FI2A2 ELECTROMAGNETISMO
Clase 17
Magnetostática-II

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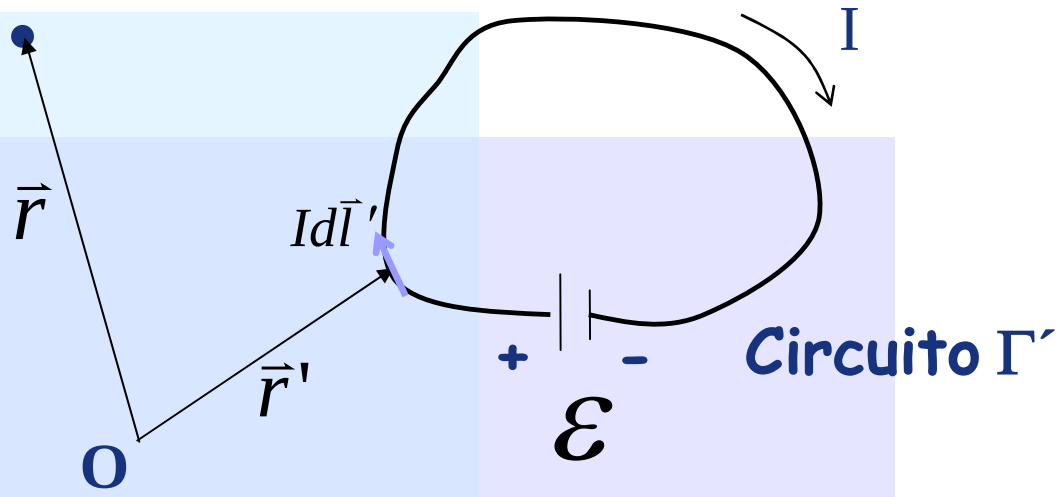
Campo magnético

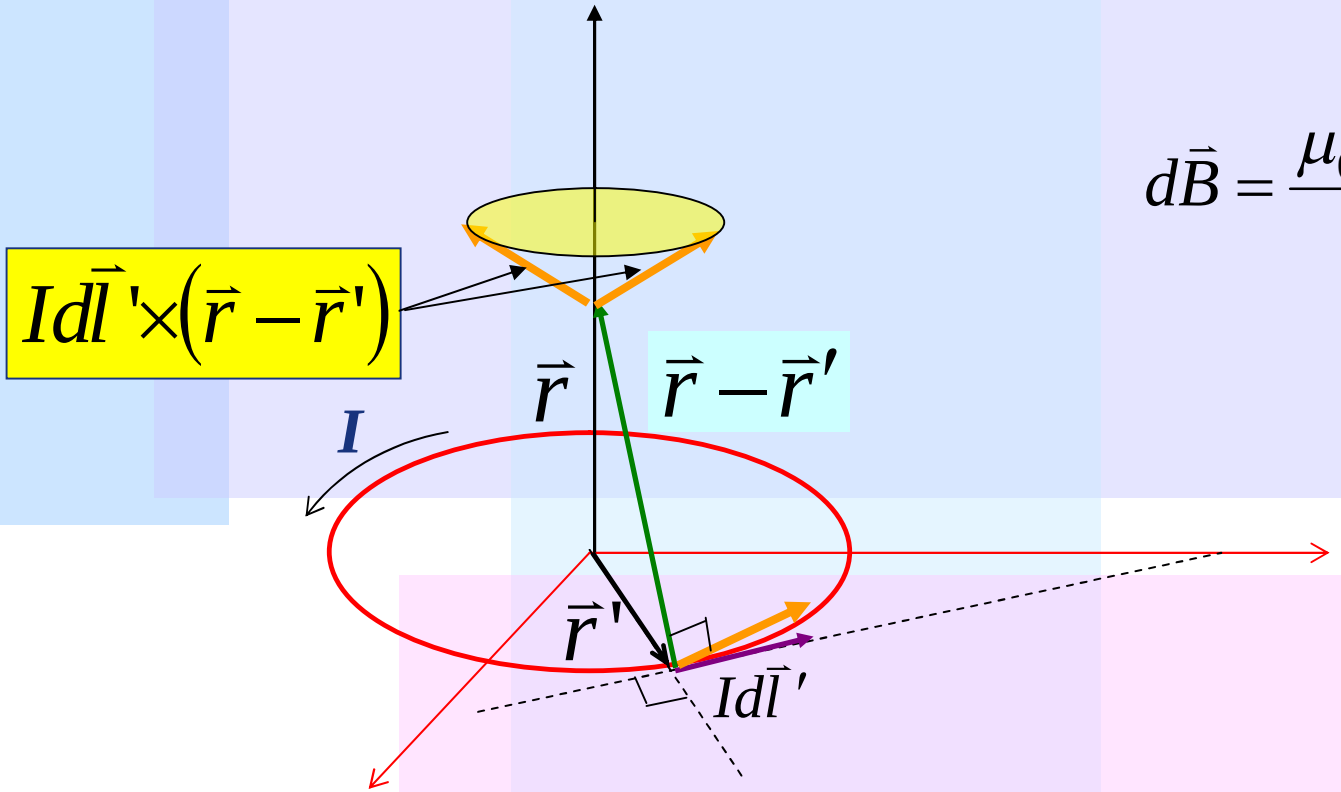


Campo producido por circuito Γ´

$$\vec{B} = \oint_{\Gamma'} \frac{\mu_0 I d\vec{l}' \times (\vec{r} - \vec{r}')}{4\pi \left\| \vec{r} - \vec{r}' \right\|^3}$$

Regla de la mano derecha

Dirección de campo está dado por el producto $Id\vec{l}\,'\times(\vec{r}-\vec{r}\,')$

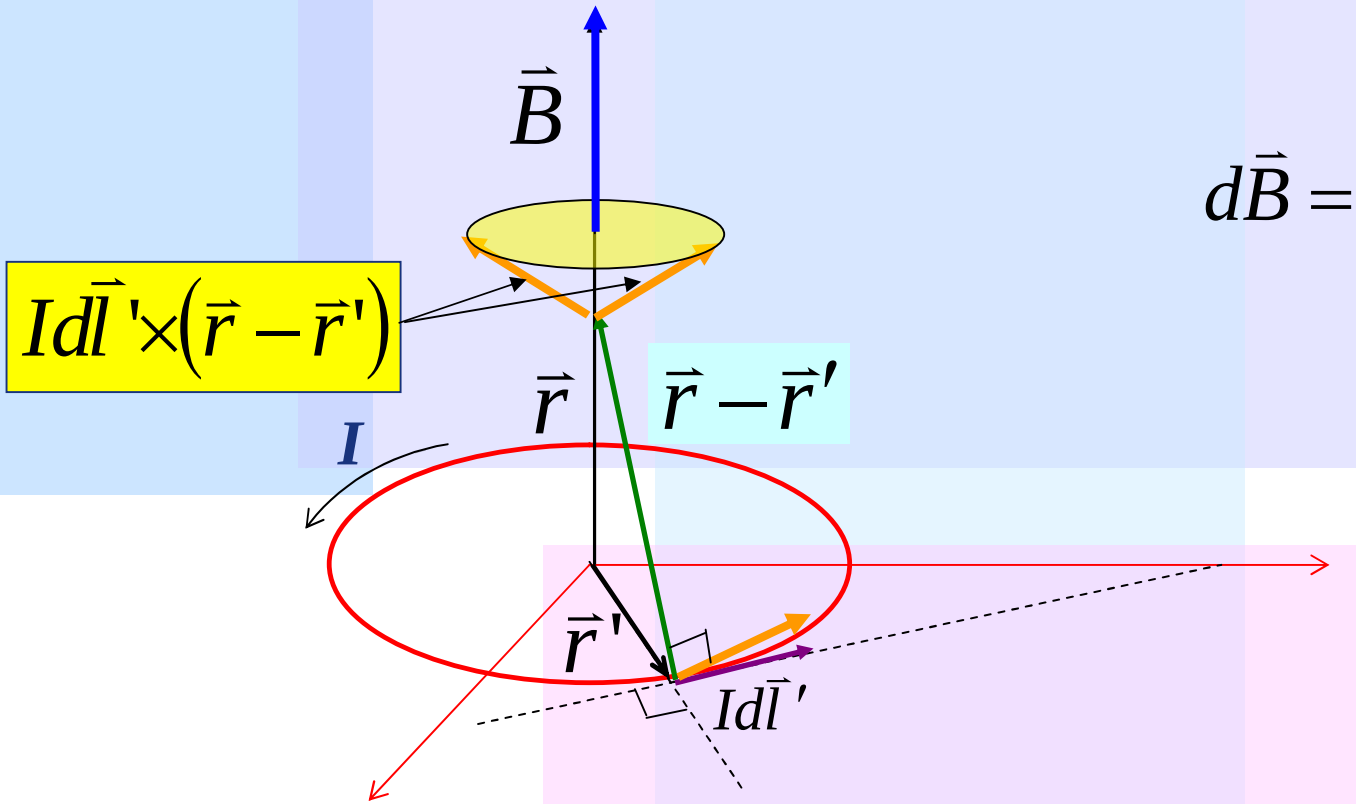


$$d\vec{B}=\frac{\mu_0 Id\vec{l}\,'\times(\vec{r}-\vec{r}\,')}{4\pi\left\|\vec{r}-\vec{r}\,'\right\|^3}$$

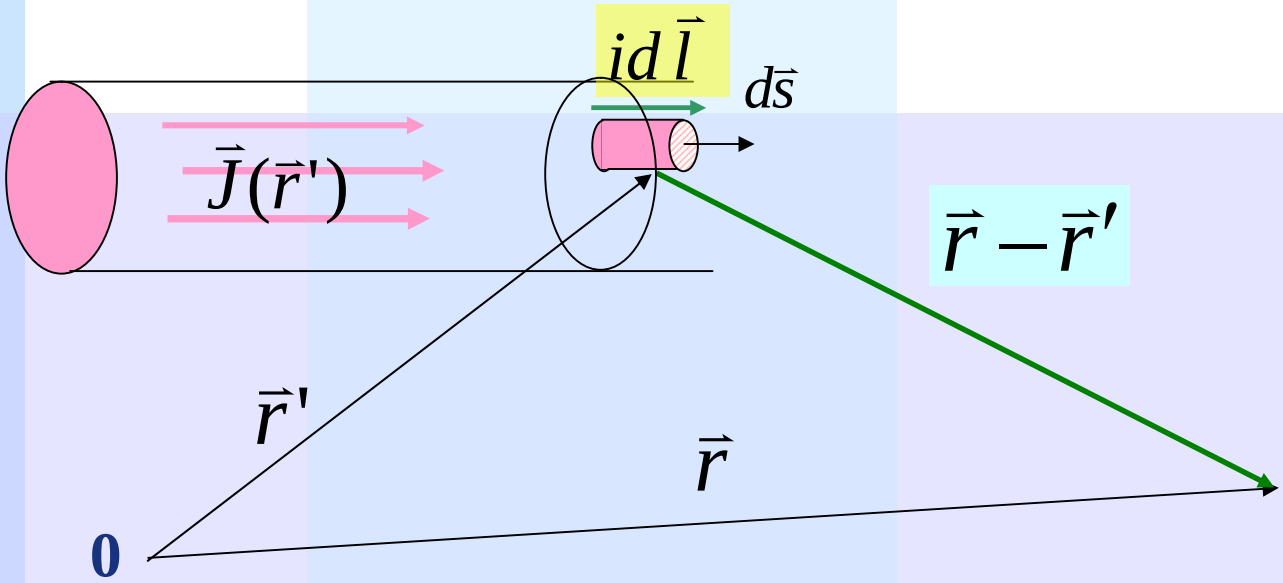
Regla de la mano derecha

Campo magnético resultante sólo tiene dirección según eje z.

$$d\vec{B} = \frac{\mu_0 I d\vec{l}\,' \times (\vec{r} - \vec{r}\,')}{4\pi \left\| \vec{r} - \vec{r}\,' \right\|^3}$$



Campo magnético de distribuciones de corriente



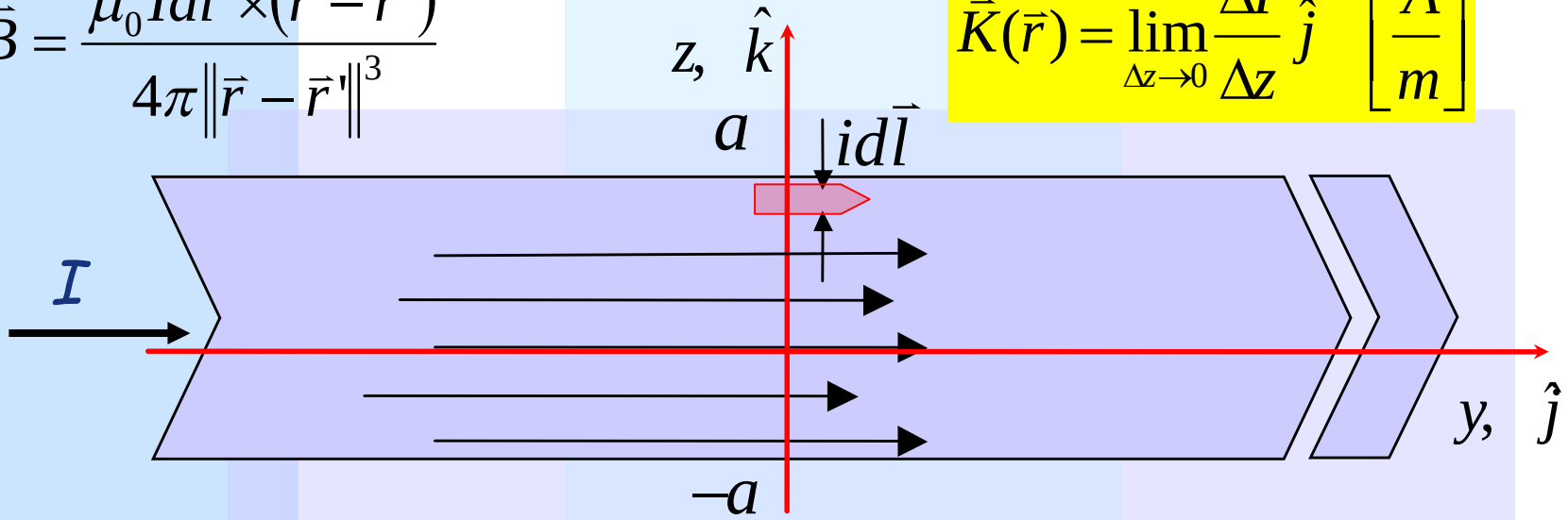
$$id\vec{l} = \vec{J}\cdot d\vec{s}\cdot d\vec{l} = \vec{J}dv'$$

$$\vec{B} = \frac{\mu_0}{4\pi}\iiint_{\Omega}\frac{\vec{J}(\vec{r}')\times(\vec{r}-\vec{r}')}{\|\vec{r}-\vec{r}'\|^3}dv'$$

Campo magnético de distribuciones de corriente

$$d\vec{B} = \frac{\mu_0 I d\vec{l}' \times (\vec{r} - \vec{r}')}{4\pi \left\| \vec{r} - \vec{r}' \right\|^3}$$

$$\vec{K}(\vec{r}) = \lim_{\Delta z \to 0} \frac{\Delta I}{\Delta z} \hat{j} \quad \left[\frac{A}{m} \right]$$

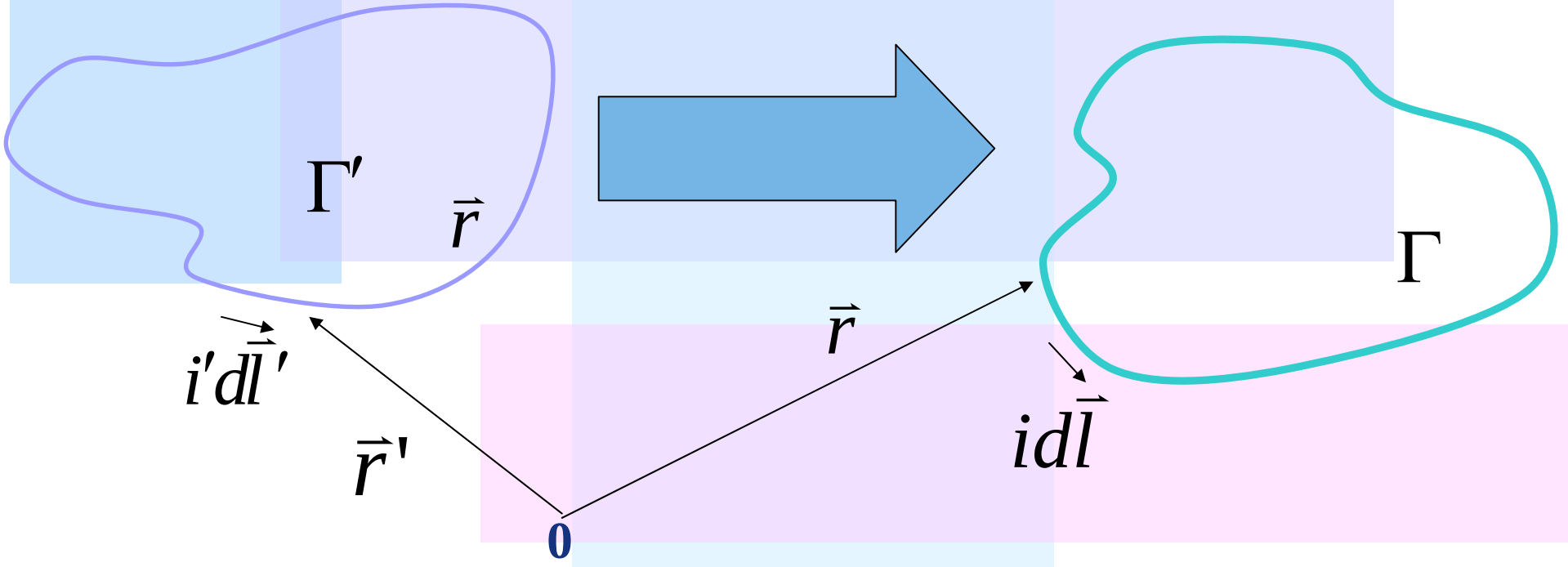


$$\vec{B} = \frac{\mu_0}{4\pi} \iint_S \frac{\vec{K}(\vec{r}') \times (\vec{r} - \vec{r}')}{\left\| \vec{r} - \vec{r}' \right\|^3} ds'$$

Ley de Biot y Savarat

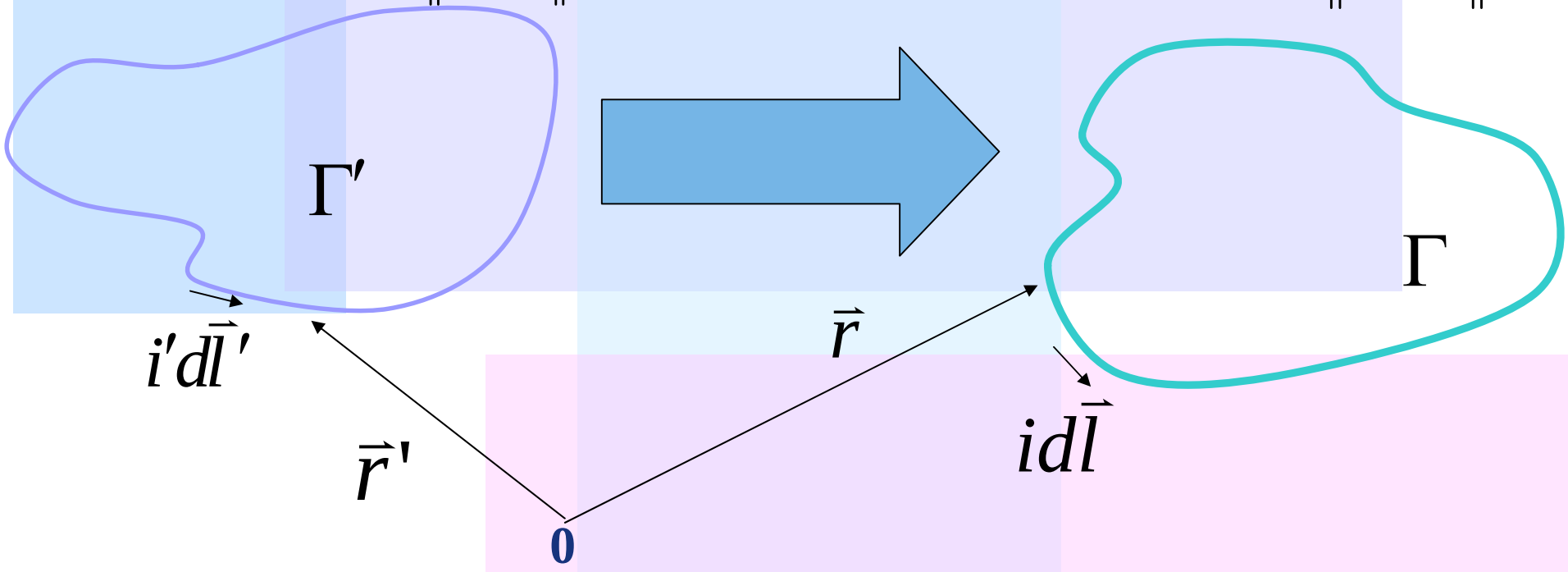
Fuerza que ejerce circuito Γ' sobre circuito Γ

$$\vec{F}_{\Gamma'\to\Gamma} = \frac{\mu_0}{4\pi}\oint_{\Gamma}\oint_{\Gamma'} \frac{I'Id\vec{l}\times\left(d\vec{l}'\times\left(\vec{r}-\vec{r}'\right)\right)}{\left\|\vec{r}-\vec{r}'\right\|^3}$$



Ley de Biot y Savarat

$$\vec{F} = \frac{\mu_0}{4\pi} \oint_{\Gamma} \oint_{\Gamma'} \frac{I' I d\vec{l} \times \left(d\vec{l}' \times \left(\vec{r} - \vec{r}' \right) \right)}{\left\| \vec{r} - \vec{r}' \right\|^3} \quad \rightarrow \quad d\vec{F} = \frac{I d\vec{l} \times \mu_0}{4\pi} \oint_{\Gamma'} \frac{I' d\vec{l}' \times \left(\vec{r} - \vec{r}' \right)}{\left\| \vec{r} - \vec{r}' \right\|^3}$$

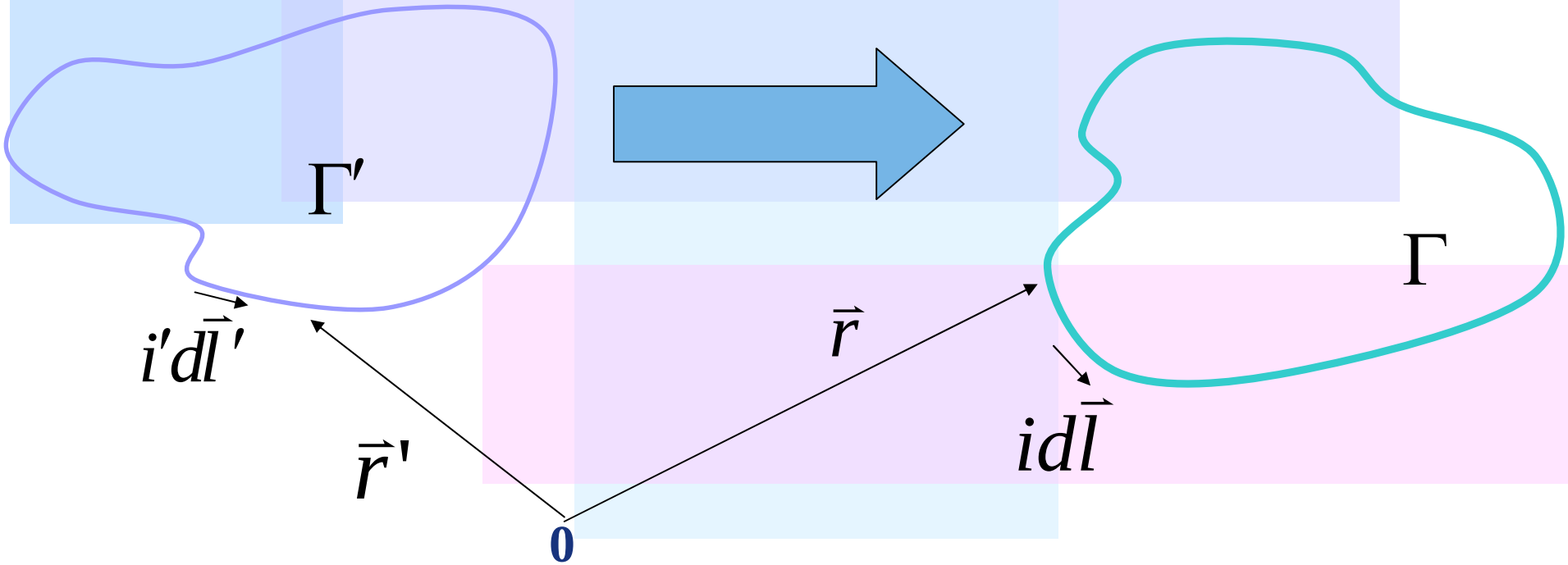


Ley de Biot y Savarat

$$d\vec{F} = \frac{Id\vec{l} \times \mu_0}{4\pi} \oint_{\Gamma'} \frac{I' d\vec{l}' \times (\vec{r} - \vec{r}')}{\|\vec{r} - \vec{r}'\|^3}$$

$$\therefore \quad d\vec{F} = Id\vec{l} \times \vec{B}(\vec{r})$$

Campo magnético producido por circuito Γ'

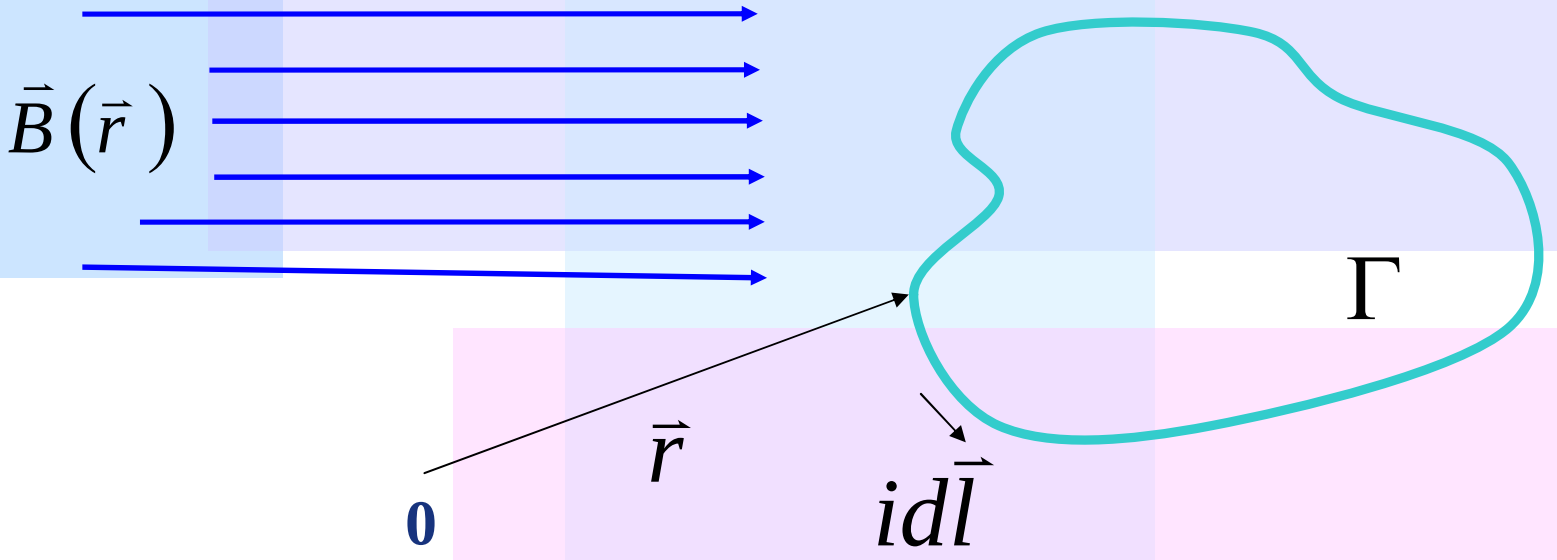


Ley de Biot y Savarat

Así, un circuito en presencia de un campo magnético experimenta una fuerza dada por la ecuación

$$d\vec{F} = Id\vec{l} \times \vec{B}(\vec{r})$$

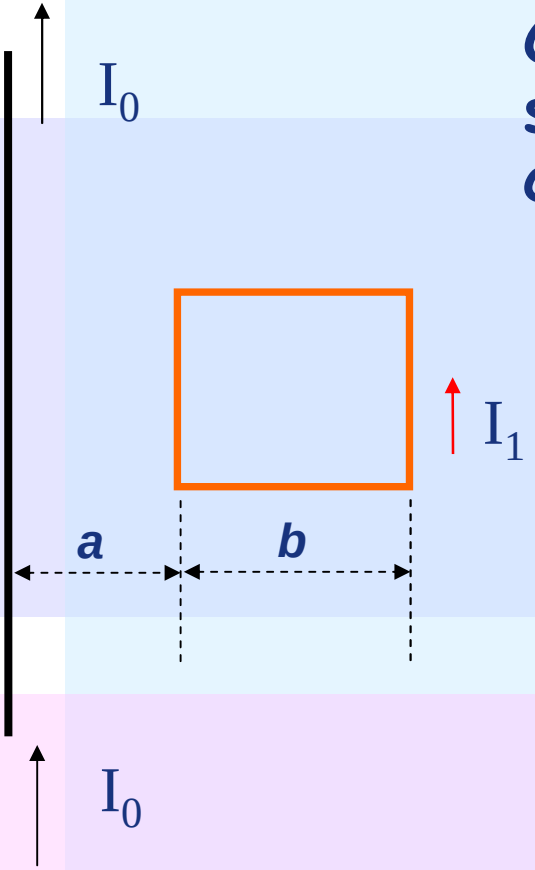
$$\therefore \quad \vec{F} = \oint_{\Gamma} d\vec{F} = \oint_{\Gamma} Id\vec{l} \times \vec{B}(\vec{r})$$



Ley de Biot y Savarat

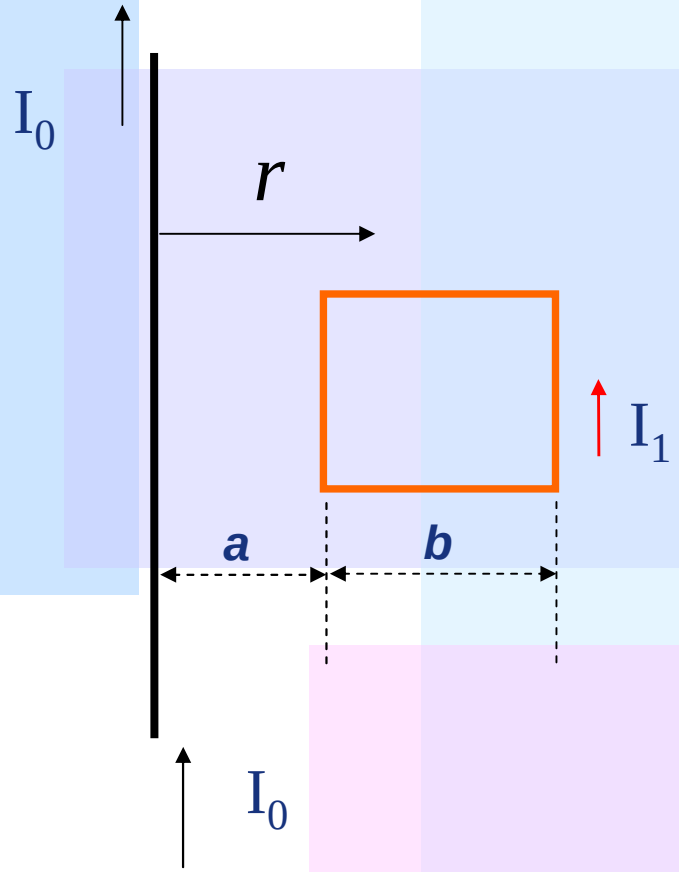
Ejemplo

Calcular la fuerza sobre la espira cuadrada



Ley de Biot y Savarat

Ejemplo



Campo producido por el conductor infinito es

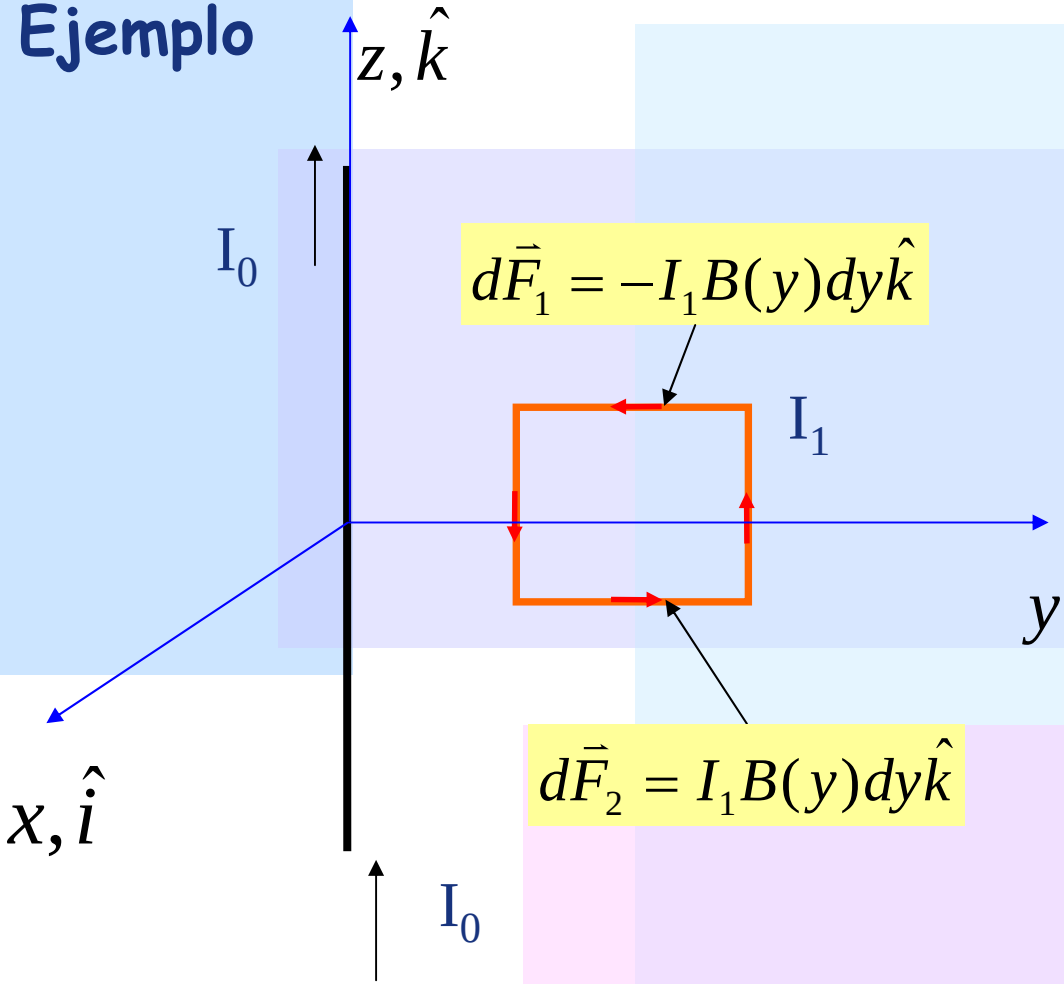
$$\vec{B} = \frac{\mu_0 I_0}{2\pi r}\hat{\theta}$$

Fuerza sobre elemento de corriente de espira cuadrada

$$d\vec{F} = I_1 d\vec{l} \times \vec{B}(\vec{r})$$

Ley de Biot y Savarat

Ejemplo



Fuerza sobre elemento de corriente de espira cuadrada

$$d\vec{F} = I_1 d\vec{l} \times \vec{B}(\vec{r})$$

$$\vec{B}(\vec{r}) = -B(y)\hat{i}$$

Claramente

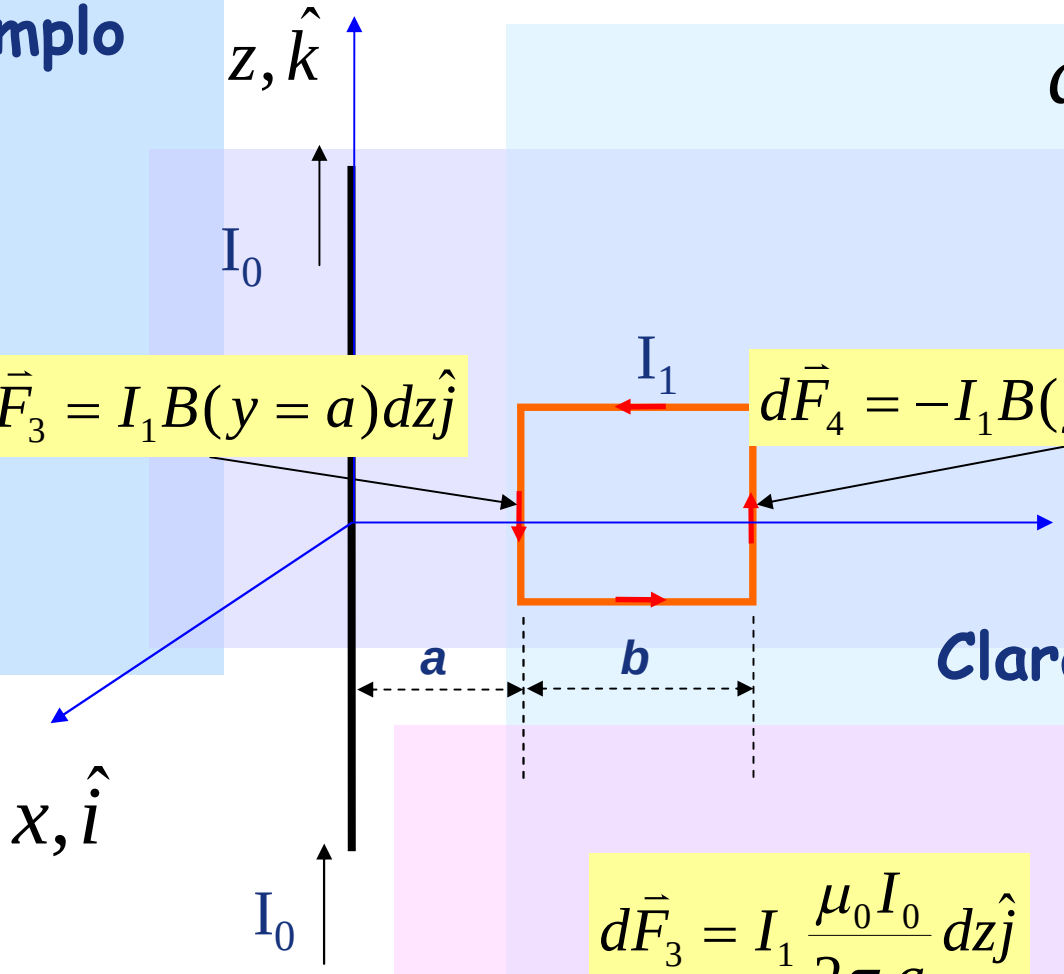
$$d\vec{F}_1 = -d\vec{F}_2$$

Ley de Biot y Savarat

Ejemplo

$$d\vec{F} = I_1 d\vec{l} \times \vec{B}(\vec{r})$$

$$\vec{B}(\vec{r}) = -B(y)\hat{i}$$



Claramente

$$d\vec{F}_3 = I_1 \frac{\mu_0 I_0}{2\pi\, a} dz\hat{j}$$

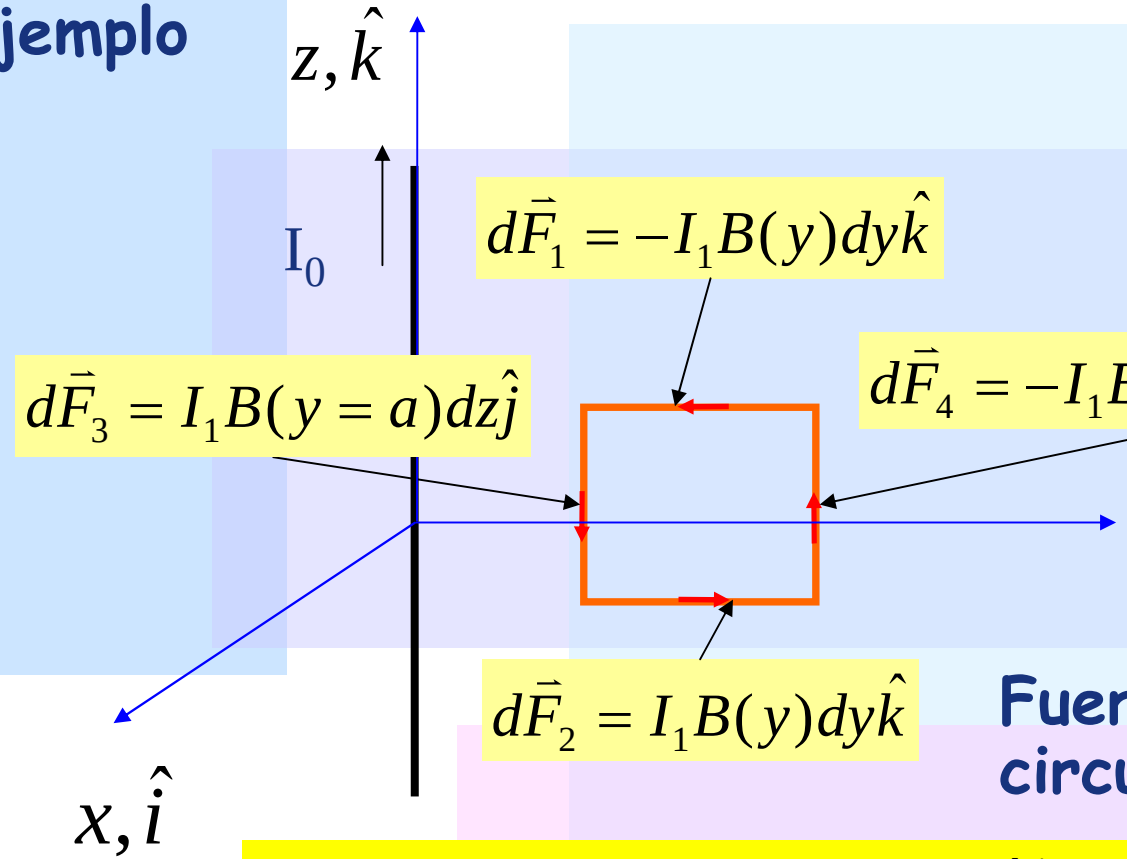
$$d\vec{F}_4 = -I_1 \frac{\mu_0 I_0}{2\pi\, a+b} dz\hat{j}$$

Ley de Biot y Savarat

Ejemplo

$$d\vec{F} = I_1 d\vec{l} \times \vec{B}(\vec{r})$$

$$\vec{B}(\vec{r}) = -B(y)\hat{i}$$



Fuerza total sobre el circuito cuadrado

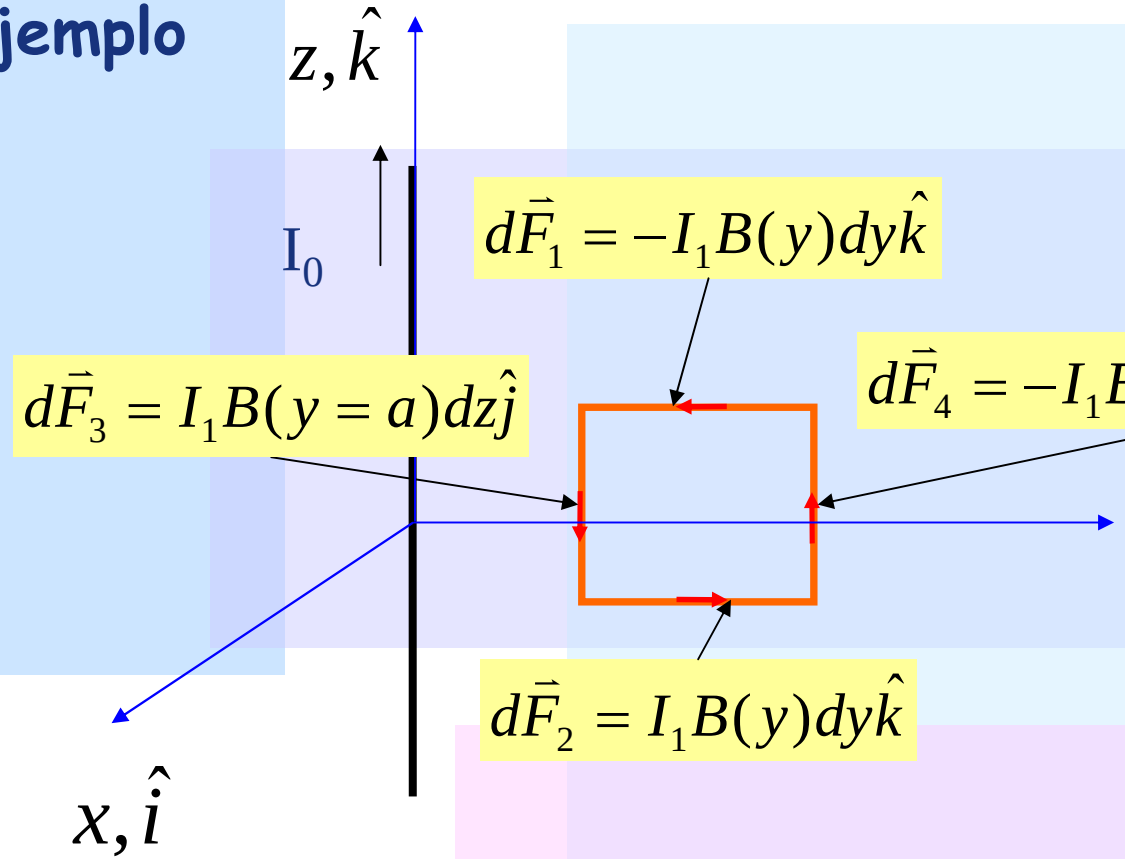
$$\therefore \quad \vec{F} = \oint_{\Gamma} d\vec{F} = \int_{y=a+b}^{y=a} d\vec{F}_1 + \int_{z=b/2}^{z=-b/2} d\vec{F}_3 + \int_{y=a}^{y=a+b} d\vec{F}_2 + \int_{z=-b/2}^{z=b/2} d\vec{F}_4$$

Ley de Biot y Savarat

Ejemplo

$$d\vec{F} = I_1 d\vec{l} \times \vec{B}(\vec{r})$$

$$\vec{B}(\vec{r}) = -B(y)\hat{i}$$

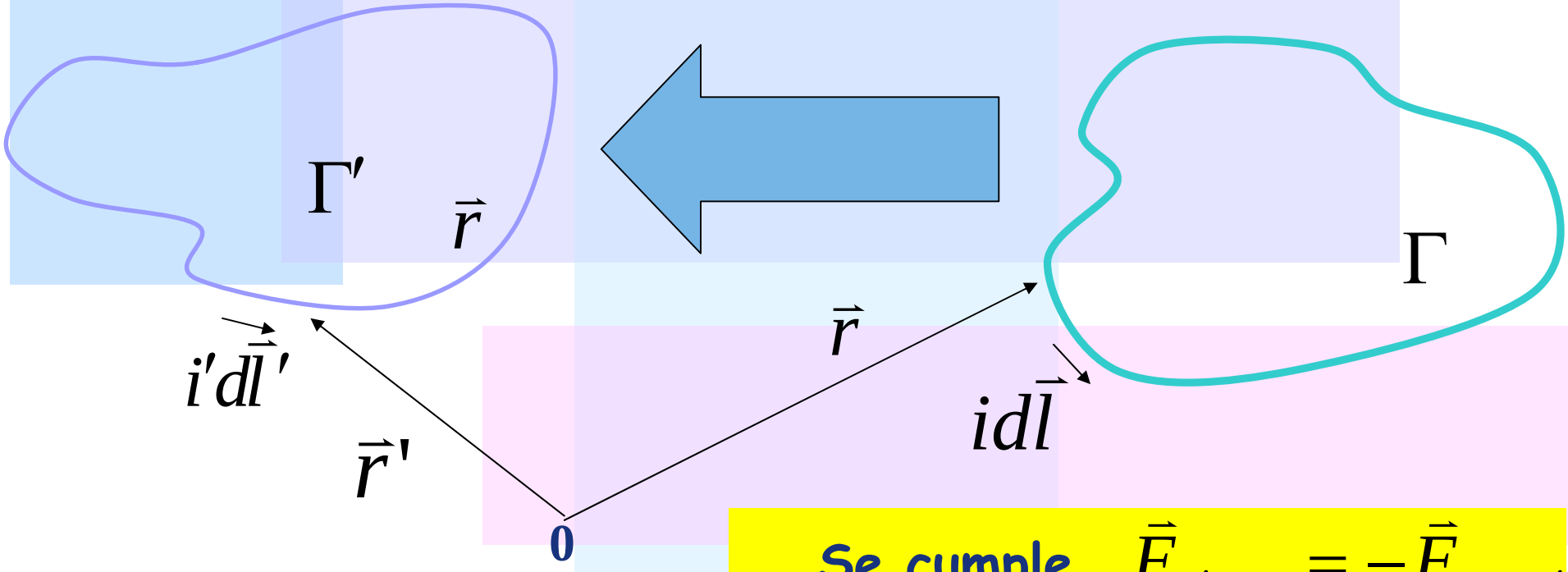


$$\vec{F} = \oint_{\Gamma} d\vec{F} = \int_{z=b/2}^{z=-b/2} \frac{\mu_0 I_1 I_0 \hat{j}}{2\pi a} dz - \int_{z=-b/2}^{z=b/2} \frac{\mu_0 I_1 I_0 \hat{j}}{2\pi(a+b)} dz = \frac{\mu_0 I_1 I_0 b^2}{2\pi(a+b)} \hat{j}$$

Ley de Biot y Savarat

Fuerza que ejerce circuito Γ sobre circuito Γ'

$$\vec{F}_{\Gamma\to\Gamma'} = \frac{\mu_0}{4\pi}\oint_{\Gamma'}\oint_{\Gamma}\frac{II'd\vec{l}'\times\left(d\vec{l}\times\left(\vec{r}-\vec{r}'\right)\right)}{\left\|\vec{r}-\vec{r}'\right\|^3} = -\frac{\mu_0}{4\pi}\oint_{\Gamma}\oint_{\Gamma'}\frac{I'Id\vec{l}\times\left(d\vec{l}'\times\left(\vec{r}-\vec{r}'\right)\right)}{\left\|\vec{r}-\vec{r}'\right\|^3}$$

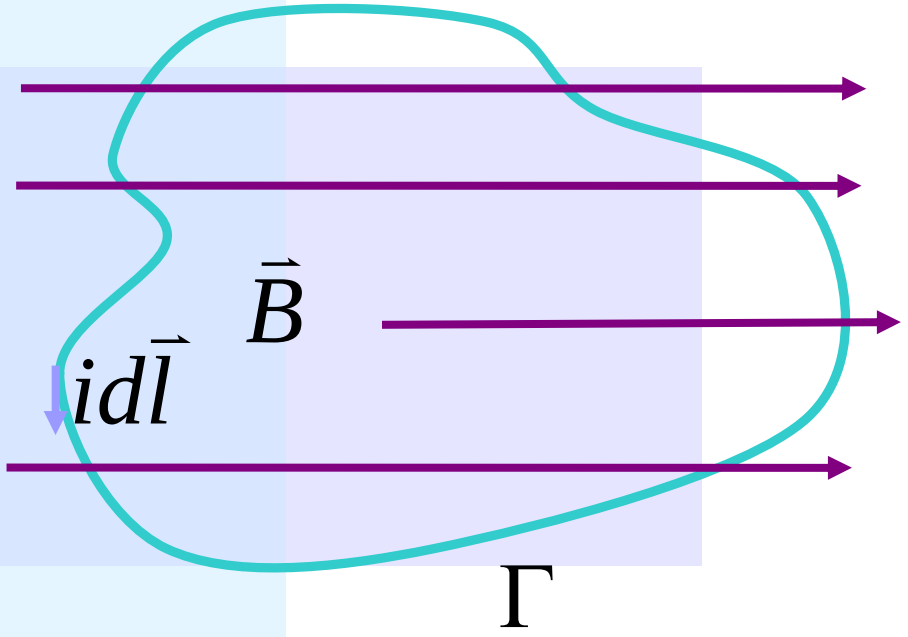


Se cumple $\vec{F}_{\Gamma'\to\Gamma} = -\vec{F}_{\Gamma\to\Gamma'}$

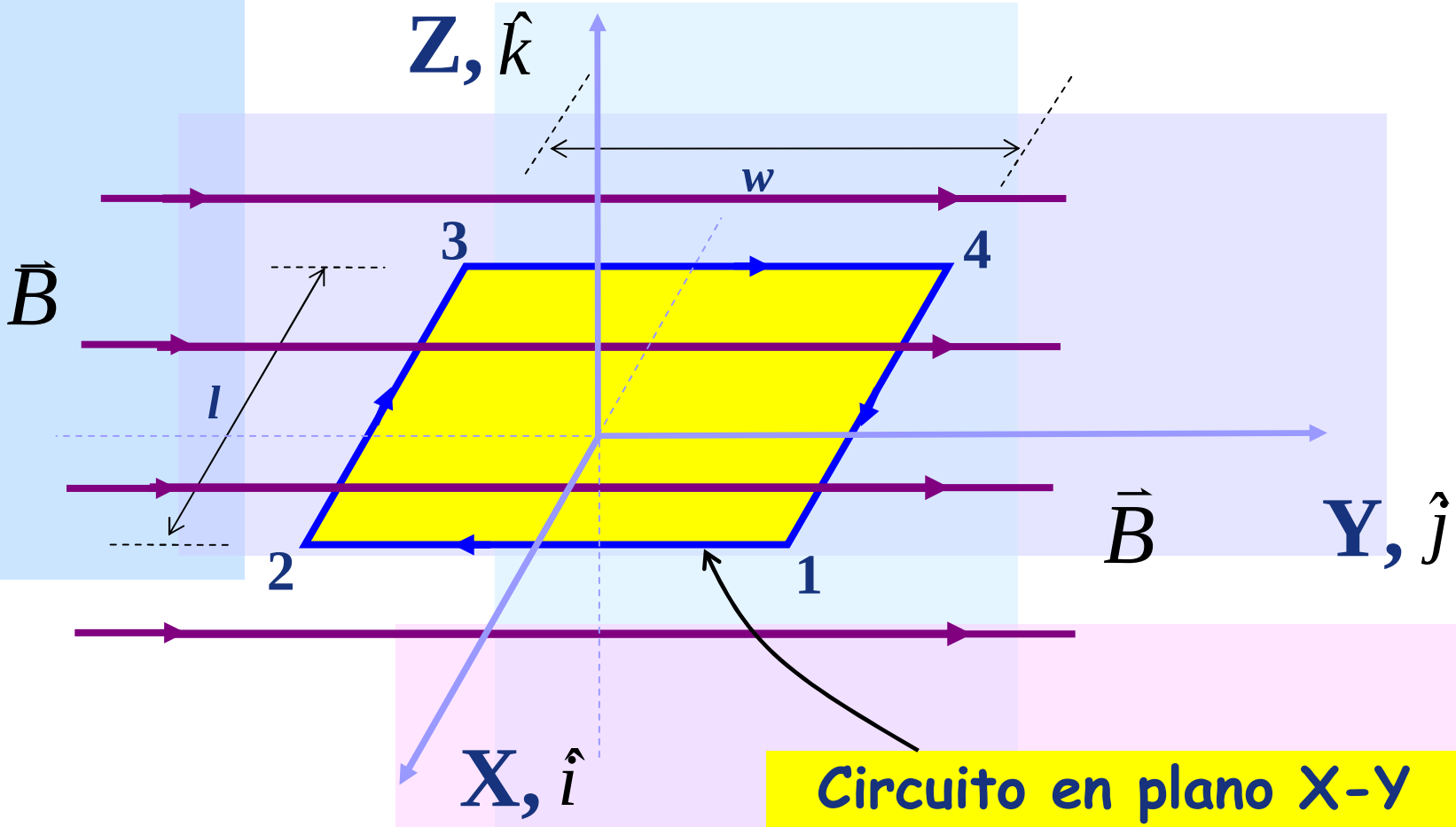
Torque Magnético

Ley de Biot y Savarat

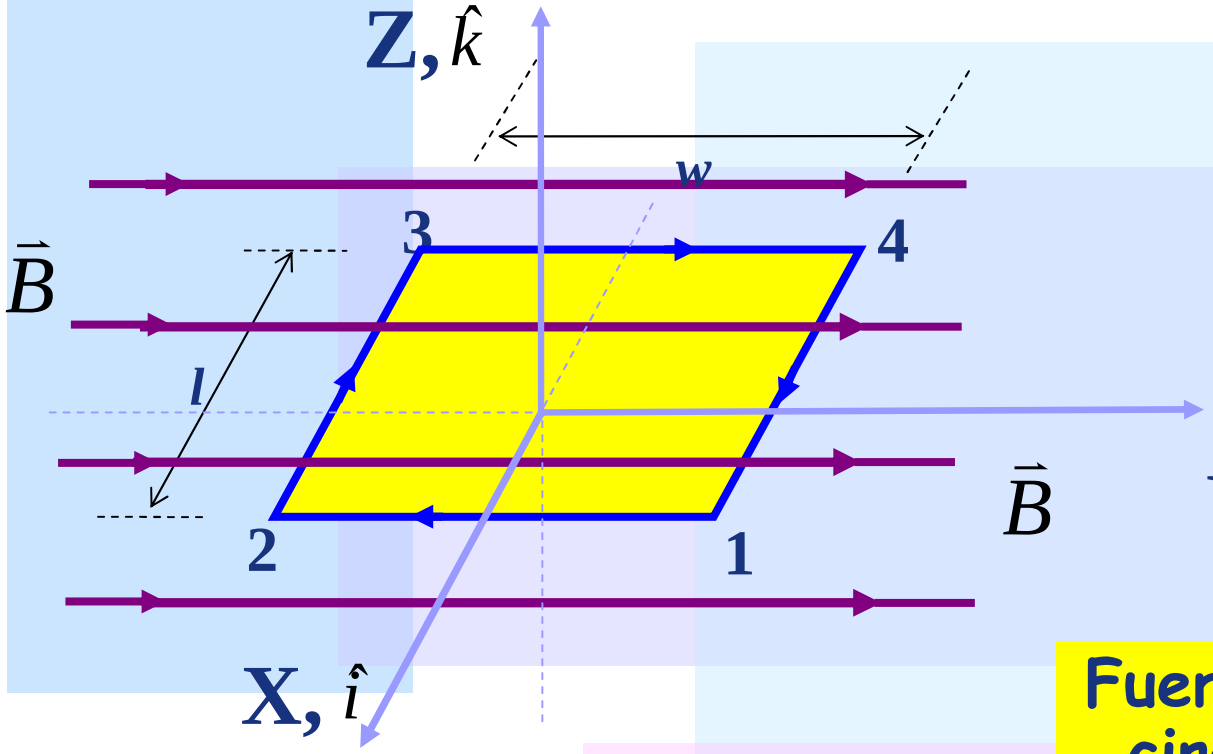
$$\therefore \quad d\vec{F} = Id\vec{l} \times \vec{B}(\vec{r})$$



Torque Magnético



Torque Magnético

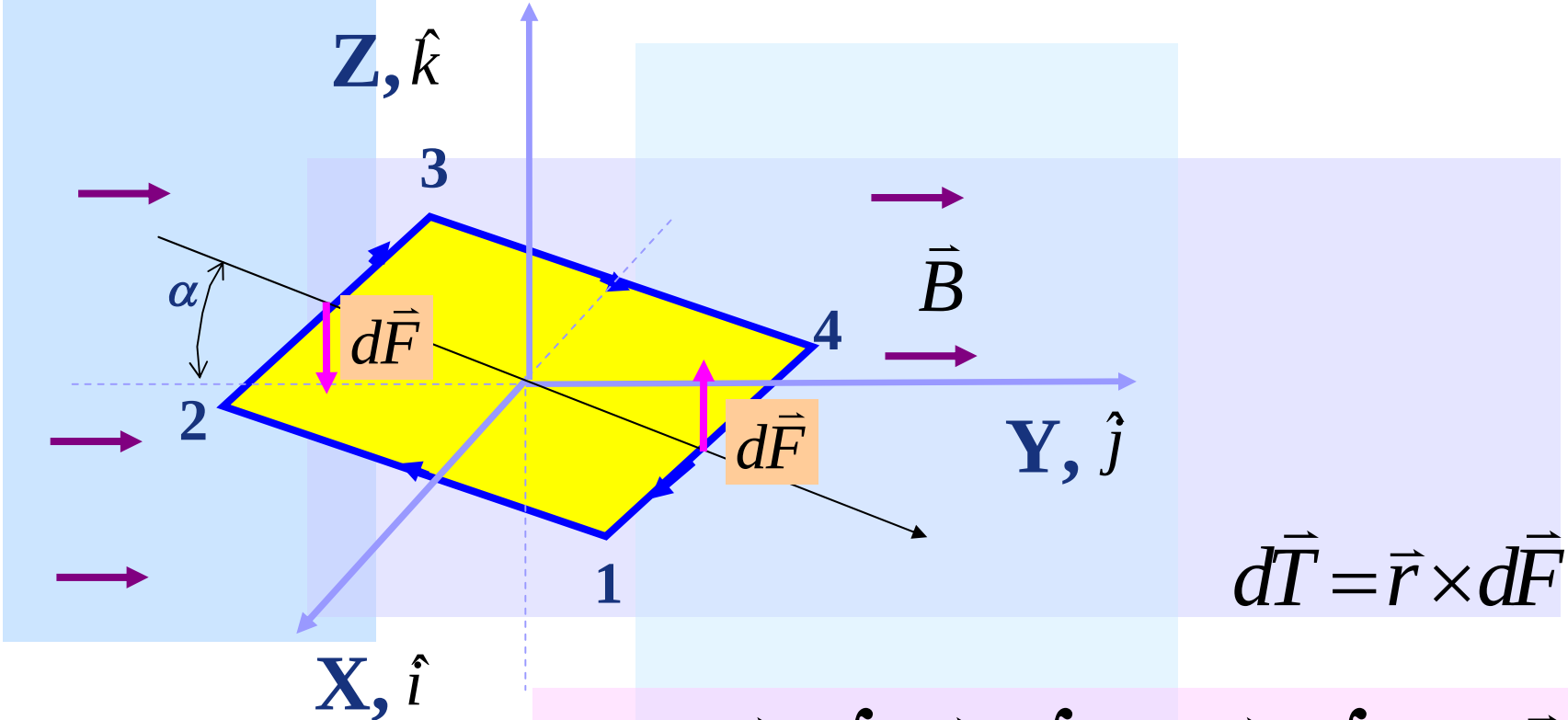


En lados 1-2 y 3-4 $Id\vec{l}$ es paralelo a $\vec{B}$
Luego F=0

Fuerza neta nula sobre el circuito si $\vec{B}$ constante

$$\vec{F} = I\int_2^3 d\vec{l}\times\vec{B} + I\int_4^1 d\vec{l}\times\vec{B} \quad\Rightarrow\quad \vec{F} = I\int_2^3 dx(-\hat{i})\times\vec{B} + I\int_4^1 dx(\hat{i})\times\vec{B}$$

Torque Magnético

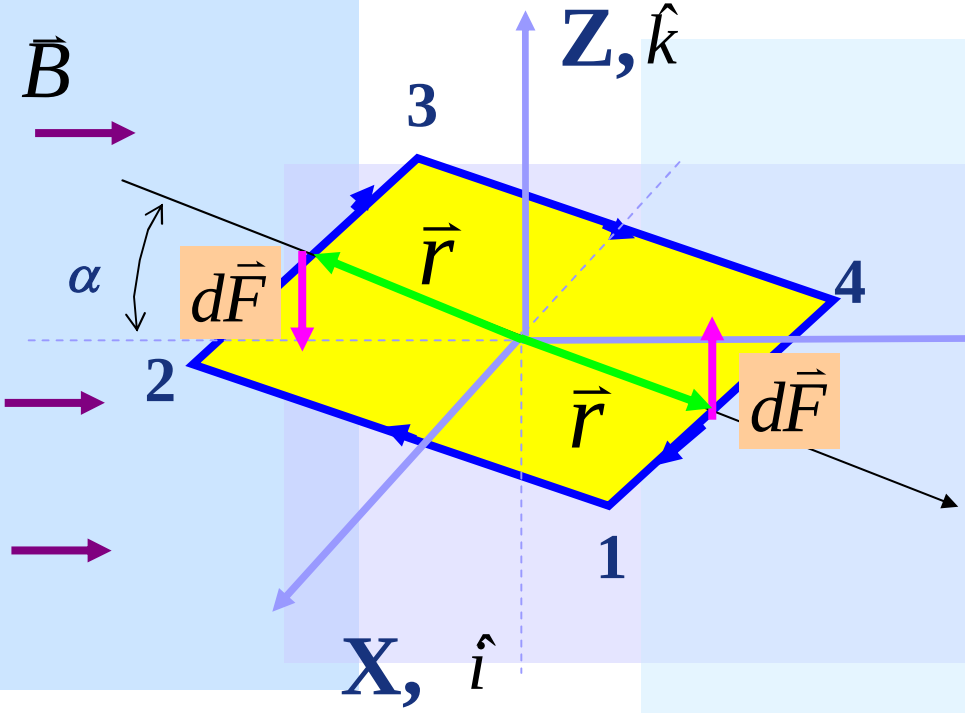


$$d\vec{T} = \vec{r} \times d\vec{F}$$

$$\vec{T} = \oint_c d\vec{T} = \oint_c \vec{r} \times d\vec{F} = \oint_c \vec{r} \times i d\vec{l} \times \vec{B}$$

Torque neto no nulo sobre el circuito

Torque Magnético

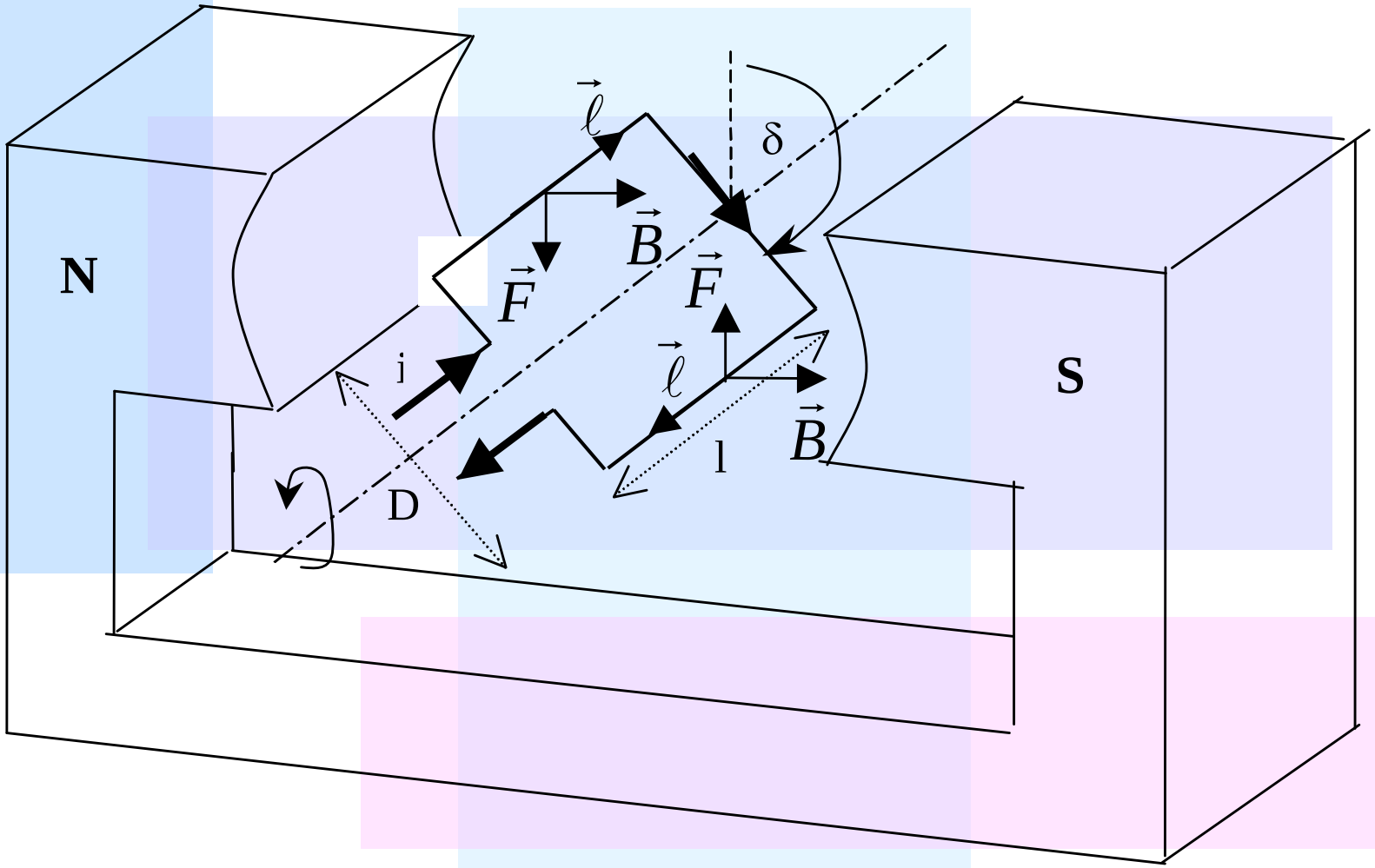


$$\vec{T} = I\int_2^3 \vec{r}\times dx\hat{i}\times\vec{B} + I\int_4^1 \vec{r}\times dx\hat{i}\times\vec{B}$$

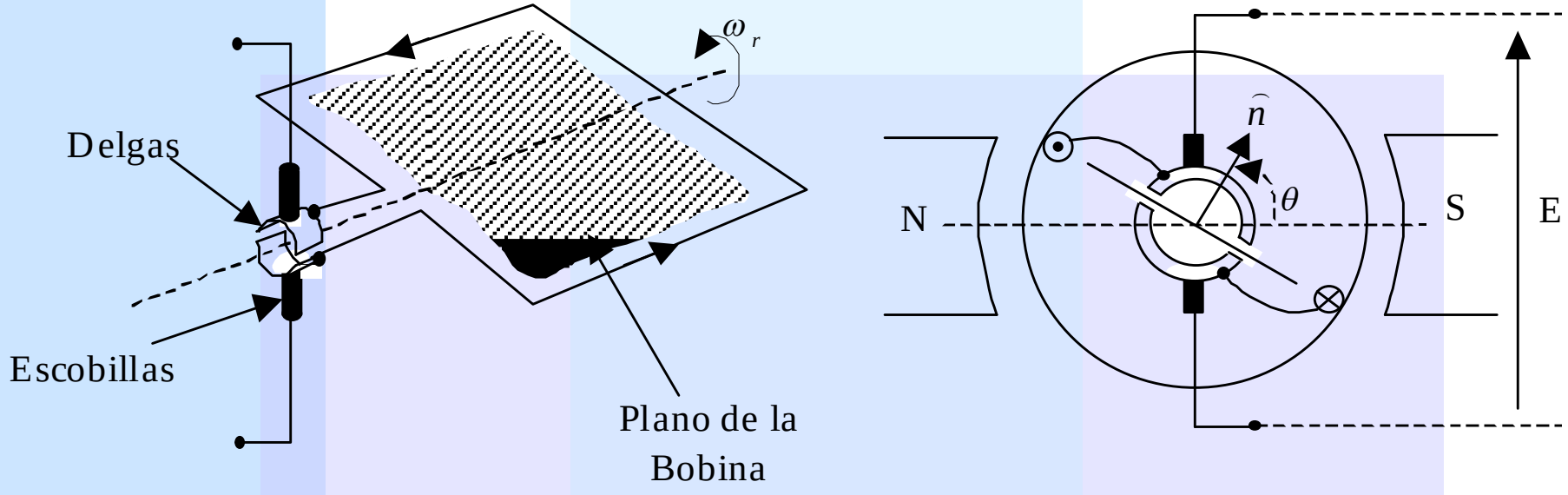
$$\vec{T} = \frac{Iwl}{2}\cos\alpha\,\hat{i} + \frac{Iwl}{2}\cos\alpha\,\hat{i}$$

Torque neto sobre el circuito $\therefore\ \vec{T} = Iwl\cos\alpha\,\hat{i}$

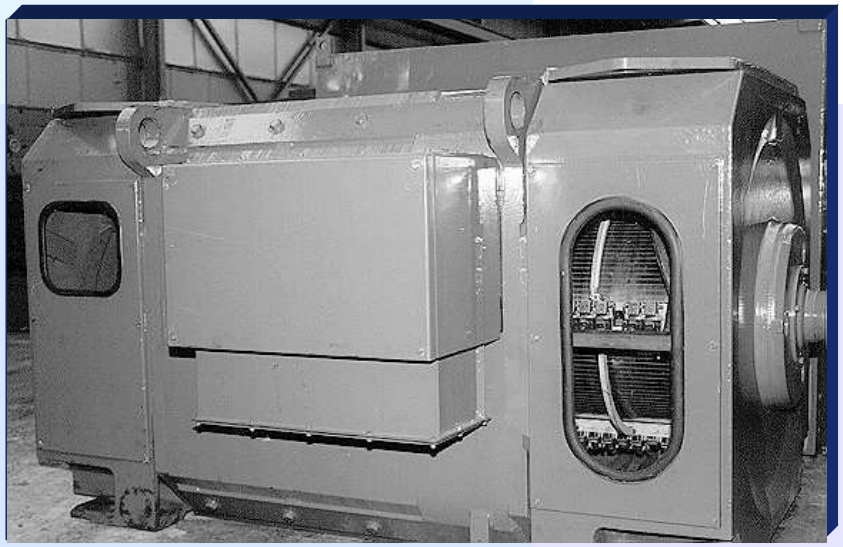
Motor elemental



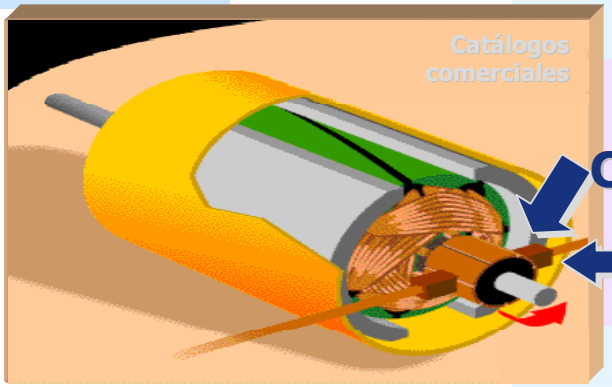
Motor elemental



Motores



Motor de CC de 6000 kW fabricado por ABB



Colector

Escobillas

Colector real