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Brane-bulk matter relation for a purely conical codimension-2 brane world

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Abstract

We study gravity on an infinitely thin codimension-2 brane world, with purely conical singularities and in the presence of an induced gravity term on the brane. We show that in this approximation, the energy momentum tensor of the bulk is strongly related to the energy momentum tensor of the brane and thus the gravity dynamics on the brane are induced by the bulk content. This is in contrast with the gravity dynamics on a codimension-1 brane. We show how this strong result is relaxed after including a Gauss-Bonnet term in the bulk.

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1 Introduction

Recently, there have been many observational and theoretical motivations for the study of theories with extra spacetime dimensions and in particular the braneworld scenario. From the observational side, the current paradigm, supported by many recent observations [1], is that most of the energy content of our universe is of the form of dark matter and dark energy. Although there have been many plausible explanations for these dark components, it is challenging to try to explain these exotic ingredients of the universe using alternative gravity theories as such of the braneworlds. From the theoretical side, such extra-dimensional braneworld models are ubiquitous in theories like string or M-theory. Since these theories claim to give us a fundamental description of nature, it is important to study what kind of gravity dynamics they predict. The hope is to propose such modified gravity theories, which share many common features with general relativity, but at the same time give alternative non-conventional cosmology.

The essence of the braneworld scenario is that the Standard Model, with its matter and gauge interactions, is localized on a three-dimensional hypersurface (called brane) in a higher-dimensional spacetime. Gravity propagates in all spacetime (called bulk) and thus connects the Standard Model sector with the internal space dynamics. This idea, although quite old [2], gained momentum the last years [3, 4] because of its connection with string theory. The cosmology of this and other related models with one transverse to the brane extra dimension (codimension-1 brane models) is well understood (for a review see [5]). In the cosmological generalization of [4], the early times (high energy limit) cosmological evolution is modified by the square of the matter density on the brane, while the bulk leaves its imprints on the brane by the "dark radiation" term [6]. The presence of a bulk cosmological constant in [4] gives conventional cosmology at late times (low energy limit) [6].

In the above models there are strong theoretical arguments for including in the gravitational action extra curvature terms in addition to the higher dimensional Einstein-Hilbert term. The localized matter fields on the brane, which couple to bulk gravitons, can generate via quantum loops a localized four-dimensional kinetic term for gravitons [7]. The latter comes in the gravitational action as a four-dimensional scalar curvature term localized at the position of the brane (induced gravity) [8] [1]. In addition, curvature square terms in the bulk, in the Gauss-Bonnet combination, give the most general action with second-order field equations in five dimensions [10]. This correction is also motivated by string theory, where the Gauss-Bonnet term corresponds to the leading order quantum correction to gravity, and its presence guarantees a ghost-free action [11]. Let us note, however, that if the curvature squared terms are to play an important role in the low energy dynamics, one may face a difficulty when interpreting gravity as an effective field theory (in the sense that even higher dimensional operators would seem to be also relevant).

If curvature corrections are included in [4], the presence of the 3-brane gives similar

[1] There has been a lot of discussion about the potential problem of the extra polarization states of the massive gravitons regarding phenomenology (discontinuity problem), but the particular model seems to be consistent in a non-trivial way [9].

modifications to the standard cosmology. In the case of a pure Gauss-Bonnet term in the bulk, the early times cosmology is modified by a term proportional to the matter density on the brane to the power two thirds [12]. If an induced gravity term is included, the conventional cosmology is modified by the square root of the matter density on the brane at low energies [13]. Finally, if both curvature corrections are present, the cosmological evolution is affected mainly by the induced gravity corrections at late times.

In six dimensions and for codimension-2 braneworlds, the gravity dynamics appear even more radical and still a good understanding of cosmology and more generally gravity in such theories is missing. The most attractive feature of codimension-2 braneworlds is that the vacuum energy (tension) of the brane instead of curving the brane world-volume, merely induces a deficit angle in the bulk solution around the brane [14] (see also [15] for string-like defects in six dimensions). This looks very promising in relation to the cosmological constant problem, although in existing models, nearby curved solutions cannot be excluded [16], unless one allows for singularities more severe than conical in particular supersymmetric models [17]. It was soon realized [18] that one can only find nonsingular solutions if the brane energy momentum tensor is proportional to its induced metric, which means simply that it is pure tension. A non-trivial energy momentum tensor on the brane causes singularities in the metric around the braneworld, which necessitates the introduction of a cut-off (brane thickness) [19, 20, 21].

An alternative approach to study the gravitational dynamics of matter on infinitely thin branes is to modify the gravitational action as discussed previously. Indeed, it was shown in [22] that the inclusion of a Gauss-Bonnet term in the gravitational action allows a non-trivial energy momentum tensor on the brane, and in the thin brane limit, four dimensional gravity is recovered as the dynamics of the induced metric on the brane. The peculiar characteristic of this way to obtain four dimensional gravity for codimension-2 branes, is that, apart from the inclusion of a (deficit angle independent) cosmological constant term, there appear to be no corrections to the Einstein equations coming from the extra dimensions in the purely conical case. Another possibility, discussed in [23], is to study (instead of conical 3-branes) codimension-2 branes sitting at the intersection of codimension-1 branes in the presence again of a bulk Gauss-Bonnet term.

In this paper, we show that results similar to the Gauss-Bonnet case, *i.e.* four dimensional gravity for an arbitrary energy momentum tensor, can be obtained if we include in the action an induced gravity correction term instead. Again, in the purely conical case, there appear to be no corrections to the Einstein equations coming from the extra dimensions. The most important observation is that the brane and bulk energy momentum tensors are strongly related and any cosmological evolution on the brane is dictated by the bulk content. We also see how this correlation is relaxed in the case where bulk Gauss-Bonnet terms and brane induced gravity terms are combined. Thus, the necessary presence of extra curvature terms in the gravitational action in order to give non-trivial gravitational dynamics on a codimension-2 brane, leads to a realistic cosmological evolution on the brane in the thin brane limit, only if a Gauss-Bonnet term is included. However, let us note that the most physical way to investigate the dynamics of codimension-2 branes, is by giving thickness to the brane [20, 21].

The paper is organized as follows. In Sec.2 we present a six-dimensional induced gravity model and we discuss its gravitational dynamics on the brane. In Sec.3 we compare our results with the five-dimensional induced gravity case and discuss their differences. In Sec.4 we extent our analysis including the Gauss-Bonnet term in the bulk and finally in Sec.5 we summarize our results and conclude.

2 Induced gravity in six dimensions

We consider a six-dimensional theory with general bulk dynamics encoded in a Lagrangian $\mathcal{L}_{Bulk}$ and a 3-brane at some point $r = 0$ of the two-dimensional internal space with general dynamics $\mathcal{L}_{brane}$ in its world-volume. If we include an induced curvature term localized at the position of the brane, the total action is written as:

$$\begin{aligned} \mathcal{S} &= \frac{M_6^4}{2}\left[\int d^6x\sqrt{G}R^{(6)} + r_c^2\int d^4x\sqrt{g}R^{(4)}\ \frac{\delta(r)}{2\pi L}\right] \\ &+ \int d^6x\mathcal{L}_{Bulk} + \int d^4x\mathcal{L}_{brane}\ \frac{\delta(r)}{2\pi L}\ . \end{aligned} \tag{1}$$

In the above action, M_6 is the six-dimensional Planck mass, M_4 is the four-dimensional one and $r_c = M_4/M_6^2$ the cross over scale between four-dimensional and six-dimensional gravity. The above induced term has been written in the particular coordinate system in which the metric is

$$ds_6^2 = g_{\mu\nu}(x,r)dx^\mu dx^\nu + dr^2 + L^2(x,r)d\theta^2\ , \tag{2}$$

where $g_{\mu\nu}(x,0)$ is the braneworld metric and x^μ denote four non-compact dimensions, $\mu = 0,...,3$, whereas r,θ denote the radial and angular coordinates of the two extra dimensions (the r direction may or may not be compact and the θ coordinate ranges form 0 to 2π). Capital M,N indices will take values in the six-dimensional space. Note, that we have assumed that there exists an azimuthal symmetry in the system, so that both the induced four-dimensional metric and the function L do not depend on θ. The normalization of the δ-function is the one discussed in [24].

To obtain the braneworld equations we expand the metric around the brane as

$$L(x,r) = \beta(x)r + O(r^2)\ . \tag{3}$$

At the boundary of the internal two-dimensional space where the 3-brane is situated, the function L behaves as $L'(x,0) = \beta(x)$, where a prime denotes derivative with respect to r. As we will see in the following, the demand that the space in the vicinity of the conical singularity is regular, imposes the supplementary conditions that $\partial_\mu\beta = 0$ and $\partial_r g_{\mu\nu}(x,0) = 0$.

The Einstein equations which are derived from the above action in the presence of the 3-brane are

$$G_M^{(6)N} + r_c^2 G_\mu^{(4)\nu} \delta_M^\mu \delta_\nu^N \frac{\delta(r)}{2\pi L} = \frac{1}{M_6^4} \left[T_M^{(B)N} + T_\mu^{(br)\nu} \delta_M^\mu \delta_\nu^N \frac{\delta(r)}{2\pi L} \right] , \tag{4}$$

with $G_M^{(6)N}$ and $G_\mu^{(4)\nu}$ the six-dimensional and the four-dimensional Einstein tensors respectively, $T_M^{(B)N}$ the bulk energy momentum tensor and $T_\mu^{(br)\nu}$ the brane one.

The six-dimensional Ricci tensor components can be written in terms of the four-dimensional ones, the extrinsic curvature $K_{\mu\nu}$ and the L function as [21]

$$\begin{aligned}
R_\mu^{(6)\nu} &= -\frac{(\sqrt{g} L K_\mu^\nu)'}{2\sqrt{g} L} + R_\mu^{(4)\nu} - \frac{\nabla_\mu^{(4)} \partial^\nu L}{L} , && (5) \\
R_\theta^{(6)\theta} &= -\frac{(\sqrt{g} L')'}{\sqrt{g} L} - \frac{\Box^{(4)} L}{L} , && (6) \\
R_r^{(6)r} &= -\frac{L''}{L} - \frac{1}{2} K' - \frac{1}{4} K_\mu^\nu K_\nu^\mu , && (7) \\
R_{\mu r}^{(6)} &= -\frac{\partial_\mu L'}{L} + \frac{K_\mu^\nu \partial_\nu L}{2L} + \frac{1}{2} \nabla_{(4)}^\nu (K_{\mu\nu} - g_{\mu\nu} K) . && (8)
\end{aligned}$$

The extrinsic curvature in the particular gauge $g_{rr} = 1$ that we are considering, is given by $K_{\mu\nu} = g'_{\mu\nu}$. The above decomposition will be helpful in the following for finding the induced dynamics on the brane.

We will now use the fact that the second derivatives of the metric functions contain δ-function singularities at the position of the brane. The nature of the singularity then gives the following relations [22]

$$\begin{aligned}
\frac{L''}{L} &= -(1 - L') \frac{\delta(r)}{L} + \mathrm{non-singular\ terms} , && (9) \\
\frac{K'_{\mu\nu}}{L} &= K_{\mu\nu} \frac{\delta(r)}{L} + \mathrm{non-singular\ terms} . && (10)
\end{aligned}$$

From the above singularity expressions and the decomposition (5)-(7), we can match the singular parts of the Einstein equations (4) and get the following “boundary” Einstein equations

$$G_\mu^{(4)\nu}|_0 = \frac{1}{r_c^2 M_6^4} T_\mu^{(br)\nu} + \frac{2\pi}{r_c^2} (1 - \beta) \delta_\mu^\nu + \frac{2\pi L}{2 r_c^2} (K_\mu^\nu - \delta_\mu^\nu K)|_0 . \tag{11}$$

where we denote by $|_0$ the value of the corresponding function at $r = 0$.

We will now make the assumption that the singularity is purely conical. In the opposite case, there would be curvature singularities $R^{(6)} \propto 1/r$, because in the Ricci tensor $R_{\mu\nu}^{(6)}$ there are terms of the form [22]

$$R_{\mu\nu}^{(6)} = -\frac{1}{2} \frac{L'}{L} \partial_r g_{\mu\nu} + ... = -\frac{\partial_r g_{\mu\nu}}{2r} + \mathcal{O}(1) , \tag{12}$$

which in the vicinity of $r = 0$ are singular if $\partial_r g_{\mu\nu}(x, 0) \neq 0$. The absence of this type of singularities imposes the requirement that $K_{\mu\nu}|_0 = 0$. Then, the Einstein equations (11) reduce to

$$G^{(4)}_{\mu\nu}|_0 = \frac{1}{r_c^2 M_6^4} T^{(br)}_{\mu\nu} + \frac{2\pi}{r_c^2}(1-\beta) g_{\mu\nu}|_0 \ . \tag{13}$$

The four-dimensional Einstein equations (13) describe the gravitational dynamics on the brane. The effective four-dimensional Planck mass and cosmological constant are simply

$$\begin{aligned} M_{Pl}^2 \ = \ M_4^2 \ &= \ r_c^2 M_6^4 \ , \qquad (14) \\ \Lambda_4 \ &= \ \lambda - 2\pi M_6^4 (1-\beta) \qquad (15) \end{aligned}$$

where λ is the contribution of the vacuum energy of the brane fields. The normalization of Λ_4 is defined by the convention that the four dimensional Einstein equation reads $G_{\mu\nu} = \frac{1}{M_{Pl}^2}(T_{\mu\nu} - \Lambda_4 g_{\mu\nu})$. Note that in contrast to the case of [22], the four dimensional Planck mass is independent of the deficit angle.

Furthermore, it is interesting to see that contrary to the five-dimensional case, the induced gravity term in six dimensions does not introduce any correction terms, apart from a cosmological term, in the four-dimensional Einstein equations on the brane, unless singularities of other type than conical are allowed in the theory, and a regularization scheme is employed. In the latter case, the last term of the right hand side of (11) would provide information of the bulk physics. This absence of corrections in the purely conical case, is exactly what happens also in the case of the bulk Gauss-Bonnet theory [22].

What is important to note at this point, is that although we have found a "boundary" Einstein equation, there is more information about the dynamics of the theory contained in the full six-dimensional Einstein equations. As we will shortly see, the consistency of the Einstein equations (4) will give us information about the matter content of the brane.

Firstly, the (μr) component of the Einstein equations (4) evaluated at $r = 0$ and with the help of (8) gives

$$\left.\frac{\partial_\mu L'}{L}\right|_0 = -\frac{1}{M_6^4} T_{\mu r}|_0 \ . \tag{16}$$

This equation is consistent only for $\beta = L'|_0 = \text{const.}$, because otherwise $\partial_\mu L'/L$ will have an $1/r$ singularity. This also means that $\frac{\Box^{(4)} L}{L}|_0 = 0$. This does not exclude, however, the possibility that there is x dependence of L close to the brane. Only the linear term in r of the L expansion is constant.

Evaluating the (rr) component of the Einstein equation (4) at the position of the brane $r = 0$ with the help of (5)-(7), we get

$$R^{(4)}|_0 = -\frac{2}{M_6^4} T^{(B)r}_{\ r}|_0 \ . \tag{17}$$

On the other hand, the trace of the "boundary" Einstein equation (13) gives us the relation

$$R^{(4)}|_0 = -\frac{1}{r_c^2 M_6^4} T_\mu^{(br)\mu} - \frac{8\pi}{r_c^2}(1-\beta) \ , \tag{18}$$

Combining now equations (17) and (18) we arrive at a relation between the bulk and brane energy momentum tensors

$$T_r^{(B)r}|_0 = \frac{1}{2r_c^2}\left[T_\mu^{(br)\mu} + 8\pi M_6^4(1-\beta)\right] \ . \tag{19}$$

This equation is the central result of our paper. It constitutes a very strong tuning relation between brane ($T_\mu^{(br)\mu}$) and bulk ($T_r^{(B)r}|_0$) matter. It shows that, in order to have some cosmological evolution on the brane (*i.e.*, time dependent $T_{\mu\nu}^{(br)}$) and since β is constant, the bulk content should evolve as well in a precisely tuned way.

In other words, one can say that the brane cosmological evolution is induced by the bulk content. The "boundary" Einstein equation may not contain any bulk information at first sight, but the admissible brane energy momentum tensor is dictated by the bulk energy momentum tensor. It is difficult to justify why such a relation would be physically natural.

3 Comparison with five dimensions

In this section we will compare the above result with what is happening in five dimensions. The action of a general five-dimensional theory with a 3-brane at the point $r = 0$ of the extra dimension, and with an induced curvature term localized on it, is

$$\begin{aligned}\mathcal{S} &= \frac{M_5^3}{2}\left[\int d^5x\sqrt{G}R^{(5)} + r_c\int d^4x\sqrt{g}R^{(4)}\ \delta(r)\right] \\ &+ \int d^5x\mathcal{L}_{Bulk} + \int d^4x\mathcal{L}_{brane}\ \delta(r) \ . \end{aligned}\tag{20}$$

In the above action, M_5 is the five-dimensional Planck mass, M_4 is the four dimensional one and $r_c = M_4^2/M_5^3$ the cross over scale of the five-dimensional theory. The above induced term has been written in the particular coordinate system in which the metric is written as

$$ds_5^2 = g_{\mu\nu}(x,r)dx^\mu dx^\nu + dr^2 \ , \tag{21}$$

where the x^μ denote the usual four non-compact dimensions, $\mu = 0, ..., 3$, whereas r denotes the radial extra coordinate and capital M,N indices will now take values in the five-dimensional space.

The Einstein equations which are derived from the action (20) in the presence of the 3-brane are

$$G_M^{(5)N} + r_c G_\mu^{(4)\nu} \delta_M^\mu \delta_\nu^N \delta(r) = \frac{1}{M_5^3} \left[T_M^{(B)N} + T_\mu^{(br)\nu} \delta_M^\mu \delta_\nu^N \delta(r) \right] \ , \tag{22}$$

with $T_M^{(B)N}$ the bulk energy momentum tensor and $T_\mu^{(br)\nu}$ the brane one. The five-dimensional Ricci tensor components can be written in terms of the four-dimensional ones and the extrinsic curvature as

$$\begin{aligned} R_\mu^{(5)\nu} &= -\frac{(\sqrt{g} K_\mu^\nu)'}{2\sqrt{g}} + R_\mu^{(4)\nu} \ , & (23) \\ R_r^{(5)r} &= -\frac{1}{2} K' - \frac{1}{4} K_\mu^\nu K_\nu^\mu \ , & (24) \\ R_{\mu r}^{(5)} &= \frac{1}{2} \nabla_{(4)}^\nu (K_{\mu\nu} - g_{\mu\nu} K) \ . & (25) \end{aligned}$$

The extrinsic curvature is $K_{\mu\nu} = g'_{\mu\nu}$ in the particular gauge we are using. We assume that there is a Z_2 symmetry around $r = 0$, so that we can write the extrinsic curvature as

$$K'_{\mu\nu} = K_{\mu\nu} \delta(r) + \mathrm{non-singular\ terms} \ . \tag{26}$$

Then, matching the singular parts of the Einstein equations (22) using (23)-(25) we get the following "boundary" Einstein equations

$$G_\mu^{(4)\nu}|_0 = \frac{1}{r_c M_5^3} T_\mu^{(br)\nu} + \frac{1}{2r_c} (K_\mu^\nu - \delta_\mu^\nu K)|_0 \ . \tag{27}$$

These four-dimensional Einstein equations describe the gravitational dynamics on a codimension-1 brane with induced gravity, and the extrinsic curvature terms in the right hand side of the equations can be considered as corrections to the four-dimensional standard Einstein equations.

We can get further information evaluating the Einstein equations (22) at the position of the brane $r = 0$. The (μr) component of the Einstein equation (22) evaluated at $r = 0$, with the help of (25) gives

$$\nabla_{(4)}^\nu (K_{\mu\nu} - g_{\mu\nu} K)|_0 = \frac{2}{M_5^3} T_{\mu r}^{(B)} \ . \tag{28}$$

There is no condition coming from this equation, in contrast with what is happening in six dimensions, in which the $T_{\mu r}^{(B)}$ component of the bulk energy momentum tensor is restricted. The (rr) component of the Einstein equation (22) evaluated at $r = 0$, with the help of (24) gives

$$R^{(4)}|_0 + \frac{1}{4} (K_\mu^\nu K_\nu^\mu - K^2)|_0 = -\frac{2}{M_5^3} T_r^{(B)r}|_0 \ . \tag{29}$$

From the above equation we see that the bulk matter content does not necessarily dictate the brane cosmological evolution. This is because the extrinsic curvature on the brane $K_{\mu\nu}$ can be non-trivial and it is this one which plays the most crucial role in the cosmology. In other words, in five dimensions it is the freedom of the brane to bend in the extra dimension which makes the evolution not tuned to the bulk matter content. The absence of such bending in six dimensions (imposed by singularity arguments) gives the bulk the crucial role for how the brane evolves.

4 Inclusion of a Gauss-Bonnet term

In this section we will introduce a Gauss-Bonnet term in the six-dimensional action (1) and see how the above results are modified. In this case the action (1) is augmented by the term

$$S_{GB} = \frac{M_6^4 \alpha}{2} \int d^6x (R^{(6)\ 2} - 4R^{(6)\ 2}_{MN} + R^{(6)\ 2}_{MNK\Lambda}) \ . \tag{30}$$

Then the variation of the above action introduces an extra term in the left hand side of the Einstein equations (4),

$$\begin{aligned} H_M^N &= -\alpha \Big[\frac{1}{2}\delta_M^N (R^{(6)\ 2} - 4R^{(6)\ 2}_{K\Lambda} + R^{(6)\ 2}_{ABK\Lambda}) - 2R^{(6)} R^{(6)N}_M \\ &\quad +4R^{(6)}_{MP} R^{NP}_{(6)} \quad +4R^{(6)\ \ \ N}_{KMP} R^{KP}_{(6)} - 2R^{(6)}_{MK\Lambda P} R^{NK\Lambda P}_{(6)}\Big] \ . \end{aligned} \tag{31}$$

Equating the singular terms of the Einstein equations by the standard procedure of section 2, and demanding that the singularity is purely conical, we obtain the following "boundary" Einstein equations

$$G^{(4)}_{\mu\nu}|_0 = \frac{1}{M_6^4(r_c^2 + 8\pi(1-\beta)\alpha)} T^{(br)}_{\mu\nu} + \frac{2\pi(1-\beta)}{r_c^2 + 8\pi(1-\beta)\alpha} g_{\mu\nu}|_0 \ . \tag{32}$$

Equation (32) describes the gravitational dynamics on a codimension-2 brane when both induced gravity and Gauss-Bonnet correction terms are present. The effective four-dimensional Planck mass and cosmological constant are simply

$$M_{Pl}^2 = M_6^4(r_c^2 + 8\pi(1-\beta)\alpha) \ , \tag{33}$$

$$\Lambda_4 = \lambda - 2\pi M_6^4 (1-\beta) \ , \tag{34}$$

where λ is the brane tension. Note that the Planck mass this time can depend on the deficit angle. This is an effect of solely the bulk Gauss-Bonnet term.

Evaluating the the (rr) component of the Einstein equation at the position of the brane $r = 0$ we obtain the following relation

$$R^{(4)}|_0 + \alpha(R^{(4)\ 2} - 4R^{(4)\ 2}_{\kappa\lambda} + R^{(4)\ 2}_{\alpha\beta\kappa\lambda})|_0 = -\frac{2}{M_6^2} T_r^{(B)r}|_0 \ . \tag{35}$$

From (35) we see that there can be no relation between the extra dimensional component $T_r^{(B)r}|_0$ of the bulk energy momentum tensor at the position of the brane with the brane energy momentum tensor $T_\mu^{(br)\mu}$. This is due to the appearance of the Riemann curvature, which cannot be evaluated from previous equations (the Ricci tensor and scalar can be substituted from (32)). Instead, using (32) and (35), one can *solve* for $R^{(4)\ 2}_{\alpha\beta\kappa\lambda}|_0$ as a function of the brane and bulk matter at the position of the brane.

5 Conclusions

In this work, we have considered the gravitational dynamics of conical codimension-2 branes of infinitesimal thickness. The assumption of the absence of metric singularities more severe than conical, imposes a strong constraint in the allowed matter on the brane. In particular we have considered theories with codimension-2 branes which are augmented with an induced gravity term on the brane and a Gauss-Bonnet term in the bulk. These additions where known in the literature to give enough freedom for the brane dynamics to admit general matter.

The crucial observation of this paper is that the dynamics of the latter theories is not exhausted by studying the "boundary" Einstein equation, which is exactly four-dimensional and bears no information of the internal space (modulo a cosmological constant contribution). In the case of a pure brane induced gravity term, the higher dimensional Einstein equations evaluated at the position of the brane, give a very precise and strong relation between the matter on the brane and the matter in the bulk in the vicinity of the brane. So, for example, in a cosmological setting, in order to have a cosmological evolution of general energy density and pressure on the brane, the bulk matter should organize itself so that the above-mentioned brane-bulk matter relation is satisfied. In other words, the bulk energy content is the primary factor for the cosmological evolution on the brane. Alternatively, for a static matter distribution on the brane to be possible, there should exist its bulk matter "image".

This matter constraint is due to the fact that the absence of severe singularities criterion prevents the brane to bend in the internal space. The presence of this bending was the reason why in five dimensions the energy density on the brane was unrelated to that of the bulk. The latter influenced the cosmological evolution on the brane (*e.g.,* through the "dark radiation" term) but did not completely determine the evolution on the brane as in the six-dimensional case.

This strong relation, that we have noted in this paper, can be avoided with the inclusion of a bulk Gauss-Bonnet term. An even more natural way to achieve this is to relax the requirement of purely conical branes and admit general brane solutions with an appropriate regularization (thickening of the brane), so that the singularities are smoothened. In view of the difficulties related to the Gauss-Bonnet term in the context of effective field theory, this work points out that the thickening of the brane is the most physical direction that one should follow in order to discuss the dynamics of codimension-2 branes.

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