Pauta Control 1

Problema (1)

La velocidad tiene componentes

$$v_r = \dot{r}, \quad v_\theta = r\dot{\theta} = 0, \quad v_\phi = r\sin\alpha\dot{\phi}$$

luego el Lagrangiano es

$$L = \frac{1}{2}m(\dot{r}^2 + r^2\sin^2\alpha\dot{\phi}^2) - mgr\cos\alpha, \qquad \text{(1 p)}$$

de donde

$$\begin{aligned} \ddot{r} - r\sin^2\alpha\phi^2 + g\cos\alpha &= 0 \\ \frac{d}{dt}m(r^2\sin^2\alpha\dot{\phi}) &= 0 \qquad \text{(a) 1 p)} \end{aligned}$$

El Hamiltoniano es

$$\begin{aligned} H &= E = \frac{1}{2}m(\dot{r}^2 + r^2\sin^2\alpha\dot{\phi}^2) + mgr\cos\alpha \\ p_r &= m\dot{r} \\ p_\phi &= mr^2\sin^2\alpha\dot{\phi} \end{aligned}$$

luego

$$H = \frac{p_r^2}{2m} + \frac{p_\phi^2}{2mr^2\sin^2\alpha} + mgr\cos\alpha$$

y las ecuaciones de Hamilton son

$$\begin{aligned} p_r &= m\dot{r} \\ p_\phi &= (m\sin^2\alpha r^2)\dot{\phi} \\ \dot{p}_r &= \frac{p_\phi^2}{mr^3\sin^2\alpha} - mg\cos\alpha \\ \dot{p}_\phi &= 0 \qquad \text{(b) 1 p)} \end{aligned}$$

Cantidades conservadas (corregir error: $\dot{r}(0) = 0$)

$$\begin{aligned} p_\phi &= mr^2\sin^2\alpha\dot{\phi} = mr_0^2\sin^2\alpha\Omega \qquad \text{(c) 1 p)} \\ H &= E = \frac{1}{2}m(\dot{r}^2 + r^2\sin^2\alpha\dot{\phi}^2) + mgr\cos\alpha = \frac{1}{2}mr_0^2\sin^2\alpha\Omega^2) + mgr_0\cos\alpha \end{aligned}$$

Extremos de r.Haga $\dot{r} = 0$

$$\begin{aligned} \frac{1}{2}mr^2\sin^2\alpha\dot{\phi}^2 + mgr\cos\alpha &= \frac{1}{2}mr_0^2\sin^2\alpha\Omega^2 + mgr_0\cos\alpha \\ \frac{\Omega^2 r_0^2\sin^2\alpha}{2g\cos\alpha}(r_0 + r) &= r^2 \end{aligned}$$

llamando $p = \frac{\Omega^2 r_0^2 \sin^2\alpha}{2g\cos\alpha}$

$$r = \frac{1}{2}p + \frac{1}{2}\sqrt{p^2 + 4pr_0} \qquad \text{(d) 1 p)}$$

Si permitimos que θ varie

$$L = \frac{1}{2}m(\dot{r}^2 + r^2\sin^2\theta\dot{\phi}^2 + r^2\dot{\theta}^2) - mgr\cos\theta$$

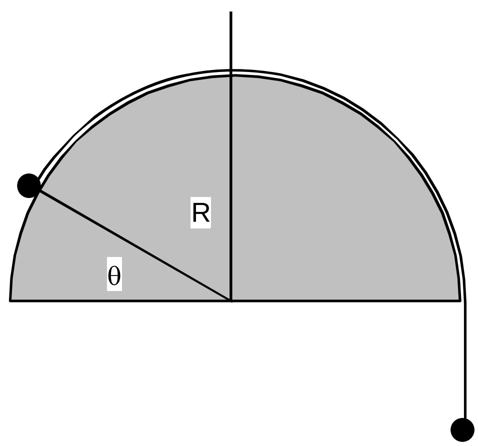
la ecuación para θ será $(\delta W = Nr\delta\theta)$

$$\frac{d}{dt}mr^2\dot{\theta} - \frac{\partial}{\partial\theta}(\frac{1}{2}mr^2\sin^2\theta\dot{\phi}^2 - mgr\cos\theta) = Nr$$

de donde haciendo $\theta = \alpha$

$$N = -mr\sin\alpha\cos\alpha\dot{\phi}^2 - mg\sin\alpha \qquad \text{(e) 1 p)}$$

Problema (2)



El lagrangiano es

$$\begin{aligned}
L &= mR^2\dot{\theta}^2 - (mgR\sin\theta - mgR\theta) \\
mR^2\ddot{\theta} - \frac{d}{d\theta}(mgR\sin\theta - mgR\theta) &= 0 \qquad \text{(a) 3 p)} \\
\ddot{\theta} &= \frac{g}{R}(\cos\theta - 1) \\
V &= mgR\sin\theta - mgR\theta) \\
V'(\theta) &= mgR(\cos\theta - 1) \\
V''(\theta) &= -mgR\sin\theta \\
V'(0) &= 0, \quad V''(0) < 0 \text{ máximo, inestable} \qquad \text{(b) 3 p)}
\end{aligned}$$

Problema (3)

a velocidad tiene componentes

$$v_r = \dot{r}, \quad v_\theta = r\dot{\theta} = 0, \quad v_\phi = r \sin\alpha\Omega$$

luego el Lagrangiano es

$$L = \frac{1}{2}m(\dot{r}^2 + r^2 \sin^2\alpha\Omega^2) - mgr\cos\alpha,$$

de donde

$$\ddot{r} - r\sin^2\alpha\Omega^2 + g\cos\alpha = 0 \qquad \text{(a) 3 p)}$$

El Hamiltoniano es

$$\begin{aligned} H &= p_r\dot{r} - L \\ p_r &= m\dot{r} \\ H &= \frac{1}{2}m(\dot{r}^2 - r^2\sin^2\alpha\Omega^2) + mgr\cos\alpha \qquad \text{(b) 3 p)} \end{aligned}$$