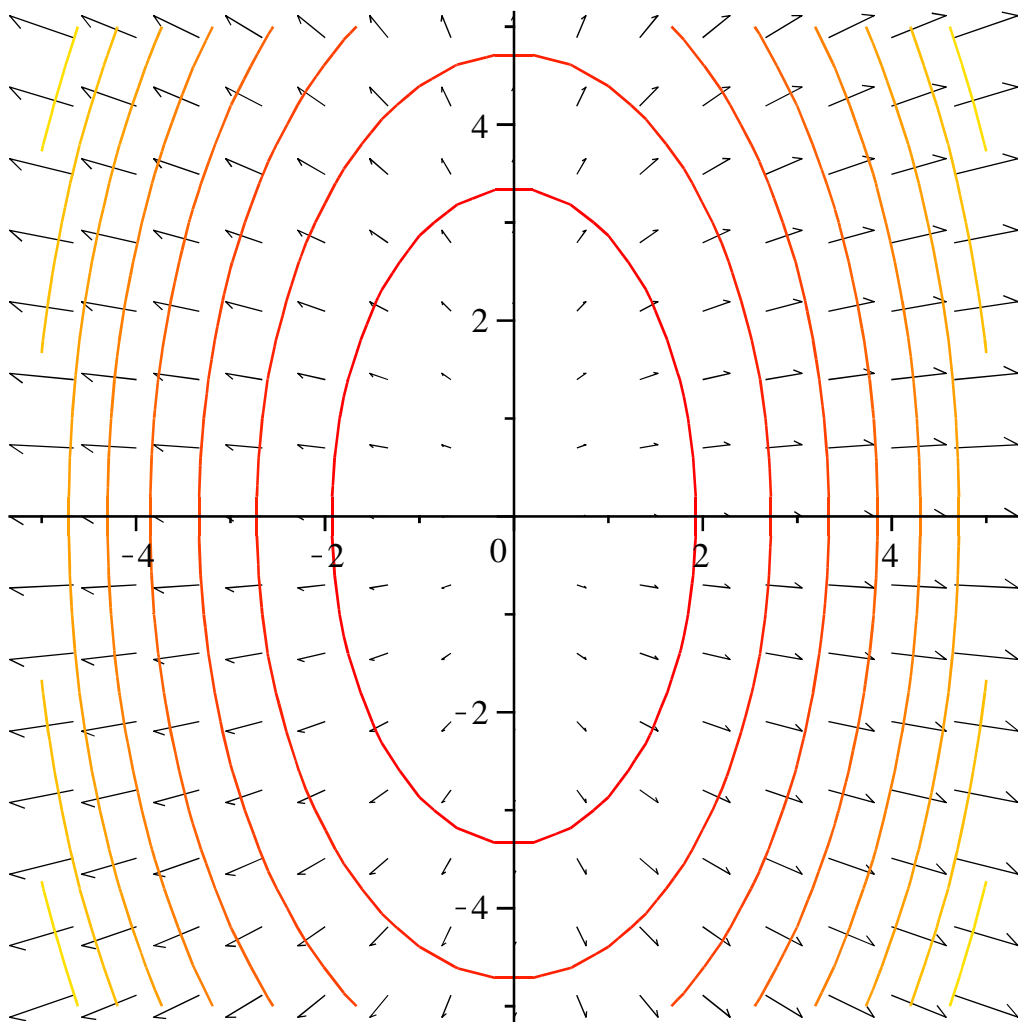
$fx := diff(f(x, y), x);$

$$3x \tag{2}$$

$fy := diff(f(x, y), y);$

$$y \tag{3}$$

$fxx := diff(fx, x);$

$$3 \tag{4}$$

$fyy := diff(fy, y);$

$$1 \tag{5}$$

$fxy := diff(fx, y);$

$$0 \tag{6}$$

$fyx := diff(fy, x);$

$$0 \tag{7}$$

$H := matrix([[fxx, fxy], [fyx, fyy]])$

$$\begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix} \tag{8}$$

$x0 := 2 : y0 := 4 :$

Iteración 0:

$[subs(x = x0, y = y0, fx), subs(x = x0, y = y0, fy)]$

$$[6, 4]$$ **(9)**

$f(x0, y0)$

$$14$$ **(10)**

$G := subs(x = x0, y = y0, evalm(H^{-1}))$

$$\begin{bmatrix} \dfrac{1}{3} & 0 \\ 0 & 1 \end{bmatrix}$$ **(11)**

$x1 := x0 - (evalm(G[1, 1]) \cdot subs(x = x0, y = y0, fx) + evalm(G[2, 1]) \cdot subs(x = x0, y = y0, fy))$

$$0$$ **(12)**

$y1 := y0 - (evalm(G[2, 1]) \cdot subs(x = x0, y = y0, fx) + evalm(G[2, 2]) \cdot subs(x = x0, y = y0, fy))$

$$0$$ **(13)**

$mejora := evalf(f(x1, y1) - f(x0, y0));$

$$-14.$$ **(14)**

$Linea3dN0 := pointplot3d([[x0, y0, f(x0, y0)], [x1, y1, f(x1, y1)]], color = black, style = line, thickness = 2,):$

$Linea2dN0 := pointplot([[x0, y0], [x1, y1]], color = black, style = line, thickness = 2,):$

$display(\{surface2, Linea3dN0\});$

$display(\{contour2d, gradiente, Linea2dN0\})$

Iteración 1:

$$[\, subs(x = x1, y = y1, fx), \, subs(x = x1, y = y1, fy)\,]$$
$$[0, 0] \tag{15}$$