Solución P3 C1

a) $\frac{\partial U}{\partial r} = -mkr \Rightarrow \partial U = -mkr\,dr \quad /\int$

$$\boxed{U = -\frac{mkr^2}{2} + c_0}$$

(0,5 p)

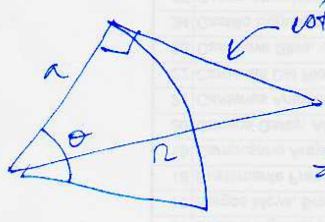
$$\begin{aligned} W &= -\Delta U \\ &= -U(r) + U(a) \\ &= \frac{mkr^2}{2} + -\frac{mka^2}{2} \Rightarrow \boxed{W = \frac{mk}{2}(r^2 - a^2)} \end{aligned}$$

(1p)

b) $$\begin{aligned} W &= \Delta K \\ &= \frac{1}{2}m(V^2 - \overset{0}{V_0^2}) \end{aligned}$$

$$\Rightarrow \boxed{\frac{1}{2}mV^2 = \frac{mk}{2}(r^2 - a^2)}$$

(0,5 p)



← este pedazo de cuerda antes estaba enrollado en un arco de circunferencia de largo $a\theta$

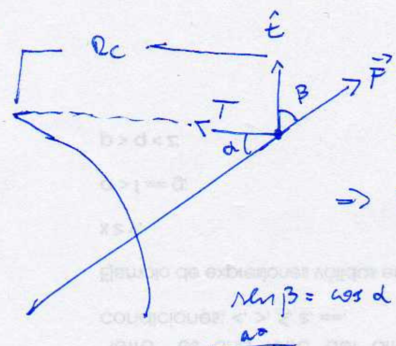
$\Rightarrow r^2 = a^2 + (a\theta)^2$

Reemplazando queda $\frac{1}{2}\cancel{m}V^2 = \frac{\cancel{m}k}{2}(a^2 + a^2\theta^2 - a^2)$

$$\Rightarrow \boxed{V = \sqrt{k}\,a\theta}$$

(1p)

c)

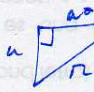


$(\beta = 90 - \alpha)$

$\sum \vec{F} = T\hat{n} + F(\cos\beta\,\hat{t} - \sin\beta\,\hat{n})$

$\Rightarrow T - F\sin\beta = \dfrac{mv^2}{R_c}$ (Intrínsecos)

$\sin\beta = \cos\alpha$



$\Rightarrow \cos\alpha = \dfrac{a\theta}{r}$ $\quad R_c = a\theta$

$\Rightarrow T - mkr \cdot \dfrac{a\theta}{r} = \dfrac{m}{R_c} k (a\theta)^2$

$T - mk\,a\theta = mka\theta \Rightarrow \boxed{T = 2mka\theta}$ (1p)

T_{max} es cuando $\theta = 2\pi \Rightarrow \boxed{T_m = 4\pi mka}$ (0,5)

d) La coordenada $\hat{t}$ de $\sum\vec{F} = m\vec{a} \Rightarrow F\cos\beta = m\dfrac{dv}{dt}$

Pero $\cos\beta = \sin\alpha = \dfrac{a}{r}$

$\Rightarrow mkr \cdot \dfrac{a}{r} = m\dfrac{d}{dt}(\sqrt{k}\,a\theta)$

$ka = \sqrt{k}\,a\dot{\theta}$

$\Rightarrow \dot{\theta} = \sqrt{k}$ /S

$\Delta\theta = \sqrt{k}\,\Delta t \Rightarrow \boxed{t = \dfrac{2\pi}{\sqrt{k}}}$ (1,5)

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