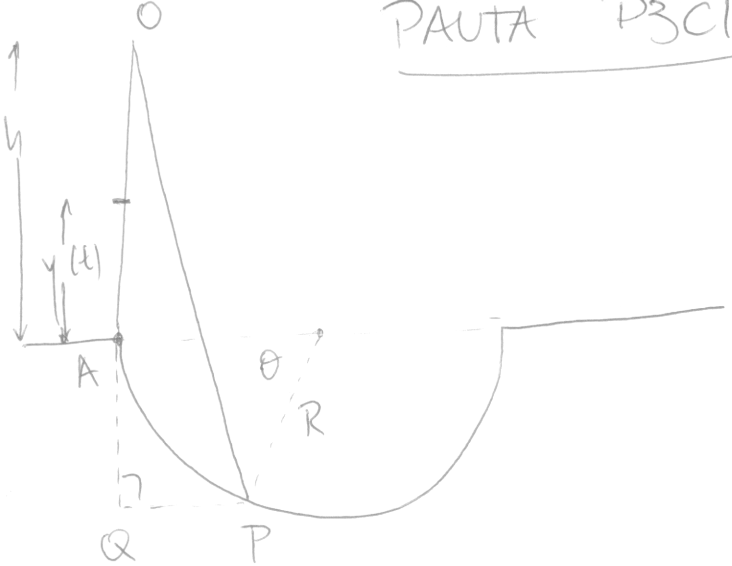
PAUTA P3C1 — Fernando Becerra Saavedra



Formemos el triángulo POQ

Notemos que

- $\overline{OP} + h - y = 2h$

  $\overline{OP} = h + y$
- $AQ = R\sin\theta$
- $QP = R - R\cos\theta$

Luego usamos pitágoras: $\overline{OP}^2 = \overline{OQ}^2 + \overline{QP}^2$

$$\Rightarrow (h+y)^2 = (h+R\sin\theta)^2 + R^2(1-\cos\theta)^2$$

$$h+y = \sqrt{(h+R\sin\theta)^2 + R^2(1-\cos\theta)^2}$$

$$y(\theta) = \sqrt{(h+R\sin\theta)^2 + R^2(1-\cos\theta)^2} - h$$

Debemos encontrar $y(t)$, pero esto encontramos $\theta = \theta(t)$, y sabemos:

$$\theta = \omega t \quad y \quad \omega = \frac{v}{R} \Rightarrow \theta = \frac{vt}{R}$$

Luego $\boxed{y(t) = \sqrt{\left(h + R\sin\left(\frac{vt}{R}\right)\right)^2 + R^2\left(1-\cos\left(\frac{vt}{R}\right)\right)^2}}$

Derivando:

$$\dot{y}(t) = \frac{1}{\cancel{2}}\,\frac{\cancel{2}\left(h+R\sin\left(\frac{vt}{R}\right)\right)R\,\frac{v}{R}\cos\left(\frac{vt}{R}\right) + \cancel{2}R^2\left(1-\cos\left(\frac{vt}{R}\right)\right)\sin\left(\frac{vt}{R}\right)\cdot\frac{v}{\cancel{R}}}{\sqrt{\left(h+R\sin\left(\frac{vt}{R}\right)\right)^2 + R^2\left(1-\cos\left(\frac{vt}{R}\right)\right)^2}}$$

$$= \frac{hv\cos\left(\frac{vt}{R}\right) + \cancel{vR\sin\left(\frac{vt}{R}\right)\cos\left(\frac{vt}{R}\right)} + vR\sin\left(\frac{vt}{R}\right) \cancel{- vR\cos\left(\frac{vt}{R}\right)\sin\left(\frac{vt}{R}\right)}}{\sqrt{\left(h+R\sin\left(\frac{vt}{R}\right)\right)^2 + R^2\left(1-\cos\left(\frac{vt}{R}\right)\right)^2}}$$

$$\boxed{\dot{y}(t) = \frac{v\left[h\cos\left(\frac{vt}{R}\right) + R\sin\left(\frac{vt}{R}\right)\right]}{\sqrt{\left(h+R\sin\left(\frac{vt}{R}\right)\right)^2 + R^2\left(1-\cos\left(\frac{vt}{R}\right)\right)^2}}}$$

Derivando esta expresión, se tiene $\ddot{y}(t)$