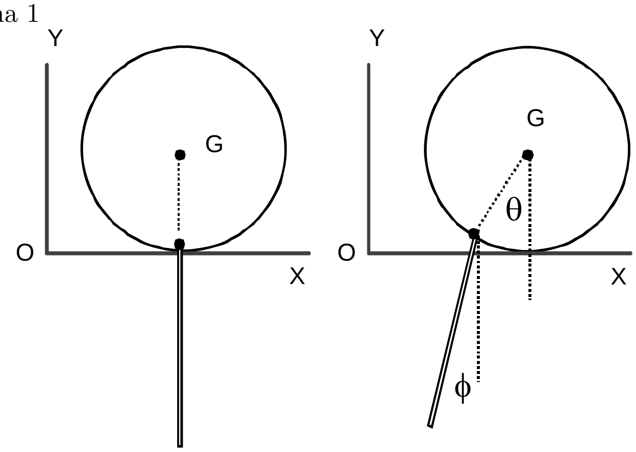
Problema 1



La energía cinética del disco

$$K_1 = \frac{1}{2}MR^2\dot{\theta}^2 + \frac{1}{2}\frac{1}{2}MR^2\dot{\theta}^2 = \frac{3}{4}MR^2\dot{\theta}^2$$

Coordenadas G de la barra

$$\begin{aligned} x &= R\theta - R\sin\theta - R\sin\phi \\ y &= R - R\cos\theta - R\cos\phi \end{aligned}$$

de aquí

$$\begin{aligned} \dot{x} &= R\dot{\theta} - R\dot{\theta}\cos\theta - R\dot{\phi}\cos\phi \\ \dot{y} &= R\dot{\theta}\sin\theta + R\dot{\phi}\sin\phi \end{aligned}$$

la energía cinética de la barra

$$K_2 = \frac{1}{2}M((R\dot{\theta} - R\dot{\theta}\cos\theta - R\dot{\phi}\cos\phi)^2 + (R\dot{\theta}\sin\theta + R\dot{\phi}\sin\phi)^2) + \frac{1}{2}\frac{1}{3}MR^2\dot{\phi}^2$$

La energía Potencial

$$V = Mg(R - R\cos\theta - R\cos\phi) \simeq MgR(\frac{\theta^2}{2} + \frac{\phi^2}{2})$$

Para oscilaciones pequeñas aproximamos

$$K_2 = \frac{1}{2}M((-R\dot{\phi})^2 + \frac{1}{2}\frac{1}{3}MR^2\dot{\phi}^2 = \frac{2}{3}MR^2\dot{\phi}^2$$

y oh! resulta un Lagrangiano sin acoplamiento

$$L = \frac{3}{4}MR^2\dot{\theta}^2 + \frac{2}{3}MR^2\dot{\phi}^2 - MgR(\frac{\theta^2}{2} + \frac{\phi^2}{2})$$

de donde

$$\begin{aligned}\ddot{\theta}+\frac{2g}{3R}\theta &= 0\\ \ddot{\phi}+\frac{3g}{4R}\phi &= 0\end{aligned}$$

las frecuencias propias serán

$$\begin{aligned}\omega_1 &= \sqrt{\frac{2g}{3R}}\\ \omega_2 &= \sqrt{\frac{3g}{4R}}\end{aligned}$$

y las coordenadas normales son θ, ϕ.

Problema 2.-

D'Alembert

$$F(x)=y(x,0)=5\sin\frac{3\pi x}{L}-10\sin\frac{7\pi x}{L}$$

es de antemano antisimétrica y de periodo $2L$ luego la usamos directamente en

$$\begin{aligned}y(x,t) &= \frac{1}{2}(F(x+vt)+F(x-vt)\\ &= \frac{1}{2}\left(5\sin\frac{3\pi(x+vt)}{L}-10\sin\frac{7\pi(x+vt)}{L}+5\sin\frac{3\pi(x-vt)}{L}-10\sin\frac{7\pi(x-vt)}{L}\right)\end{aligned}$$

Bernouilli

$$\begin{aligned}y(x,t) &= \sum_{n=1}^{\infty}(A_n\cos\frac{n\pi vt}{L}+B_n\sin\frac{n\pi vt}{L})\sin\frac{n\pi x}{L}\\ 5\sin\frac{3\pi x}{L}-10\sin\frac{7\pi x}{L} &= \sum_{n=1}^{\infty}(A_n)\sin\frac{n\pi x}{L}\\ 0 &= \sum_{n=1}^{\infty}(\frac{n\pi v}{L}B_n)\sin\frac{n\pi x}{L}\end{aligned}$$

de donde se deduce que son solamente distintos de cero

$$A_3=5,\quad A_7=-10$$

finalmente

$$y(x,t)=5\cos\frac{5\pi vt}{L}\sin\frac{5\pi x}{L}-10\cos\frac{7\pi vt}{L}\sin\frac{7\pi x}{L}$$