DERIVADAS POR DEFINICION

- **Derivada de una constante**: $f(x) = k$

$$lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = lim_{\Delta x \to 0} \frac{k-k}{\Delta x} = lim_{\Delta x \to 0} \frac{0}{\Delta x} = 0$$

- **Derivada de x** : $f(x) = x$

$$lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = lim_{\Delta x \to 0} \frac{x+\Delta x - x}{\Delta x} = lim_{\Delta x \to 0} \frac{\Delta x}{\Delta x} = lim_{\Delta x \to 0} 1 = 1$$

- **Derivada de la raíz cuadrada de x**: $f(x) = \sqrt{x}$

$$lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = lim_{\Delta x \to 0} \frac{\sqrt{x+\Delta x} - \sqrt{x}}{\Delta x} = lim_{\Delta x \to 0} \frac{\sqrt{x+\Delta x} - \sqrt{x}}{\Delta x} * \frac{\sqrt{x+\Delta x} + \sqrt{x}}{\sqrt{x+\Delta x} + \sqrt{x}} = lim_{\Delta x \to 0} \frac{x+\Delta x - x}{\Delta x.(\sqrt{x+\Delta x} + \Delta x)} =$$

$$= lim_{\Delta x \to 0} \frac{\Delta x}{\Delta x(\sqrt{x+\Delta x} + \sqrt{x})} = lim_{\Delta x \to 0} \frac{1}{\sqrt{x+\Delta x} + \sqrt{x}} = \frac{1}{2\sqrt{x}}$$

- **Derivada de 1/x**: $f(x) = \frac{1}{x}$

$$lim_{\Delta x \to 0} \frac{\frac{1}{x+\Delta x} - \frac{1}{x}}{\Delta x} = lim_{\Delta x \to 0} \frac{\frac{x-(x+\Delta x)}{x(x+\Delta x)}}{\Delta x} = lim_{\Delta x \to 0} \frac{x-x-\Delta x}{x(x+\Delta x)\Delta x} = lim_{\Delta x \to 0} \frac{-\Delta x}{x(x+\Delta x)\Delta x} = lim_{\Delta x \to 0} \frac{-1}{x(x+\Delta x)} = \frac{-1}{x^2}$$

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- **Derivada de x²**: $f(x) = x^2$

$$lim_{\Delta x \to 0} \frac{(x+\Delta x)^2 - x^2}{\Delta x} = lim_{\Delta x \to 0} \frac{x^2 + 2.x.\Delta x + \Delta x^2 - x^2}{\Delta x} = lim_{\Delta x \to 0} \frac{2.x.\Delta x + \Delta x^2}{\Delta x} =$$

$$lim_{\Delta x \to 0} \frac{(2.x + \Delta x).\Delta x}{\Delta x} = lim_{\Delta x \to 0} (2.x + \Delta x) = 2.x$$

- **Derivada de la suma**: $f(x) = u(x) + v(x) \Rightarrow f'(x) = u'(x) + v'(x)$

$$lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = lim_{\Delta x \to 0} \frac{(u(x+\Delta x) + v(x+\Delta x)) - (u(x) + v(x))}{\Delta x} = lim_{\Delta x \to 0} \frac{u(x+\Delta x) + v(x+\Delta x) - u(x) - v(x)}{\Delta x}$$

$$lim_{\Delta x \to 0} \frac{u(x+\Delta x) - u(x) + v(x+\Delta x) - v(x)}{\Delta x} = lim_{\Delta x \to 0} \frac{u(x+\Delta x) - u(x)}{\Delta x} + lim_{\Delta x \to 0} \frac{v(x+\Delta x) - v(x)}{\Delta x} = u'(x) + v'(x)$$

- **Derivada de la resta**: $f(x) = u(x) - v(x) \Rightarrow f'(x) = u'(x) - v'(x)$

$$lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = lim_{\Delta x \to 0} \frac{(u(x+\Delta x) - v(x+\Delta x)) - (u(x) - v(x))}{\Delta x} = lim_{\Delta x \to 0} \frac{u(x+\Delta x) - v(x+\Delta x) - u(x) + v(x)}{\Delta x}$$

$$lim_{\Delta x \to 0} \frac{u(x+\Delta x) - u(x) - (v(x+\Delta x) - v(x))}{\Delta x} = lim_{\Delta x \to 0} \frac{u(x+\Delta x) - u(x)}{\Delta x} - lim_{\Delta x \to 0} \frac{v(x+\Delta x) - v(x)}{\Delta x} = u'(x) - v'(x)$$

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- **Regla de la cadena**: $\left[f\left(g(x)\right)\right]' = f'\left(g(x)\right).g'(x)$

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$$lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x} = lim_{\Delta x\to 0}\frac{f\left[g(x+\Delta x)\right]-f\left[g(x)\right]}{\Delta x} = lim_{\Delta x\to 0}\frac{f\left[g(x+\Delta x)\right]-f\left[g(x)\right]}{\Delta x}.\frac{g(x+\Delta x)-g(x)}{g(x+\Delta x)-g(x)}$$

$$\cdot\ lim_{\Delta x\to 0}\frac{f\left[g(x+\Delta x)\right]-f\left[g(x)\right]}{g(x+\Delta x)-g(x)}.\frac{g(x+\Delta x)-g(x)}{\Delta x} = f'\left[g(x)\right].g'(x)$$

- **Derivada de logaritmo natural de x**: $f(x) = \mathbf{ln}(x)$

Vamos a usar las siguientes propiedades del logaritmo:

1) $\mathbf{ln}(A)-\mathbf{ln}(B) = \mathbf{ln}\left(\frac{A}{B}\right)$

2) $B.\mathbf{ln}(A) = \mathbf{ln}(A^B)$

3) $lim_{x\to 0}\left(1+\frac{1}{x}\right)^x = e$

4) $\mathbf{ln}(e) = 1$

$$lim_{\Delta x\to 0}\frac{\mathbf{ln}(x+\Delta x)-\mathbf{ln}(x)}{\Delta x} = lim_{\Delta x\to 0}\frac{\mathbf{ln}\left(\frac{x+\Delta x}{\Delta x}\right)}{\Delta x} = lim_{\Delta x\to 0}\frac{\mathbf{ln}\left(\frac{x}{\Delta x}+1\right)}{\Delta x} = lim_{\Delta x\to 0}\frac{1}{\Delta x}.\mathbf{ln}\left(1+\frac{x}{\Delta x}\right) = lim_{\Delta x\to 0}\mathbf{ln}\left(1+\frac{1}{\frac{\Delta x}{x}}\right)^{\frac{1}{\Delta x}} =$$

$$= lim_{\Delta x\to 0}\ \mathbf{ln}\left(1+\frac{1}{\frac{\Delta x}{x}}\right)^{\frac{1}{\Delta x}\frac{x}{\Delta x}\frac{\Delta x}{x}} = lim_{\Delta x\to 0}\left[\mathbf{ln}\left(1+\frac{1}{\frac{\Delta x}{x}}\right)^{\frac{\Delta x}{x}}\right]^{\frac{1}{\Delta x}\frac{\Delta x}{x}} = \mathbf{ln}\left[lim_{\Delta x\to 0}\left(1+\frac{1}{\frac{\Delta x}{x}}\right)^{\frac{x}{\Delta x}}\right]^{\frac{1}{x}} = \mathbf{ln}\, e^{\frac{1}{x}} = \frac{1}{x}\mathbf{ln}(e) = \frac{1}{x}$$

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Derivas aplicando diferenciación logarítmica

- $\boxed{\mathbf{f(x) = x^n \Rightarrow f'(x) = n.x^{n-1}}}$

$$f(x) = x^n$$
$$\ln(f(x)) = \ln(x^n)$$
$$\ln(f(x)) = n.\ln(x)$$
$$\frac{1}{f(x)}.f'(x) = n.\frac{1}{x}$$
$$f'(x) = n.\frac{1}{x}.f(x)$$
$$f'(x) = n.\frac{1}{x}.x = n.x^{n-1}$$

- $\boxed{\mathbf{y = [f(x)]^n \Rightarrow y' = n.[f(x)]^{n-1}.f'(x)}}$

$$y = [f(x)]^n$$
$$\ln(y) = \ln([f(x)]^n)$$
$$\ln(y) = n.\ln(f(x))$$
$$\frac{1}{y}.y' = n.\frac{1}{f(x)}.f'(x)$$
$$y' = n.\frac{1}{f(x)}.f'(x).y$$
$$y' = n.\frac{1}{f(x)}.f'(x).f(x) = n.[f(x)]^{n-1}.f'(x)$$

- $\boxed{y = a^x \Rightarrow y' = a^x.\ln(a)}$

$$y = a^x$$
$$\ln(y) = \ln(a^x)$$
$$\ln(y) = x.\ln(a)$$
$$\frac{1}{y}.y' = 1.\ln(a)$$
$$y' = \ln(a).y$$
$$y' = \ln(a).a^x$$

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- $\boxed{y = e^x \Rightarrow y' = e^x}$

Se resuelva igual que el caso anterior. Pero como ***ln(e)=1***, resulta ***y'=e^x***.

- $\boxed{y = a^{f(x)} \Rightarrow y' = a^{f(x)}.\ln(a).f'(x)}$

$$y = a^{f(x)}$$
$$\ln(y) = \ln(a^{f(x)})$$
$$\ln(y) = f(x).\ln(a)$$
$$\frac{1}{y}.y' = f'(x).\ln(a)$$
$$y' = f'(x).\ln(a).y$$
$$y' = f'(x).\ln(a).a^{f(x)}$$

- $\boxed{y = e^{f(x)} \Rightarrow y' = e^{f(x)}.f'(x)}$

Se resuelva igual que el caso anterior. Pero como ***ln(e)=1***, resulta ***y'=e^{f(x)}.f'(x)***

- $\boxed{y = [f(x)]^{g(x)} \Rightarrow y' = \left[g'(x).\ln(f(x)) + g(x).\frac{1}{f(x)}.f'(x)\right].[f(x)]^{g(x)}}$

$$y = [f(x)]^{g(x)}$$
$$\ln(y) = \ln\left[[f(x)]^{g(x)}\right]$$
$$\ln(y) = g(x).\ln(f(x))$$
$$\frac{1}{y}.y' = g'(x).\ln(f(x)) + g(x).\frac{1}{f(x)}.f'(x)$$
$$y' = \left[g'(x).\ln(f(x)) + g(x).\frac{1}{f(x)}.f'(x)\right].y$$
$$y' = \left[g'(x).\ln(f(x)) + g(x).\frac{1}{f(x)}.f'(x)\right].[f(x)]^{g(x)}$$

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- **Regla del producto:** $\mathbf{y = u(x).v(x) \Rightarrow y' = u'(x).v(x) + u(x).v'(x)}$

$$y = u(x).v(x)$$
$$\ln(y) = \ln(u(x).v(x))$$
$$\ln(y) = l(u(x)) + \ln(v(x))$$
$$\frac{1}{y}.y' = \frac{1}{u(x)}.u'(x) + \frac{1}{v(x)}.v'(x)$$
$$y' = \left(\frac{1}{u(x)}.u'(x) + \frac{1}{v(x)}.v'(x)\right).y$$
$$y' = \left(\frac{1}{u(x)}.u'(x) + \frac{1}{v(x)}.v'(x)\right).u(x).v(x)$$
$$y' = \frac{1}{u(x)}.u'(x).u(x).v(x) + \frac{1}{v(x)}.v'(x).u(x).v(x)$$
$$y' = u'(x).v(x) + v'(x).u(x)$$

- **Deducción de la regla del cociente**: $\mathbf{y = \frac{u(x)}{v(x)} \Rightarrow y' = \frac{u'(x).v(x) - u(x).v'(x)}{v^2(x)}}$

$$y = \frac{u(x)}{v(x)}$$
$$\ln(y) = \ln\left(\frac{u(x)}{v(x)}\right)$$
$$\ln(y) = l(u(x)) - \ln(v(x))$$
$$\frac{1}{y}.y' = \frac{1}{u(x)}.u'(x) - \frac{1}{v(x)}.v'(x)$$
$$y' = \left(\frac{1}{u(x)}.u'(x) - \frac{1}{v(x)}.v'(x)\right).y$$
$$y' = \left(\frac{1}{u(x)}.u'(x)\frac{1}{v(x)}.v'(x)\right).\frac{u(x)}{v(x)}$$
$$y' = \frac{1}{u(x)}.u'(x).\frac{u(x)}{v(x)} - \frac{1}{v(x)}.v'(x).\frac{u(x)}{v(x)}$$
$$y' = \frac{u'(x)}{v(x)} - \frac{v'(x).u(x)}{v^2(x)} = \frac{u'(x).v(x) - v'(x).u(x)}{v^2(x)}$$

Apunte Nro 0028

·Derivida del seno de x:

Vamos a usar la siguiente relación trigonométrica

$$\operatorname{sen} a - \operatorname{sen} b = 2\operatorname{sen}\left(\frac{a-b}{2}\right).\cos\left(\frac{a+b}{2}\right)$$

$$lim_{\Delta x\to 0}\frac{\operatorname{sen}(x+\Delta x)-\operatorname{sen}(x)}{\Delta x} = lim_{\Delta x\to 0}\frac{2\operatorname{sen}\left(\frac{x+\Delta x-x}{2}\right).\cos\left(\frac{x+\Delta x+x}{2}\right)}{\Delta x} = lim_{\Delta x\to 0}\frac{\operatorname{sen}\left(\frac{\Delta x}{2}\right).\cos\left(\frac{2x+\Delta x}{2}\right)}{\frac{\Delta x}{2}} =$$

$$lim_{\Delta x\to 0}\left(\frac{\operatorname{sen}(\frac{\Delta x}{2})}{\frac{\Delta x}{2}}\right).lim_{\Delta x\to 0}\cos\left(\frac{(2x+\Delta x)}{2}\right) = lim_{\Delta x\to 0}\cos\left(\frac{(2x+\Delta x)}{2}\right) = \cos\left(\frac{2x}{2}\right) = \cos(x)$$

Contraejemplo para demostrar que la continuidad de una función no implica su derivabilidad:

F(x)= $|x|$

1)Demostramos que es contínua en x=0

$$\left.\begin{array}{r} lim_{x\to 0}|x| = |0| = 0 \\ F(0) = |0| = 0 \end{array}\right\}\text{ es contínua en x=0}$$

2) Demostramos que no es derivable en x=0

$$F'(x) = lim_{h\to 0}\frac{F(x+h)-F(x)}{h} = lim_{h\to 0}\frac{F(h)-F(0)}{h} = lim_{h\to 0}\frac{|h|}{h} = \left\{\begin{array}{l} lim_{h\to 0^-}\frac{-h}{h} = -1 \\ \\ lim_{h\to 0^+}\frac{h}{h} = 1 \end{array}\right\} \Rightarrow$$

Por el teorema de unicidad del límite, si el límite existe, debe ser único. Por lo tanto este límite no existe, o sea que F(x) no es derivable en x=0