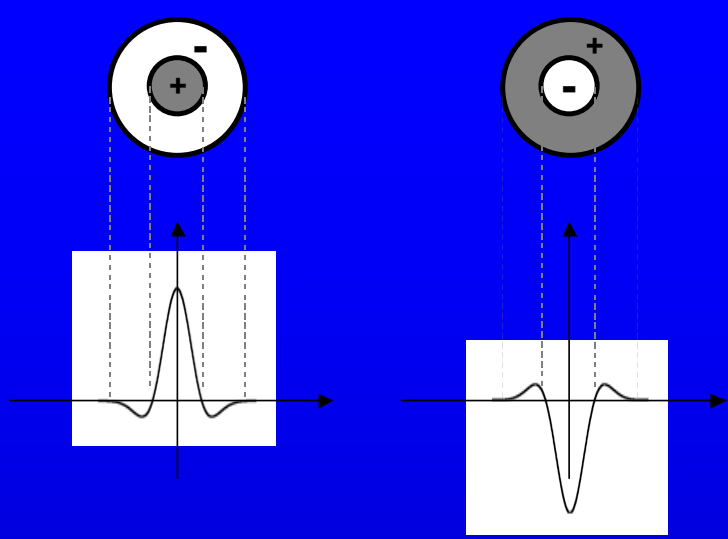
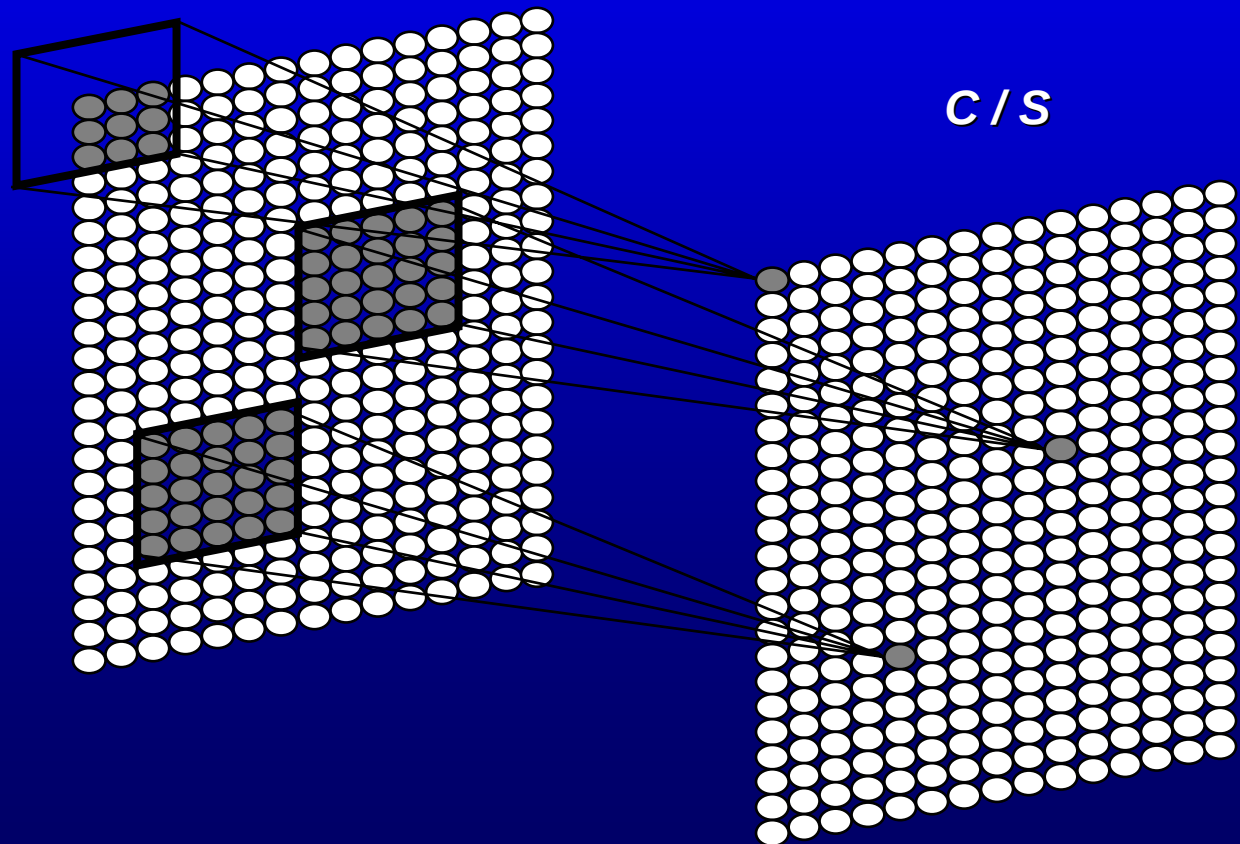
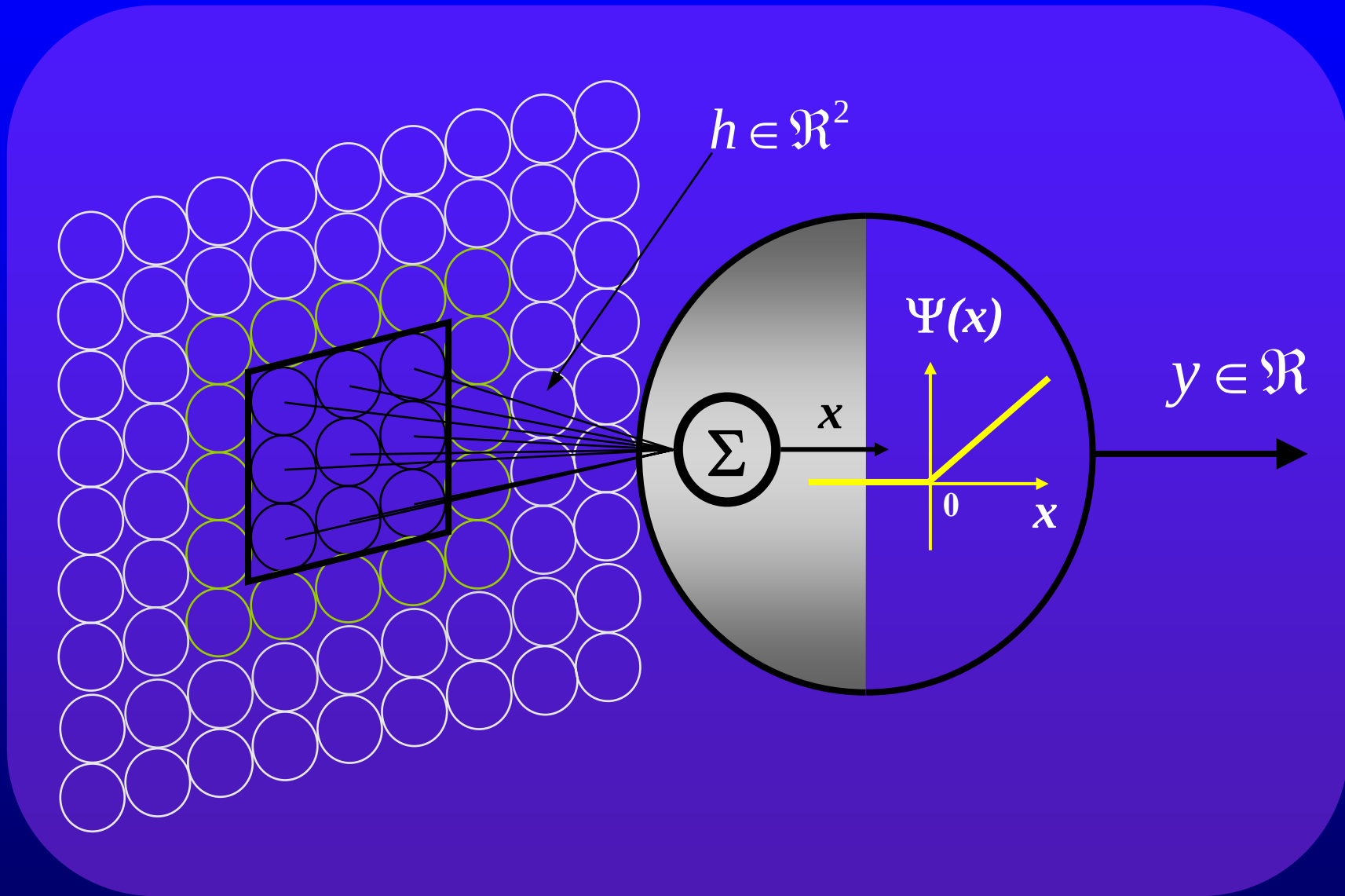




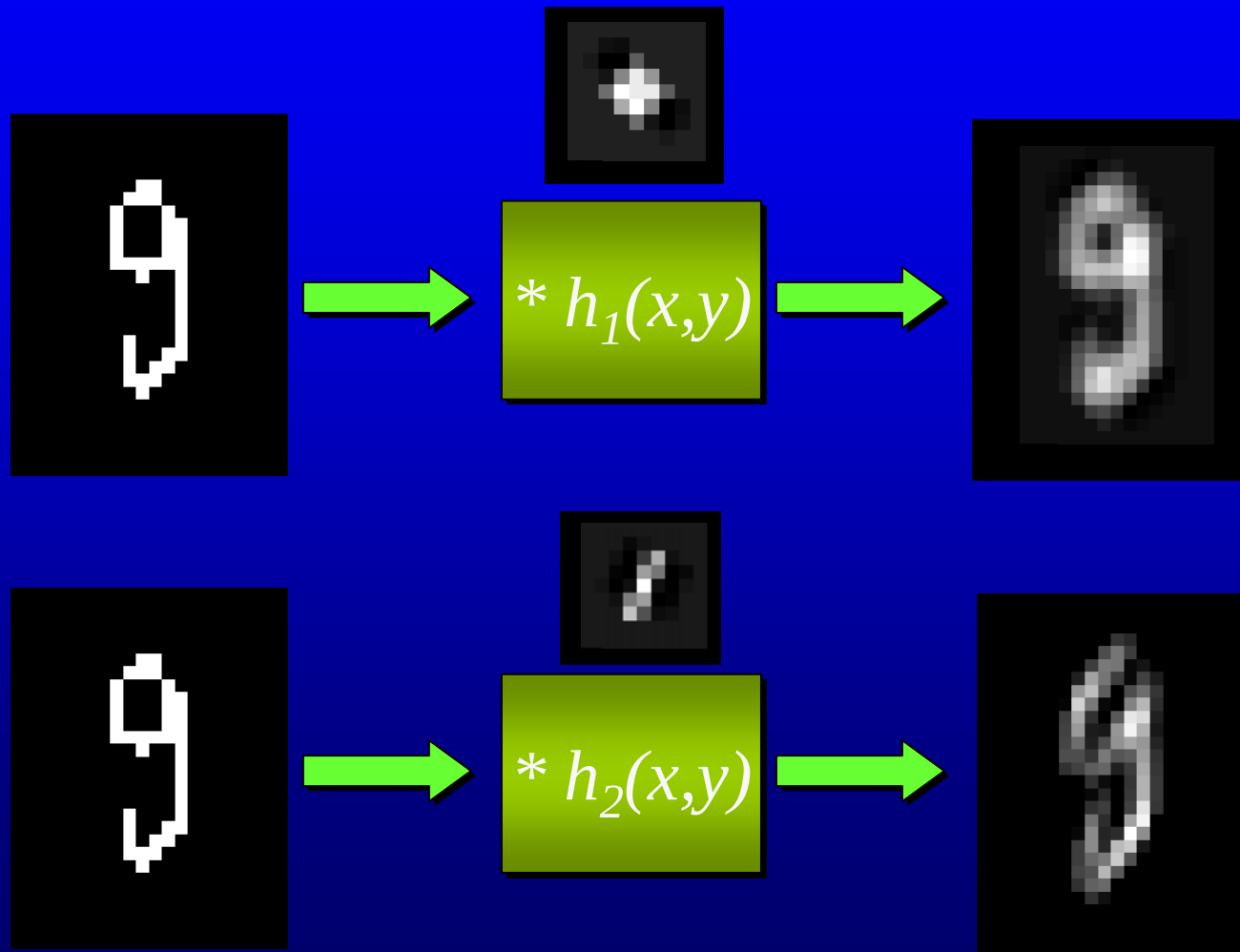
S/C



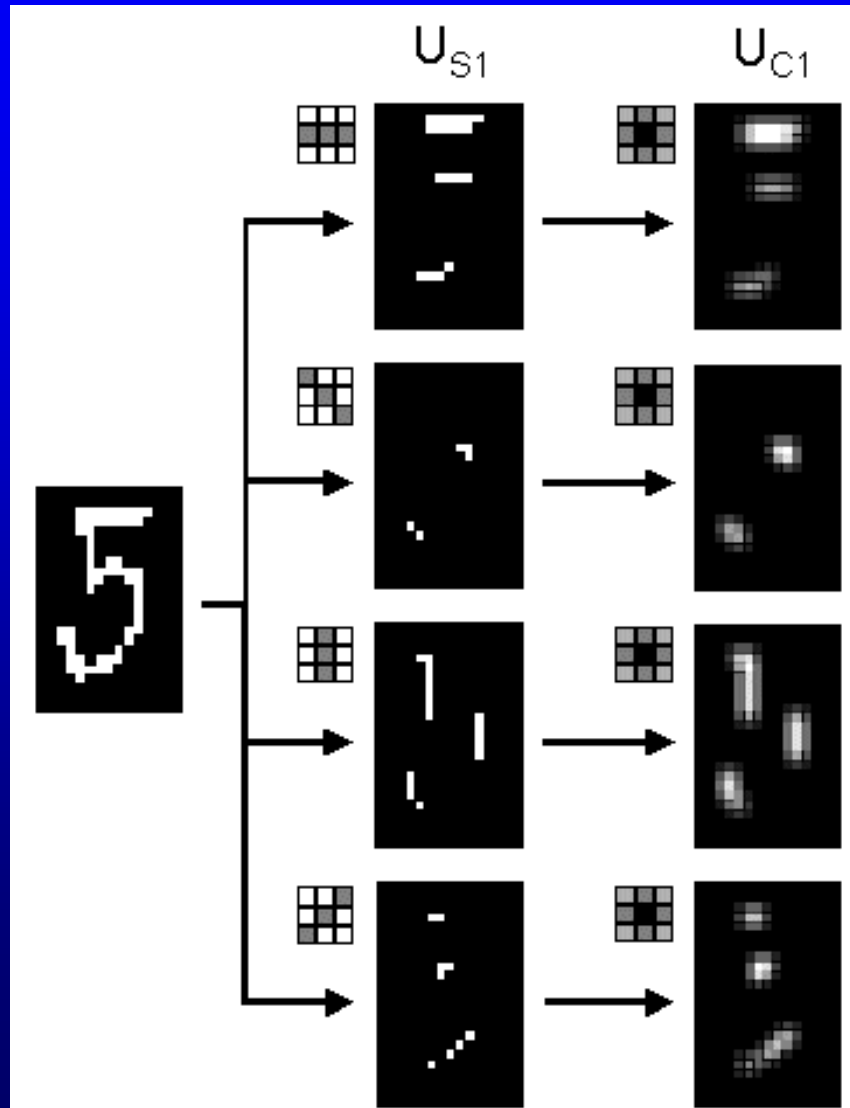
C/S

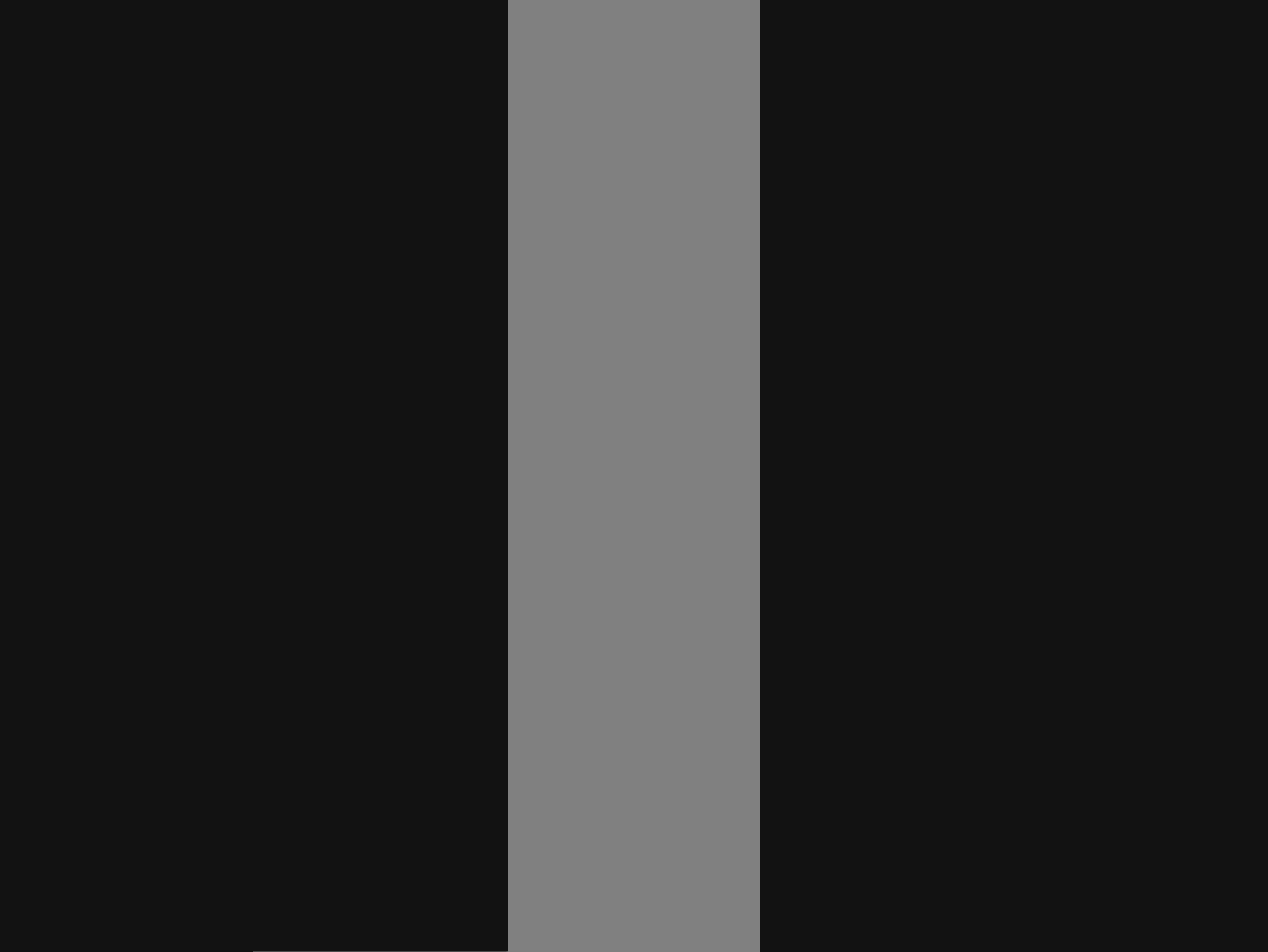


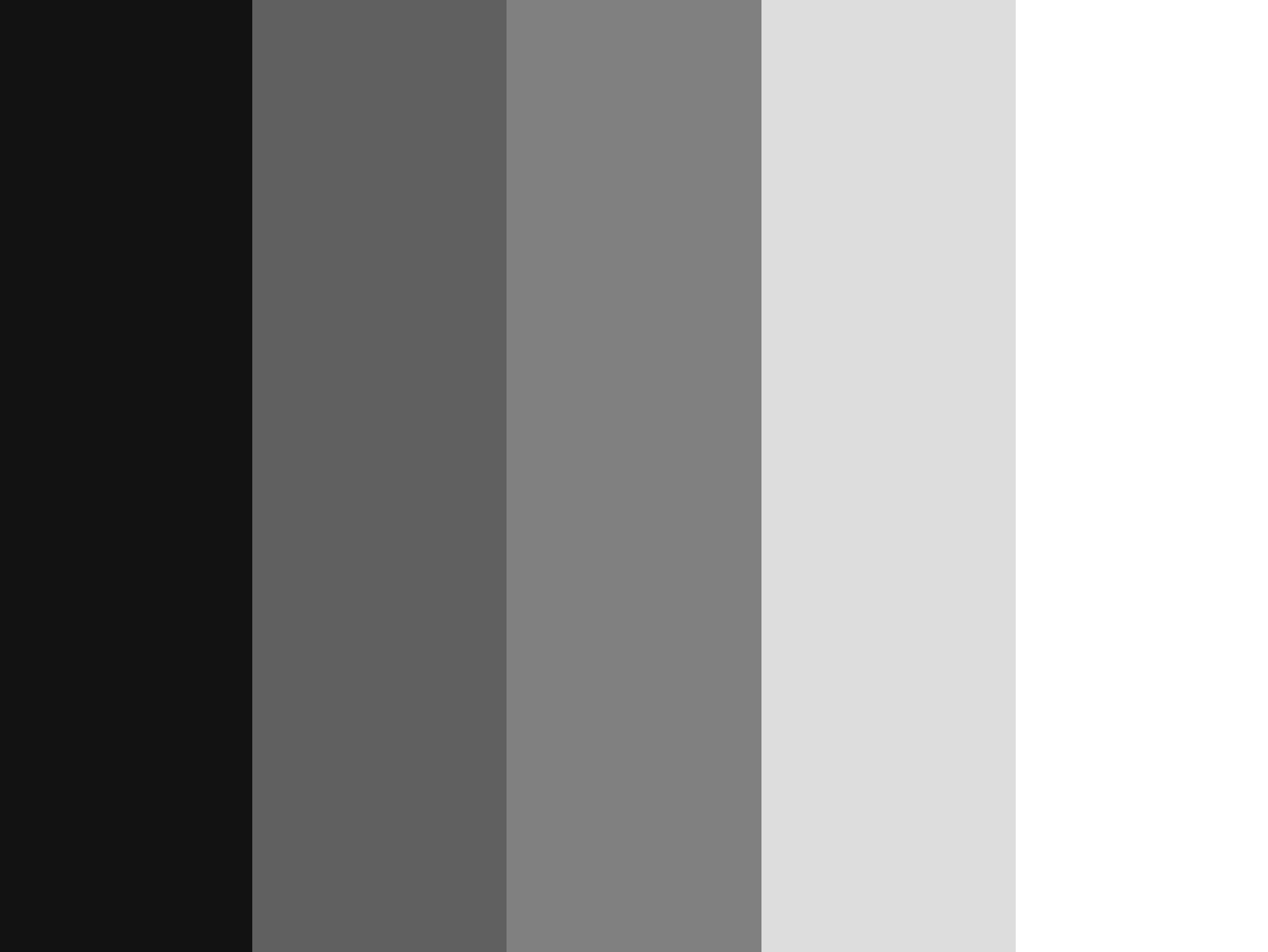
Receptive fields as filters

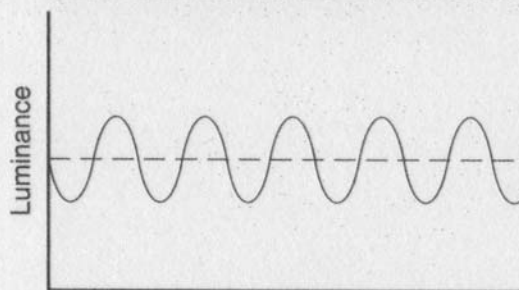
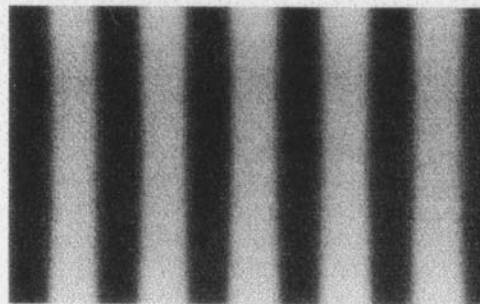


Introduction: Non-Linear Filters

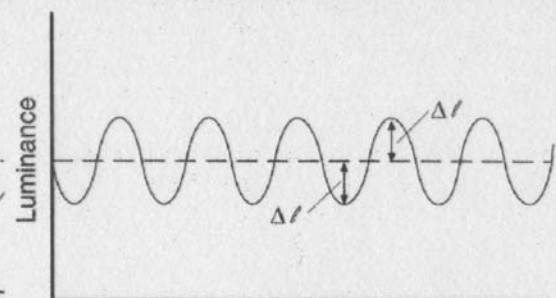
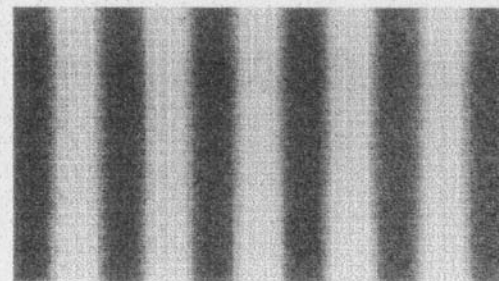




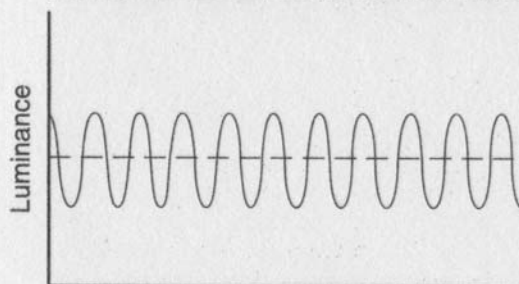
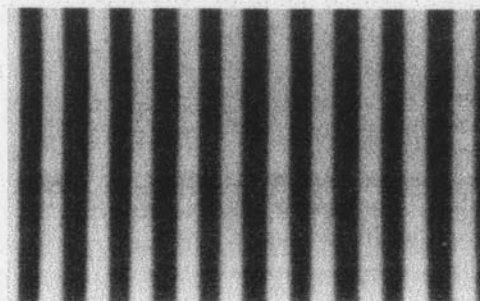




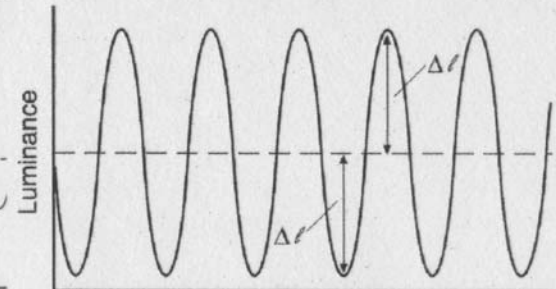
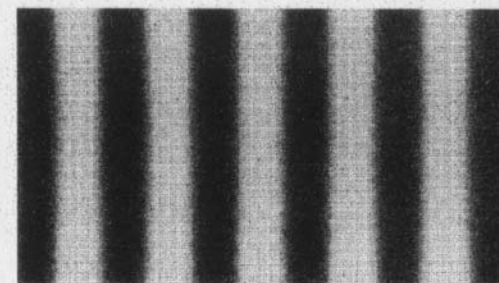
Position



Position

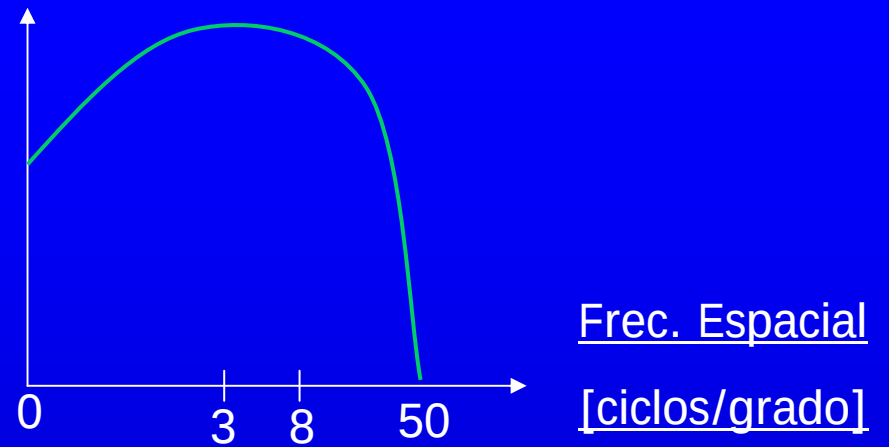


Position

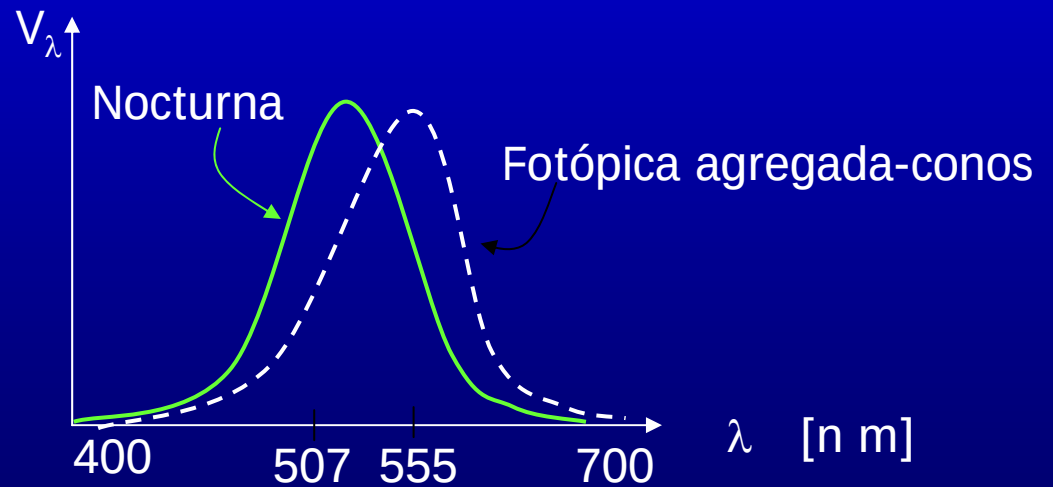


Position

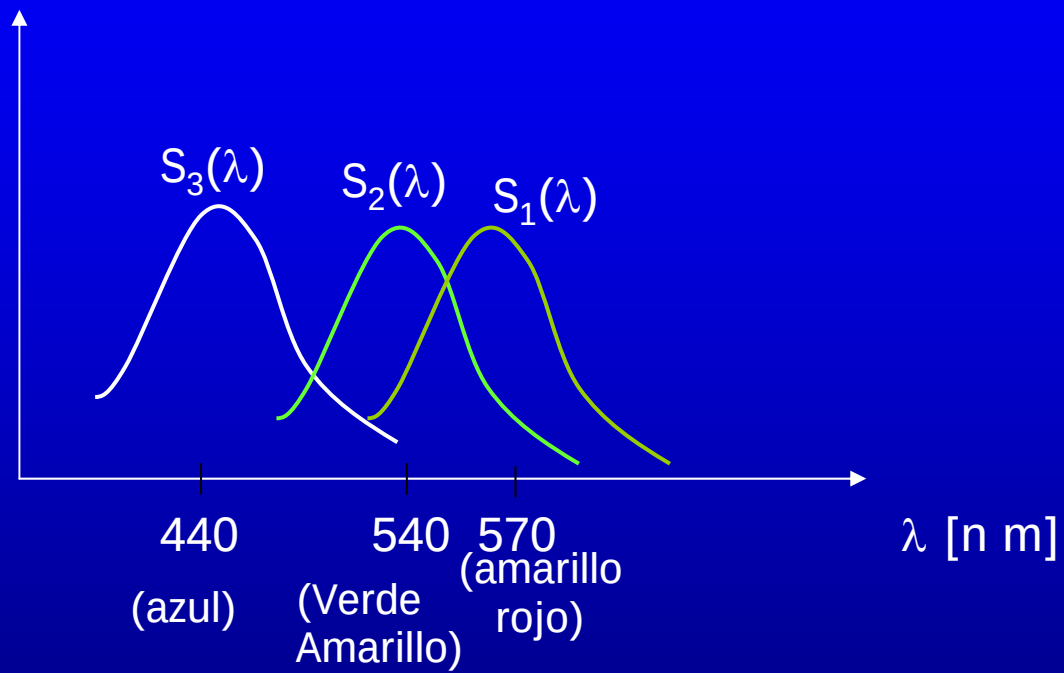
- Sensibilidad al Contraste

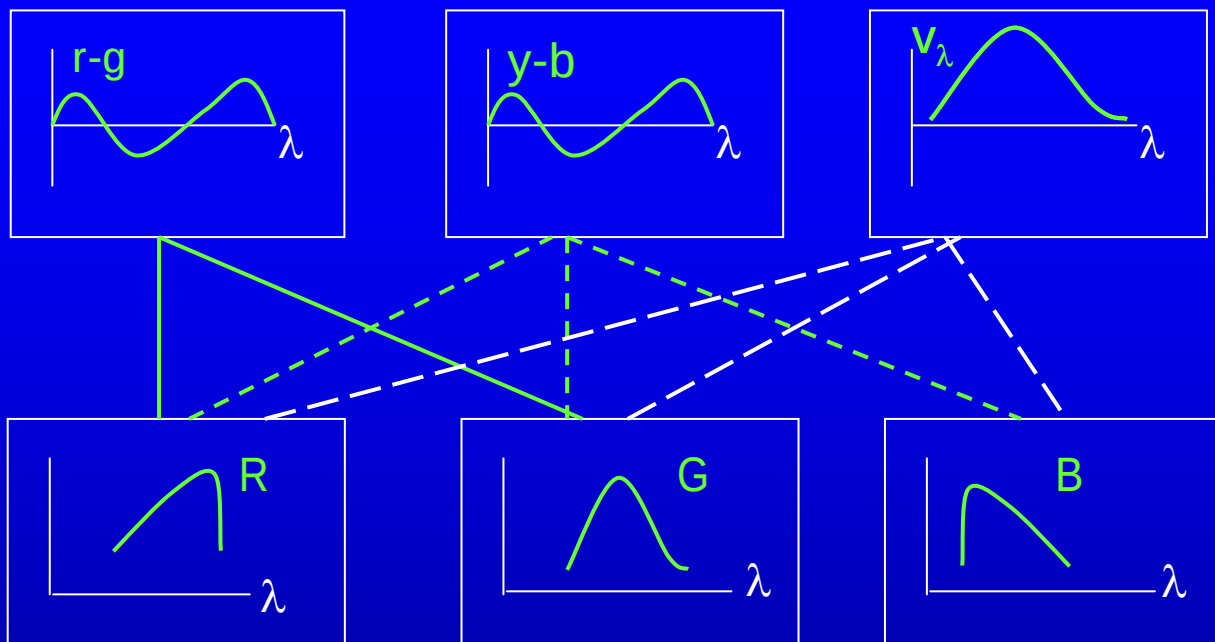


- Función eficiencia luminosa relativa

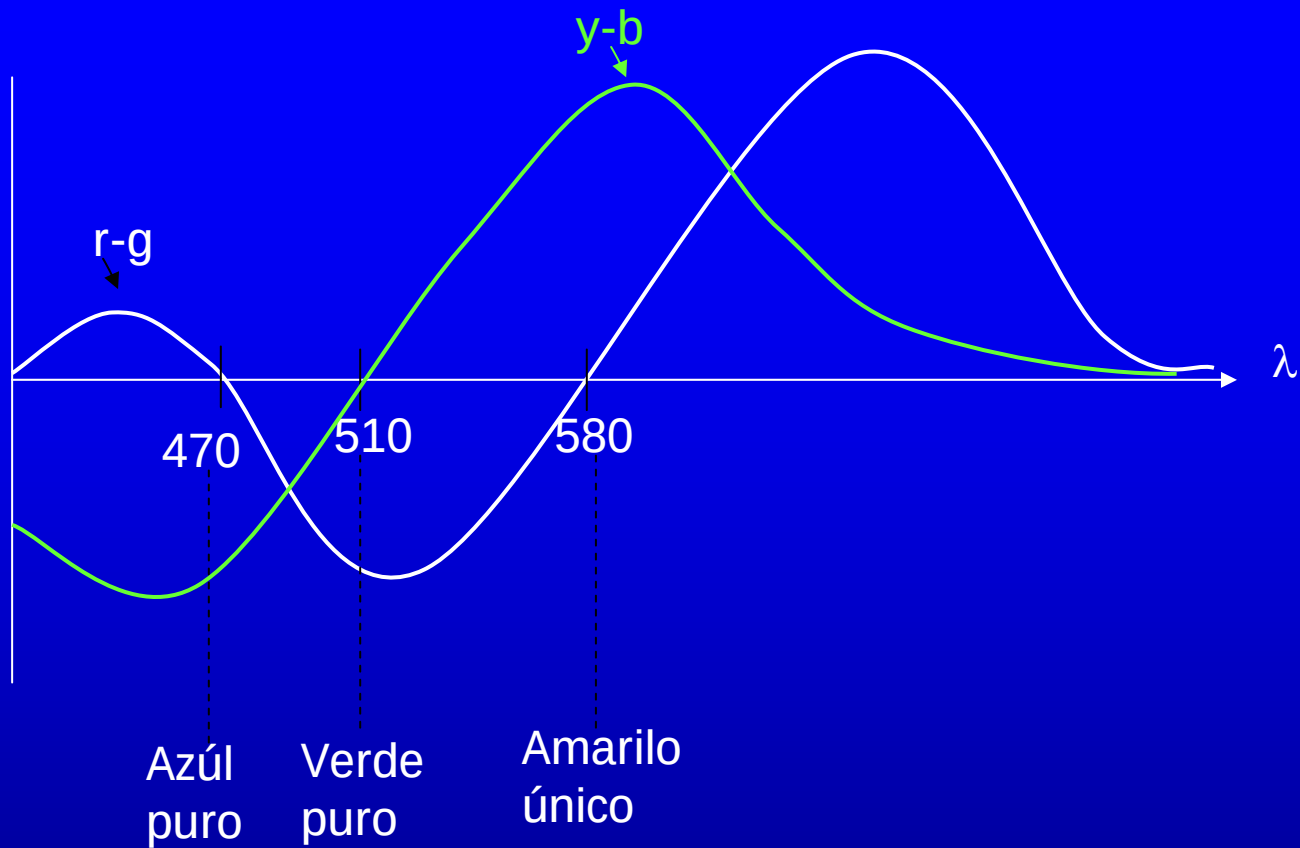


- Sensibilidad - Conos





$$\begin{bmatrix} r-g \\ y-b \\ v_\lambda \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

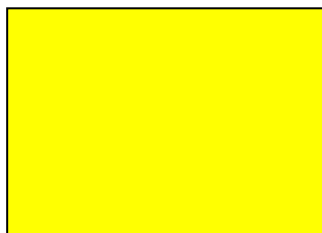
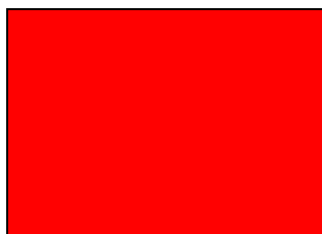


- Experimento

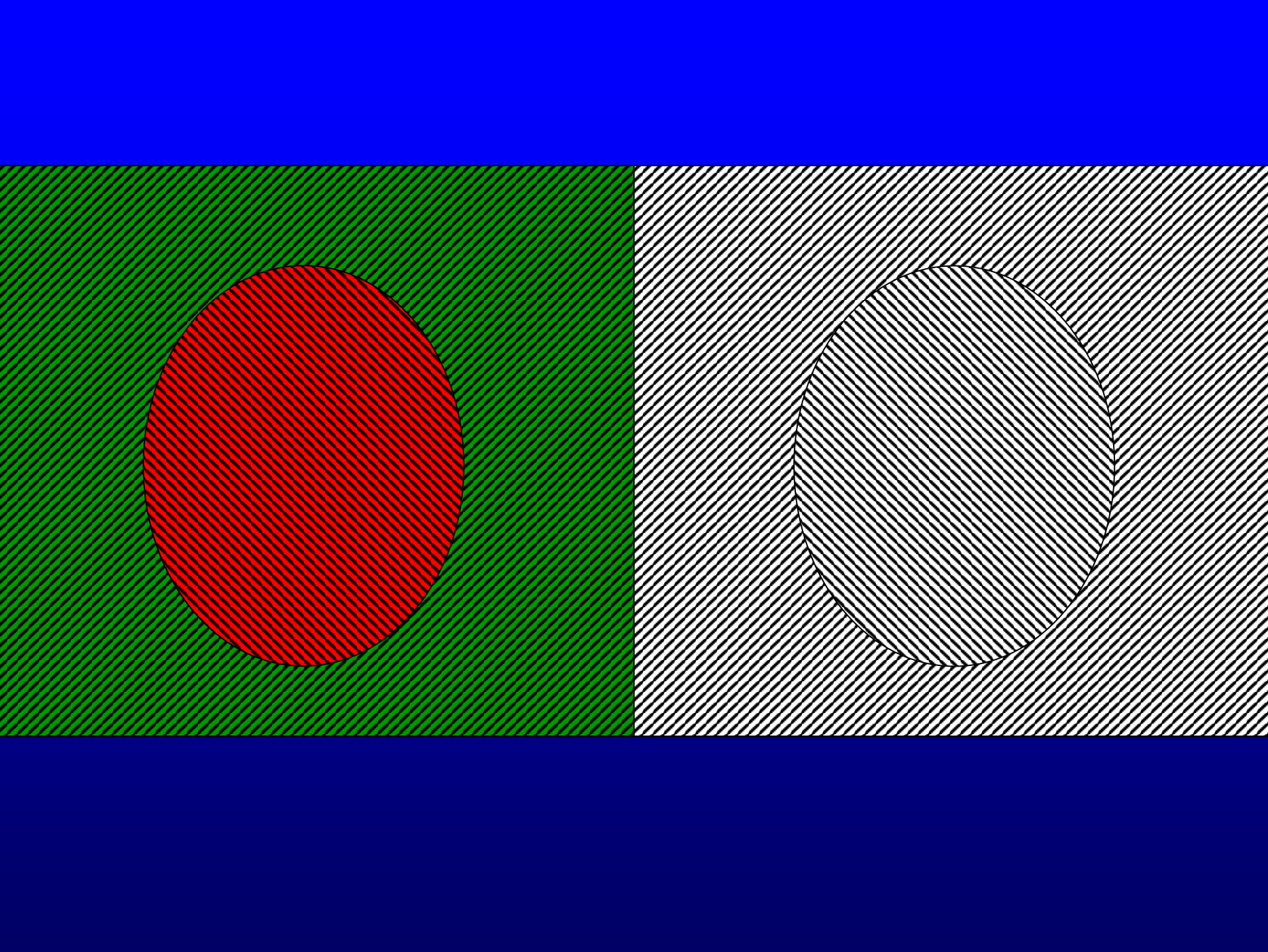
x



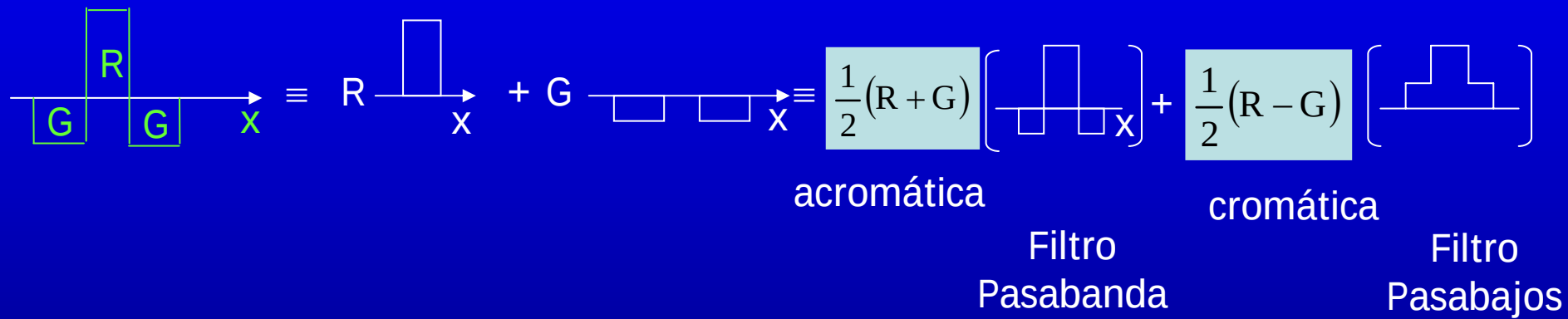
+



+

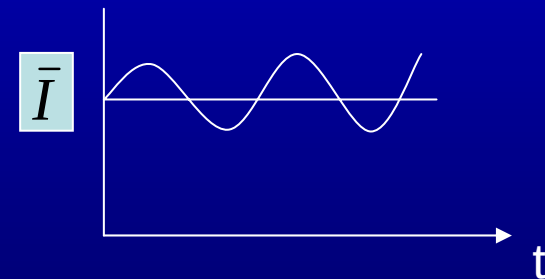
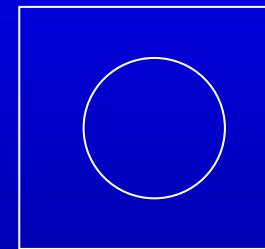
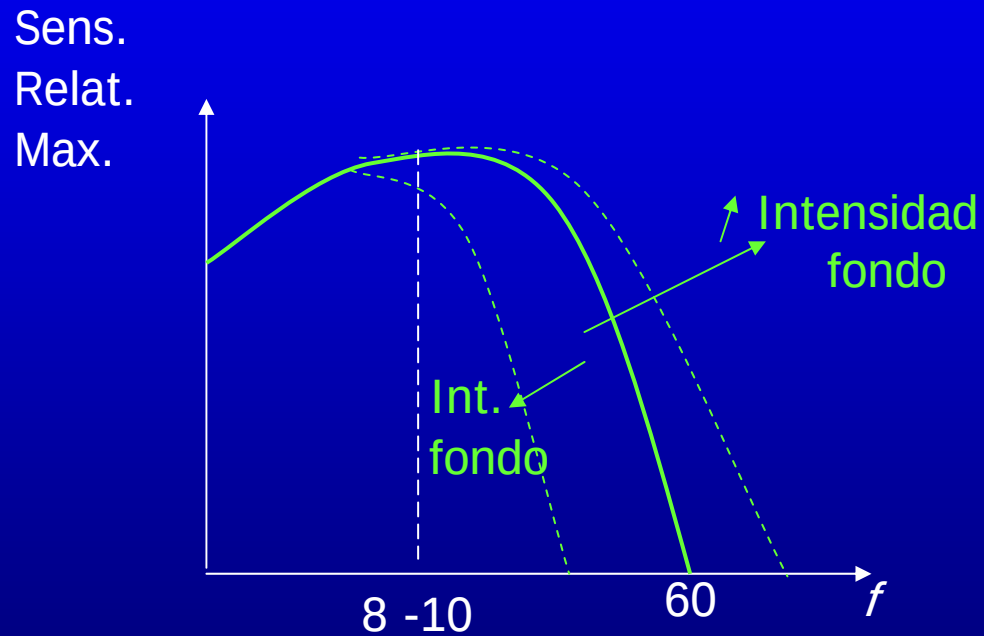


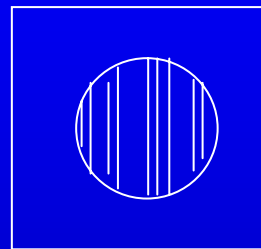
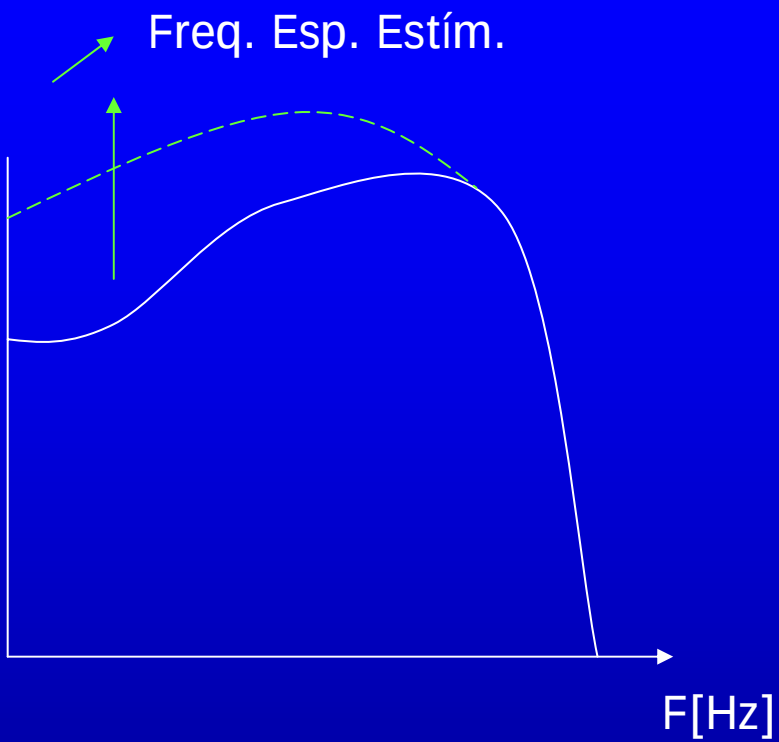
- Descomposición campo receptivo

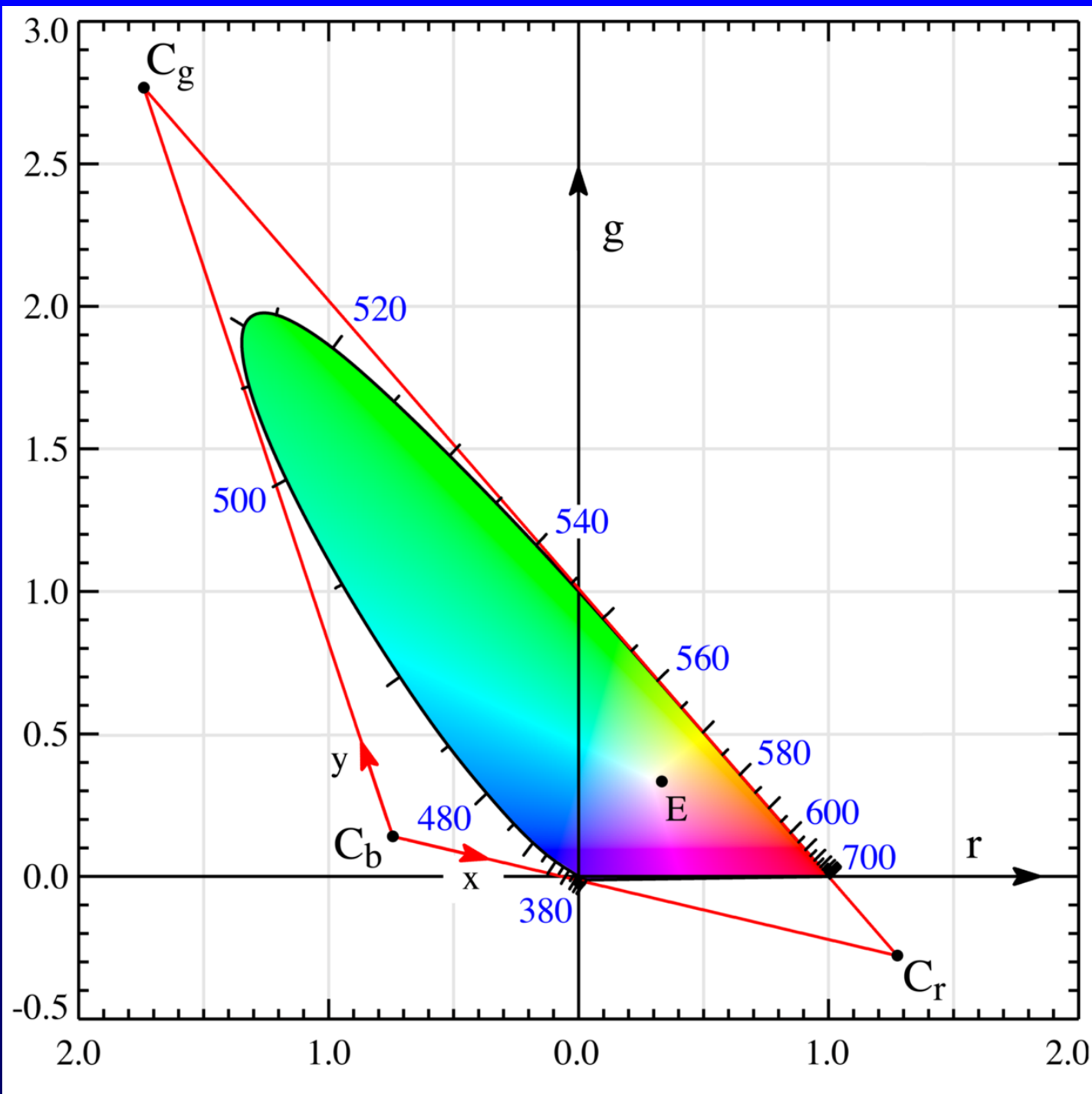


- Propiedades Temporales

- Frec. Crítica de Fusión
- Función de Sensibilidad a la variación de contraste temporal







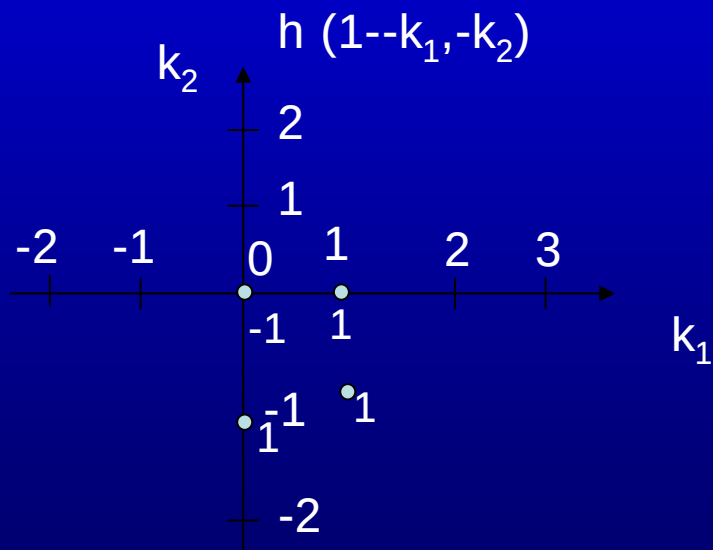
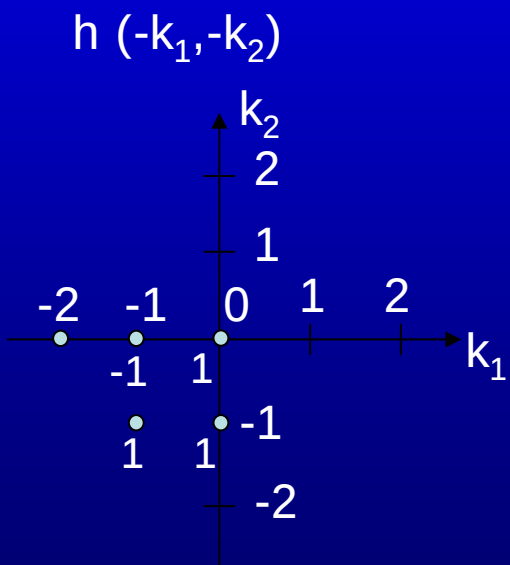
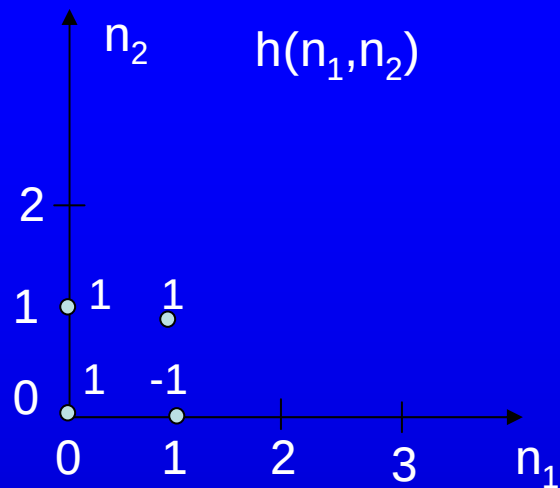
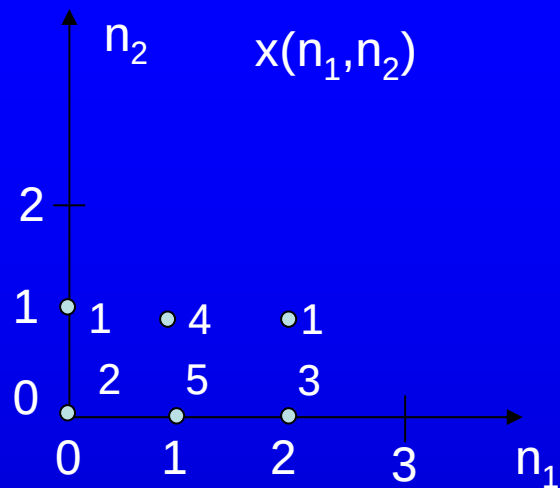
Señales 2-D

- Secuencia 2-D

$$\begin{matrix} \delta(n_1, n_2) \\ u(n_1, n_2) \end{matrix}$$

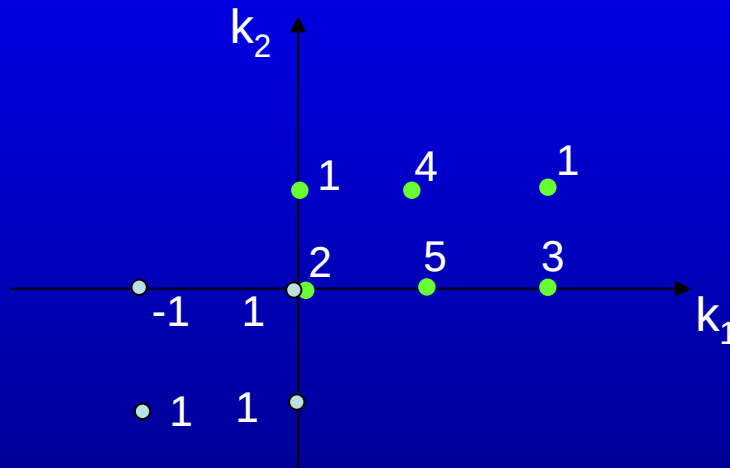
- Convolución 2-D

$$y(n_1, n_2) = x(n_1, n_2) * h(n_1, n_2) = \sum_{k_1} \sum_{k_2} x(k_1, k_2) h(n_1 - k_1, n_2 - k_2)$$

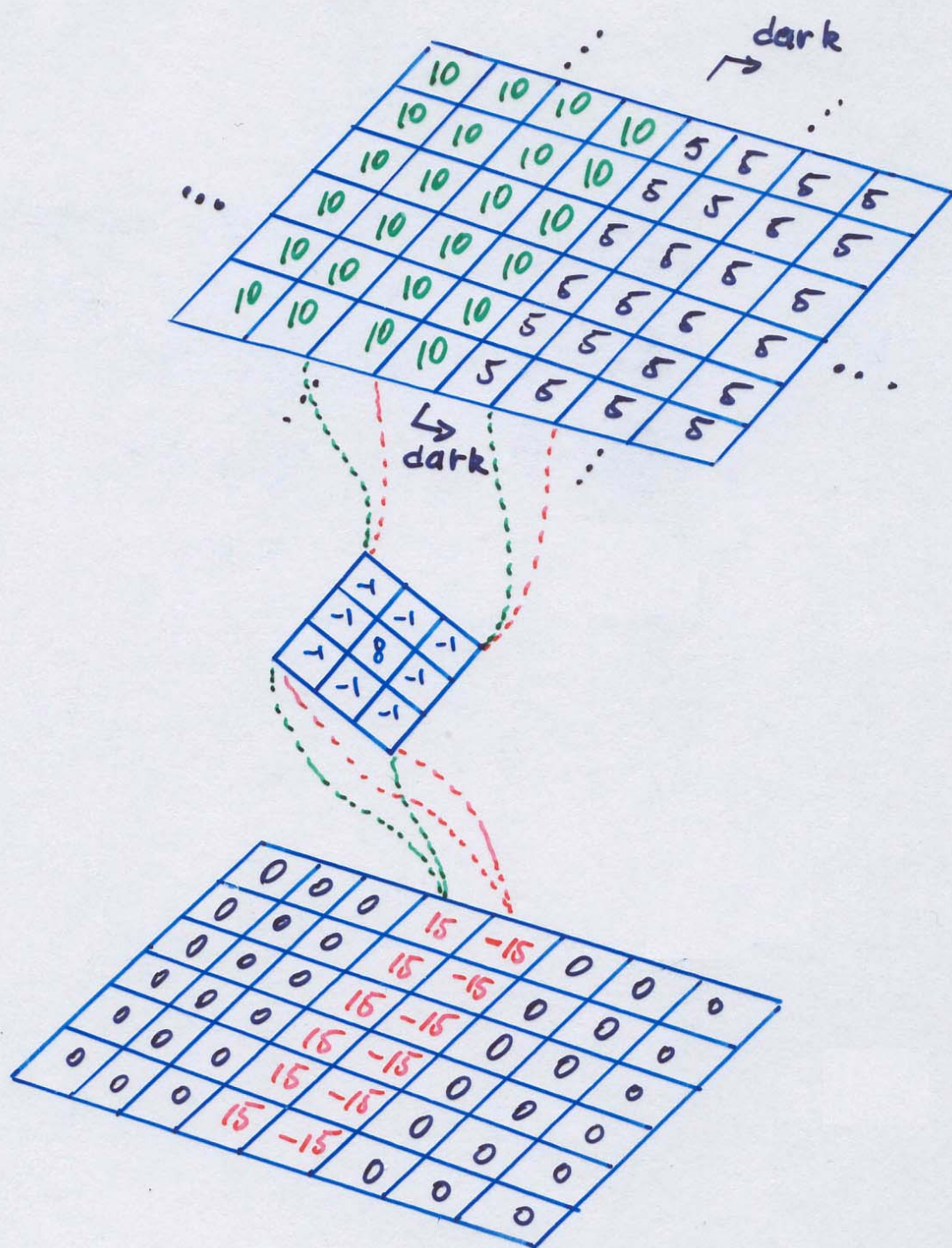


$$n_1 = n_2 = 0$$

$$y(0,0) = \sum_{k_1} \sum_{k_2} (-k_1, -k_2) x(k_1, k_2)$$



$$Y(0,0) = 1 \cdot 2 = 2$$



$$|H(\omega_1, \omega_2)|$$

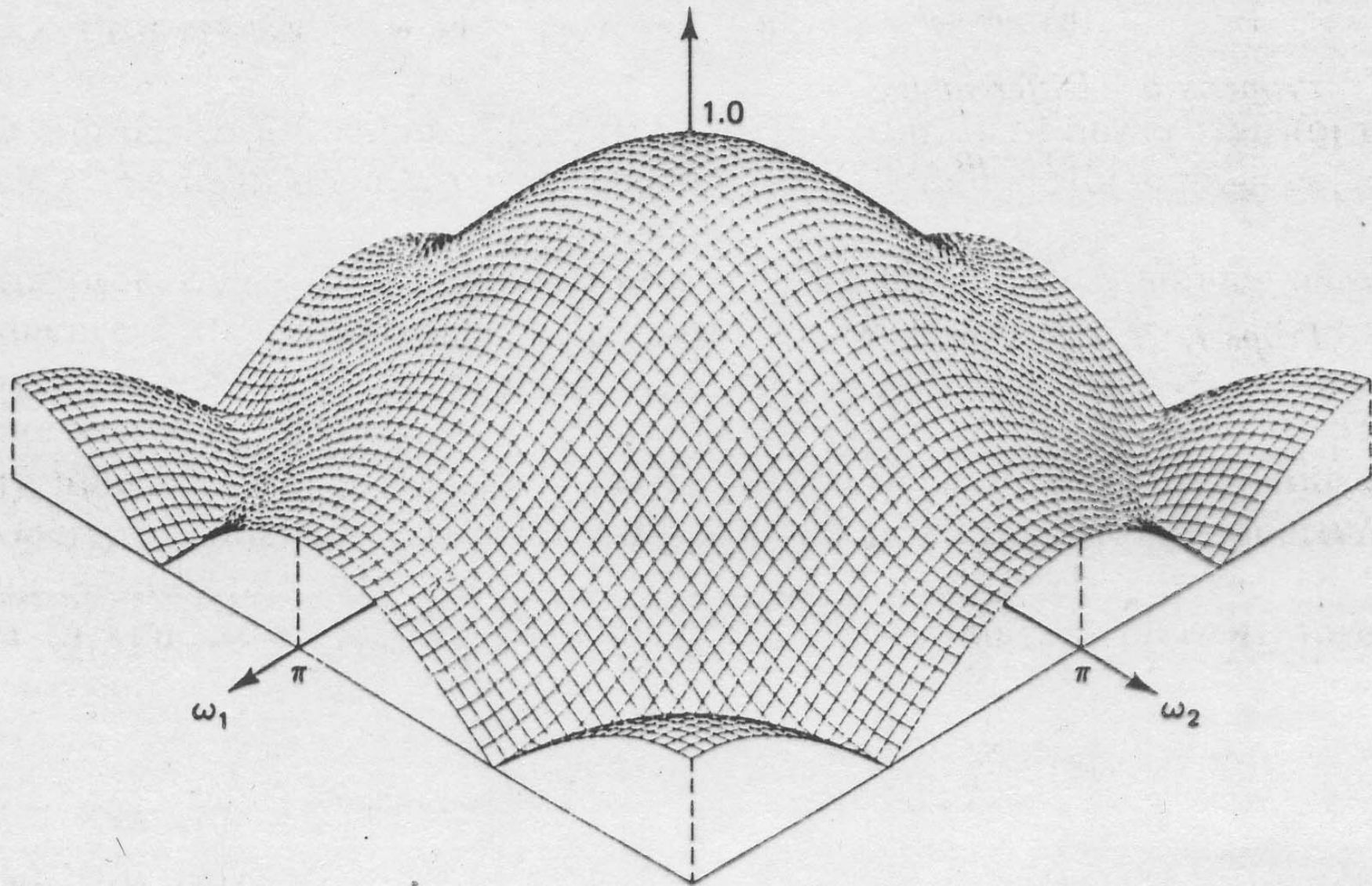
1.0

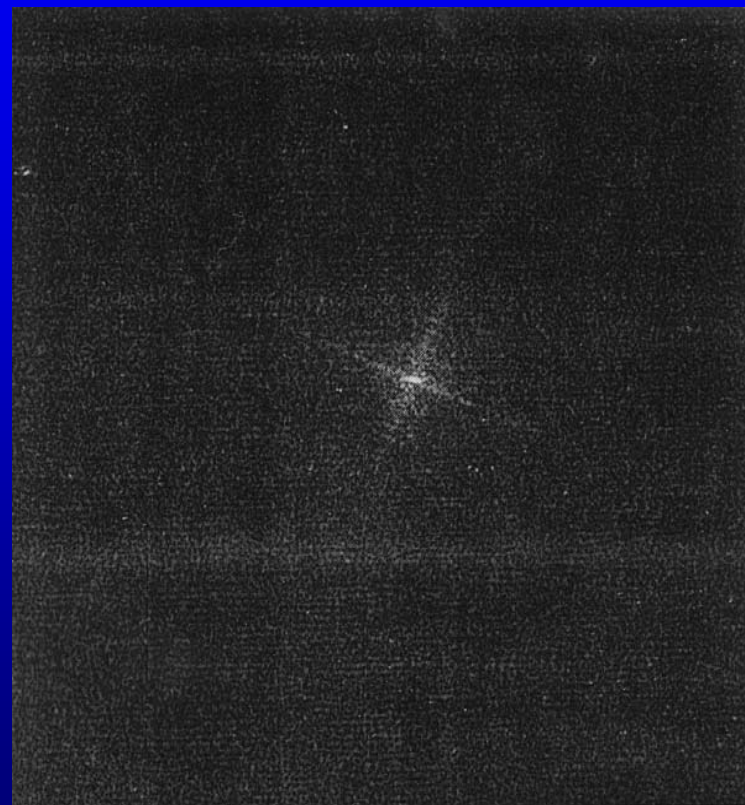
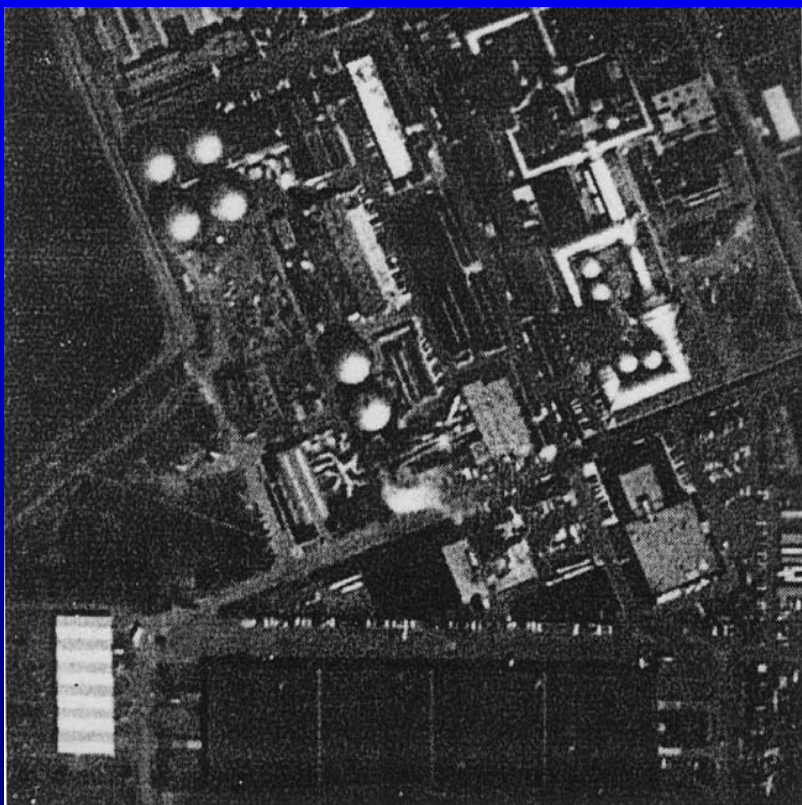
ω_1

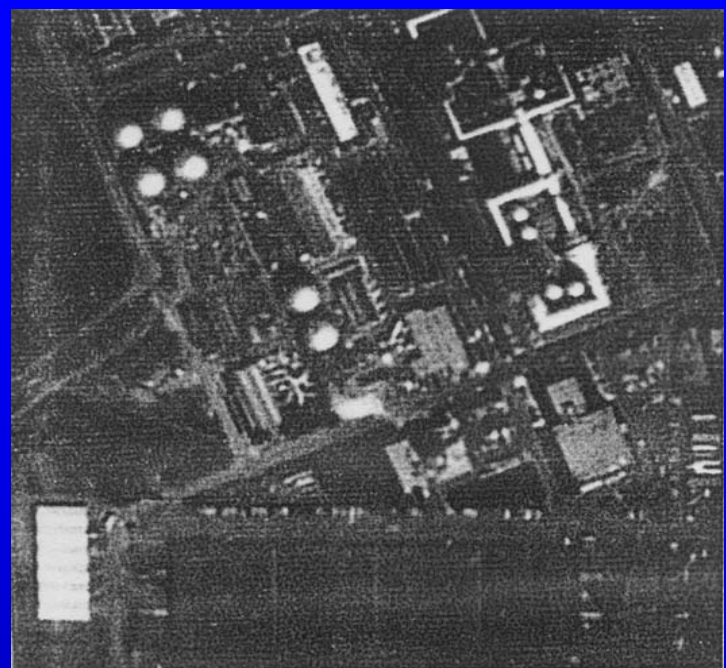
π

π

ω_2

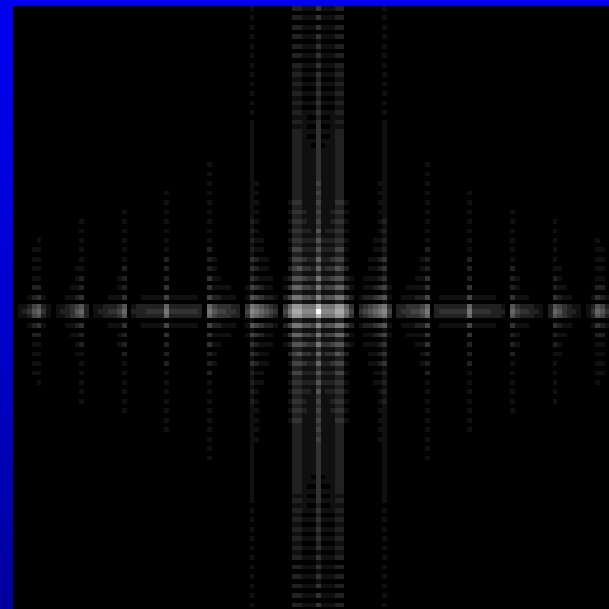








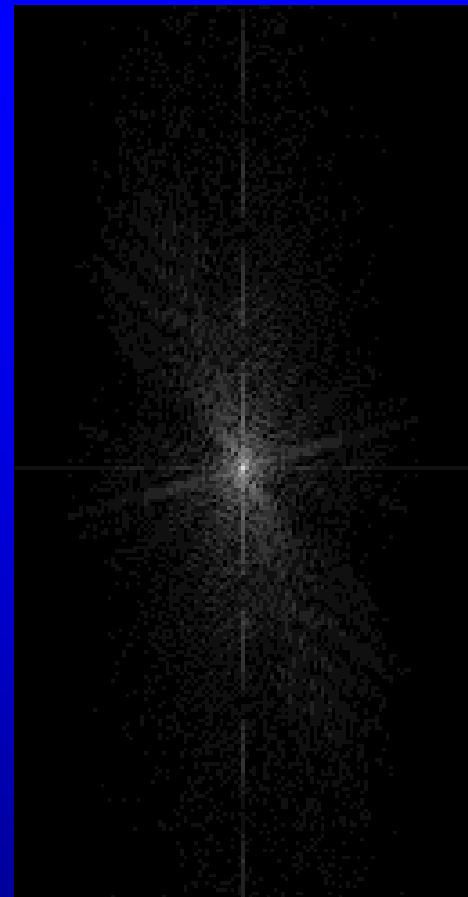
plano (n_1, n_2)



plano (k_1, k_2)



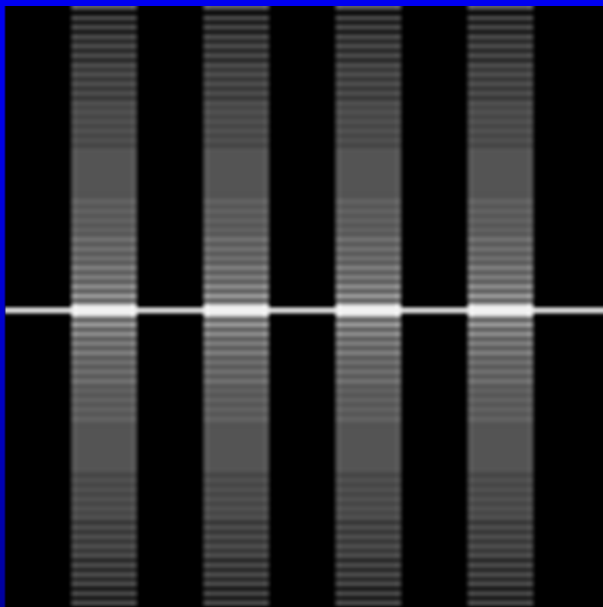
plano (n_1, n_2)



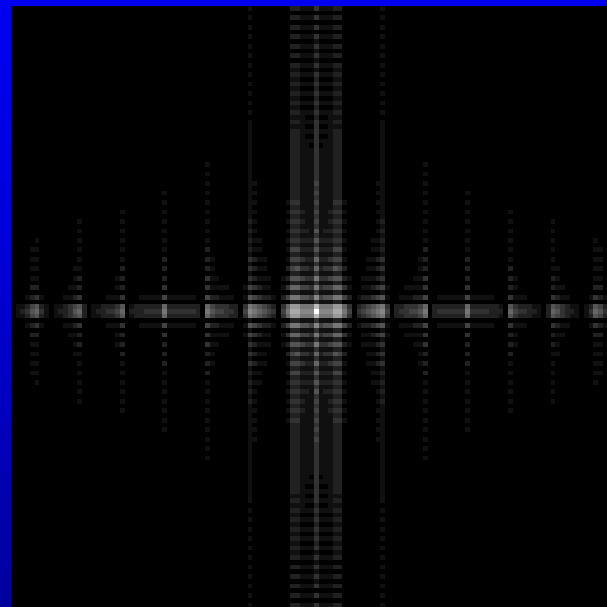
plano (k_1, k_2)



plano (n_1, n_2)



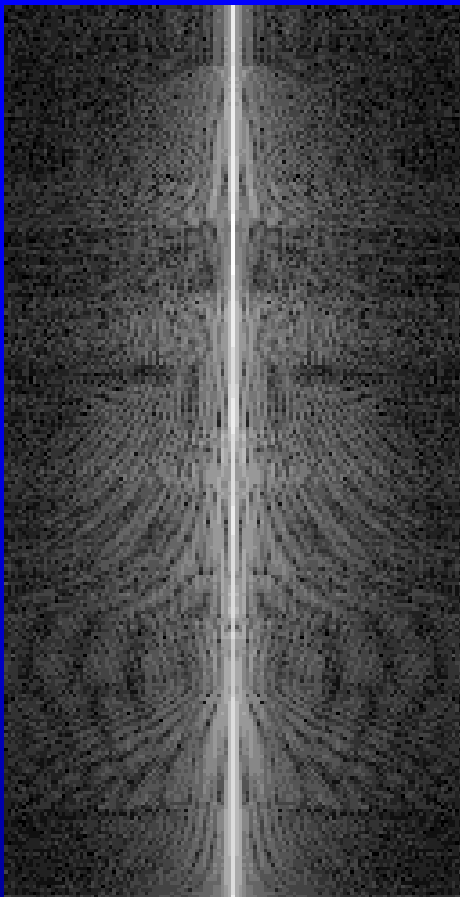
plano (k_1, n_2)



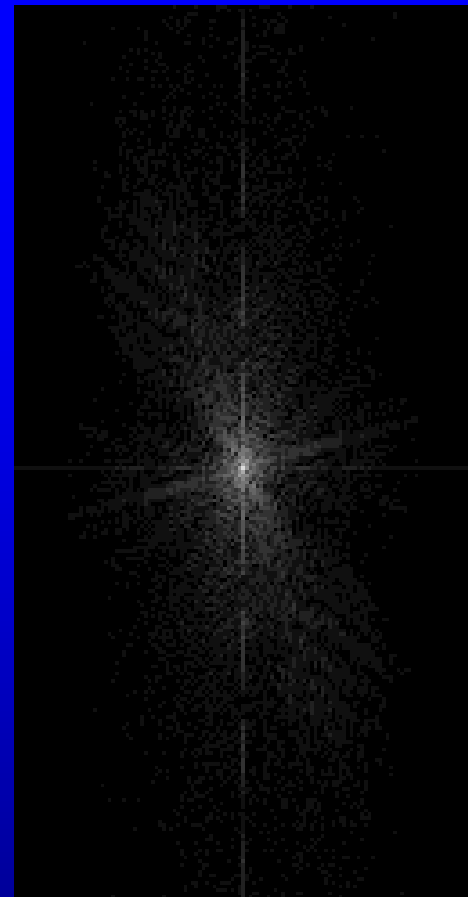
plano (k_1, k_2)



plano (n_1, n_2)

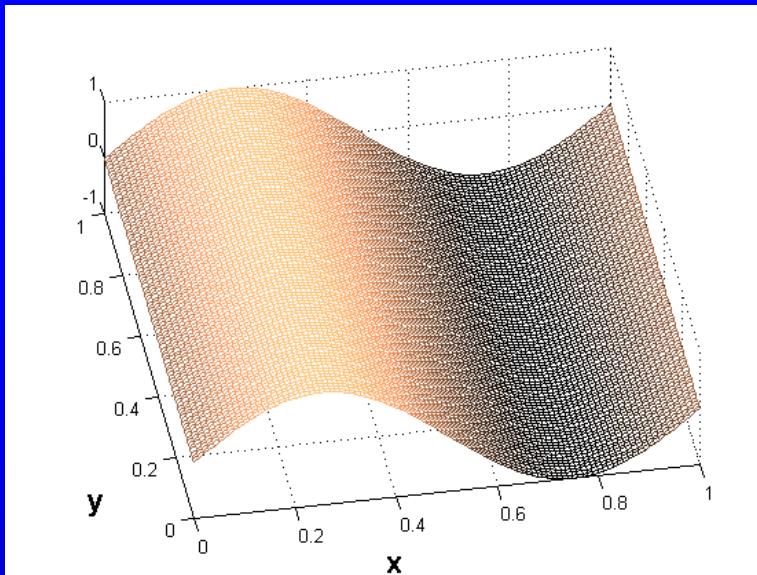


plano (k_1, n_2)

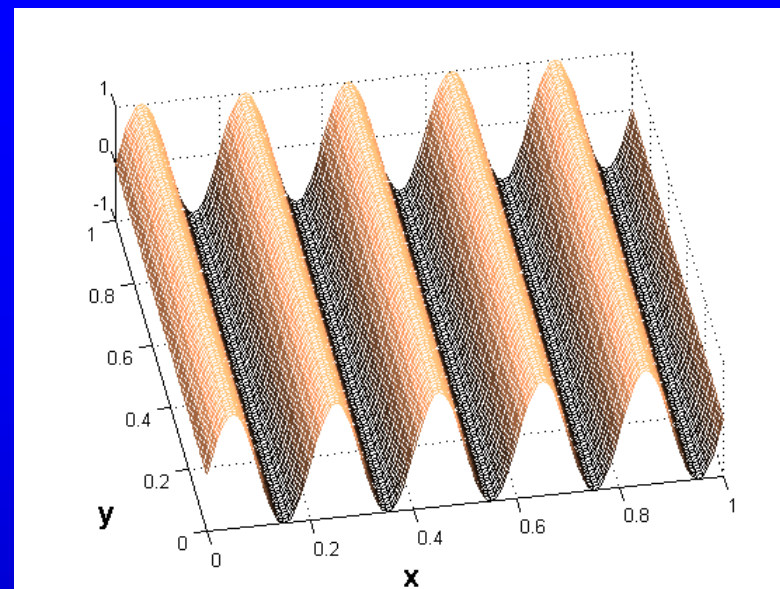


plano (k_1, k_2)

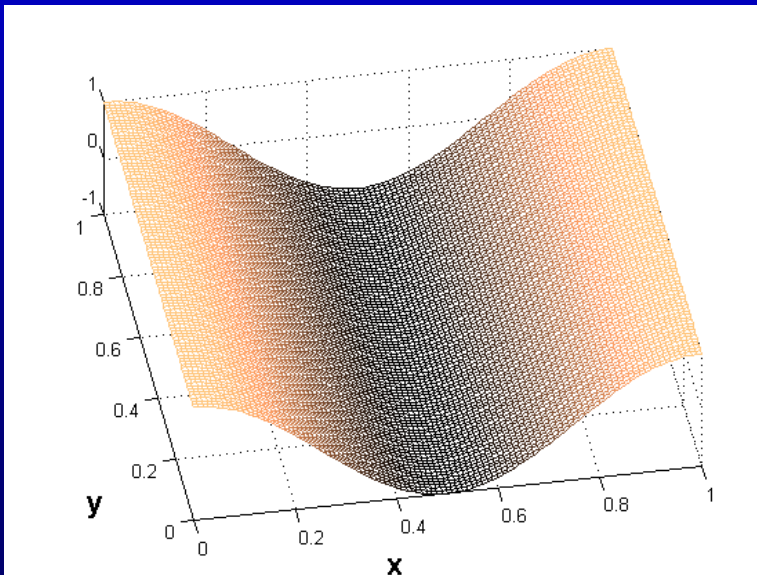
$N_1=N_2$	$(N_1 N_2)^2$ (DFT Directa)	$N_2(N_1)^2+N_1(N_2)^2$ (DFT, Des. Fila-Columna)	$N_1 N_2 \log_2(N_1 N_2)$ (FFT, Des. Fila-Columna)
8	4096	1024	35
32	1048576	65536	928
128	268435456	4194304	20786
256	4294967296	33554432	95021
512	68719476740	268435456	427595



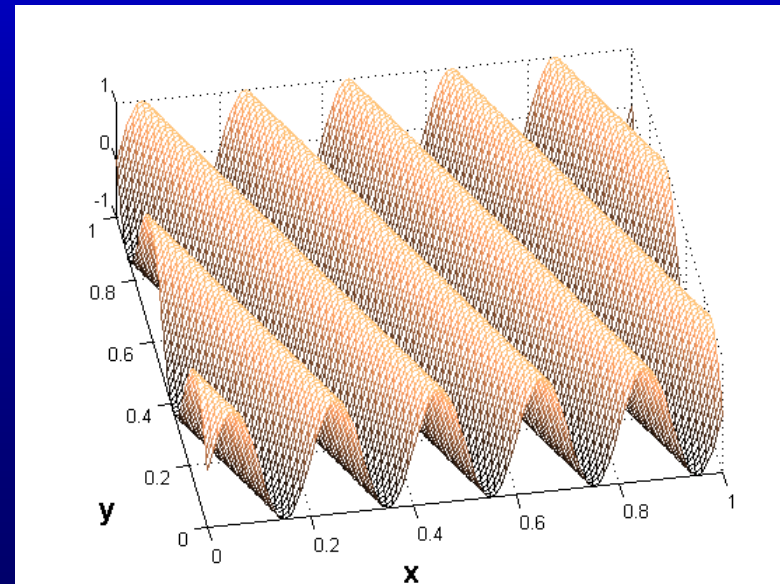
$\text{sen}(2\pi x)$



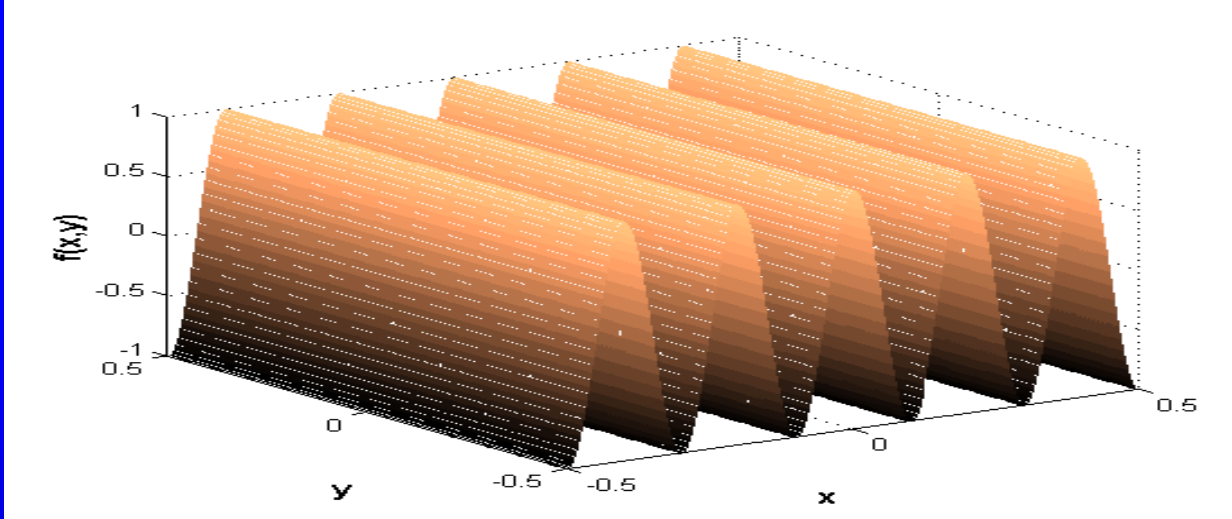
$\text{sen}(10\pi x)$



$\cos(2\pi x)$



$\text{sen}(10\pi x + 4\pi y)$



$$f(x,y) = \cos(10\pi x)$$

Fourier Transform

$$F(u,v) = 1/2 [\delta(u+5,0) + \delta(u-5,0)]$$

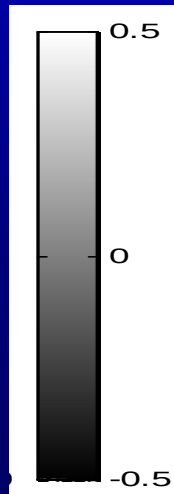
Real [F(u,v)]

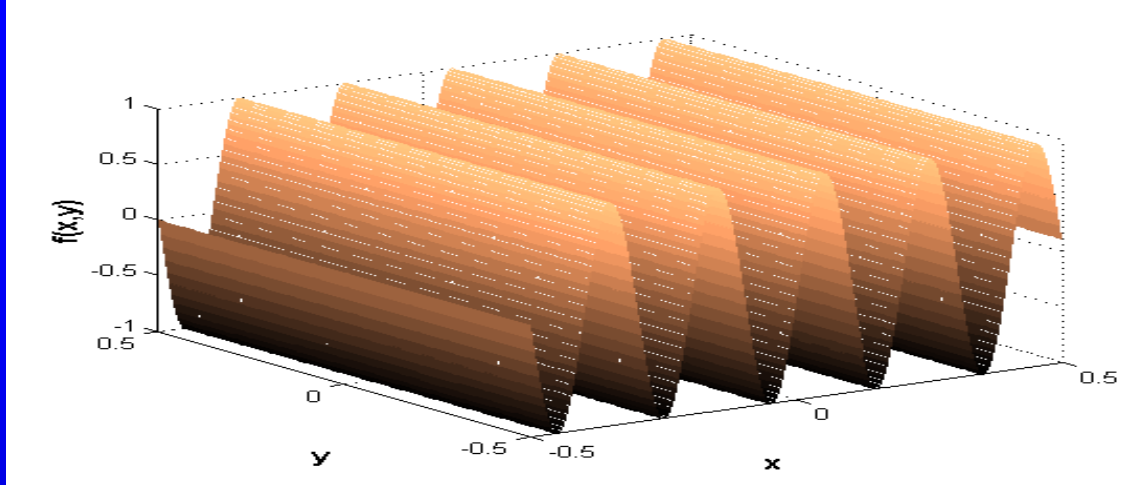
↑
v



Imaginary [F(u,v)]

↑
v





$$f(x,y) = \sin(10\pi x)$$

Fourier Transform

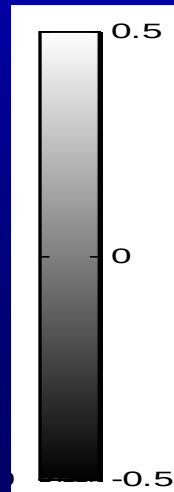
$$F(u,v) = i/2 [\delta(u+5,0) - \delta(u-5,0)]$$

Real $[F(u,v)]$

↑
v

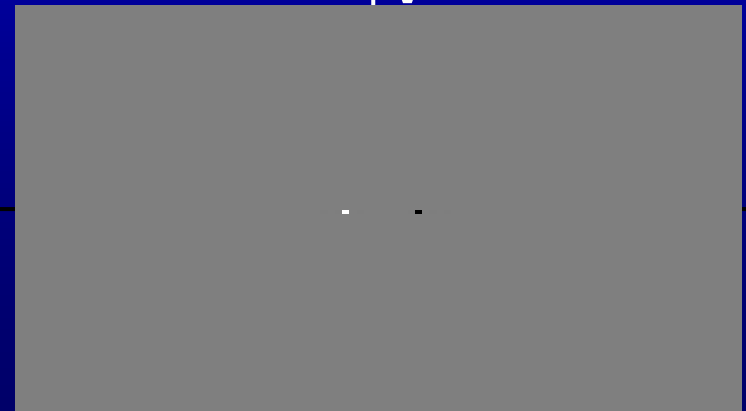


→
u

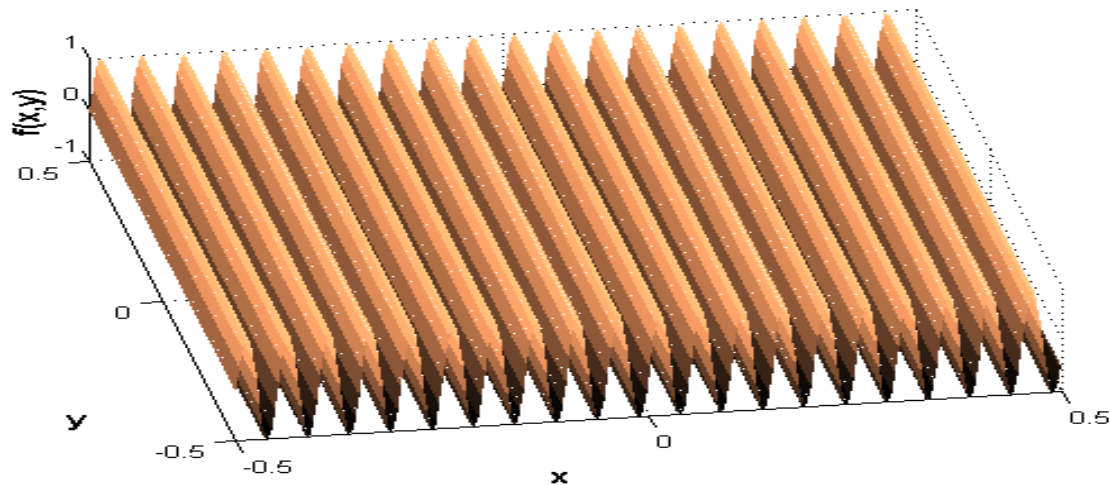


Imaginary $[F(u,v)]$

↑
v



→
u



$$f(x,y) = \sin(40\pi x)$$

Fourier Transform

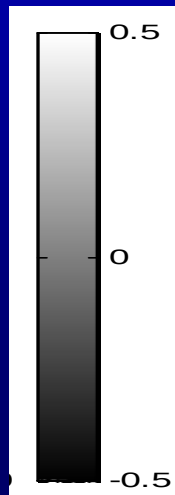
$$F(u,v) = i/2 [\delta(u+20,0) - \delta(u-20,0)]$$

Real $[F(u,v)]$

v



u



Imaginary $[F(u,v)]$

v



u



(a)



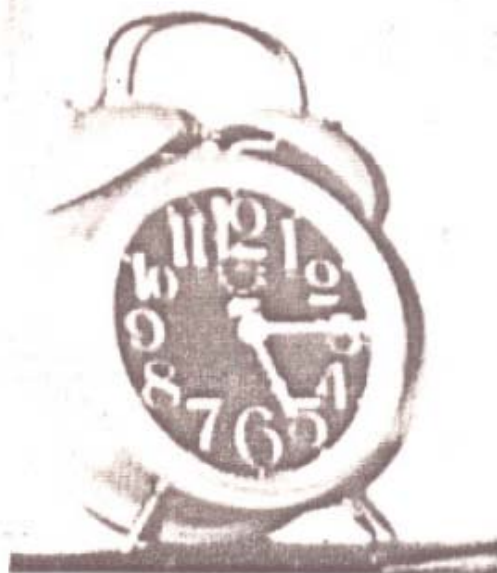


(a)



(b)



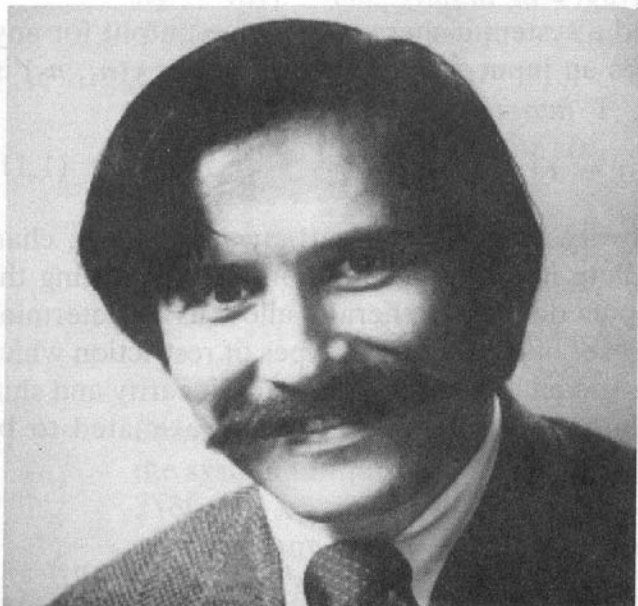


(a)

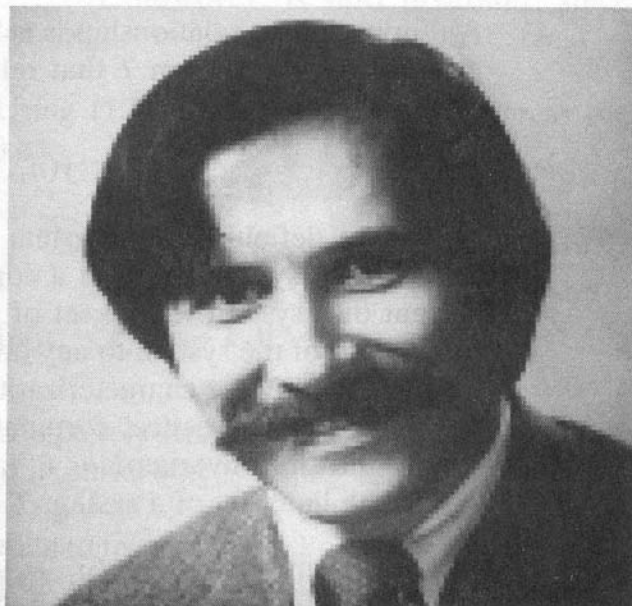


(b)

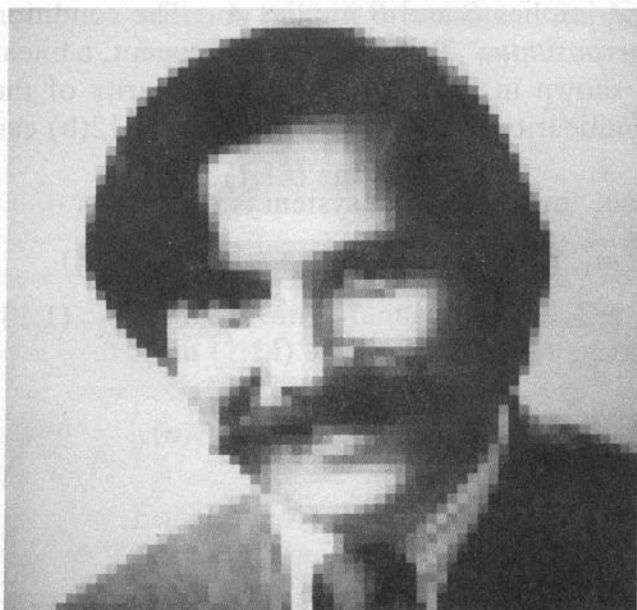




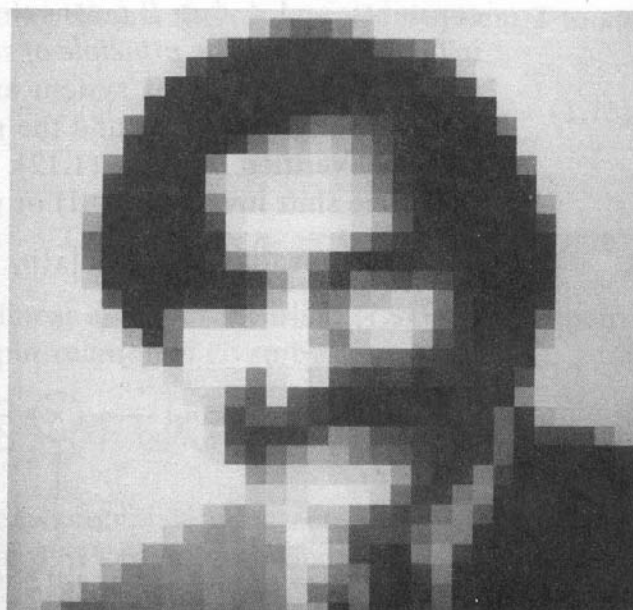
(a)



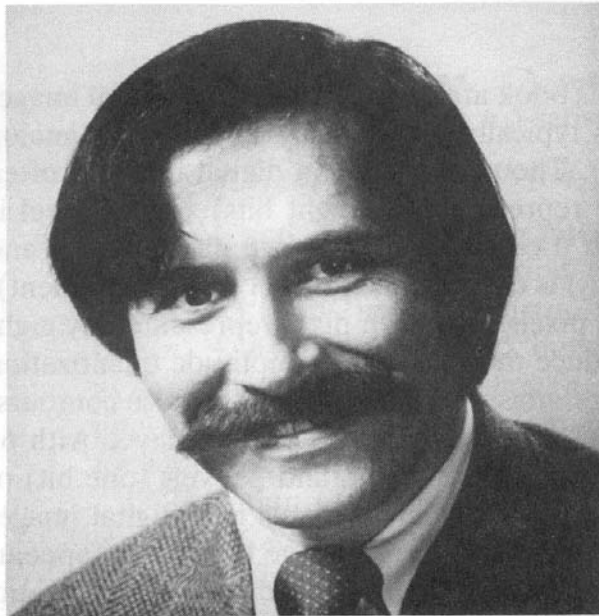
(b)



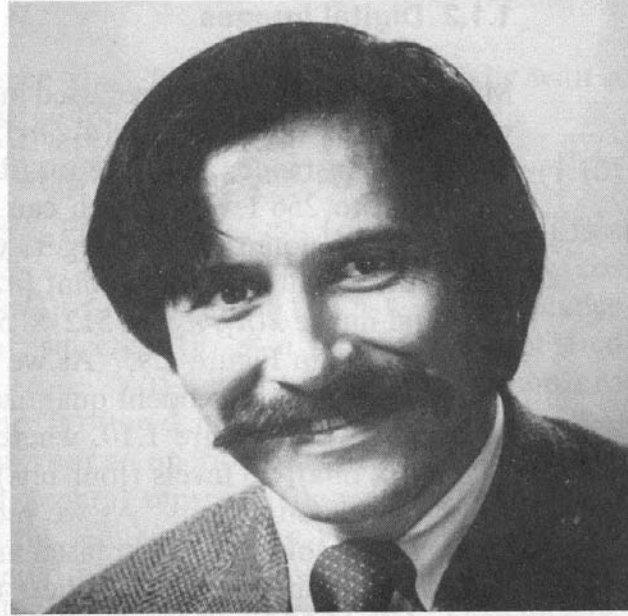
(c)



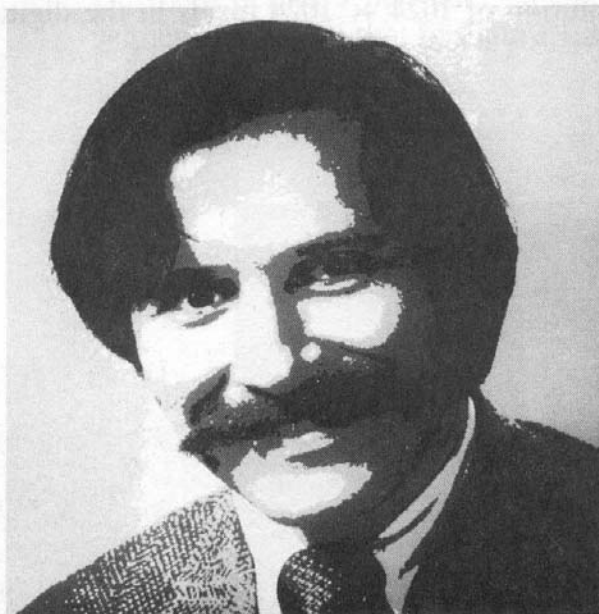
(d)



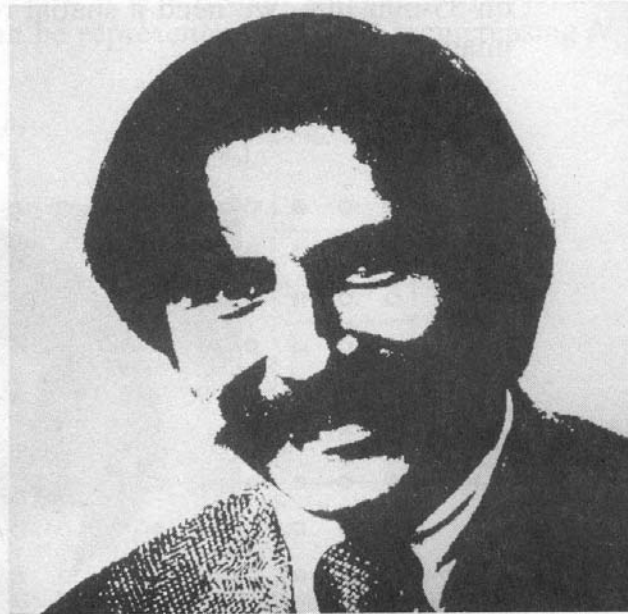
(a)



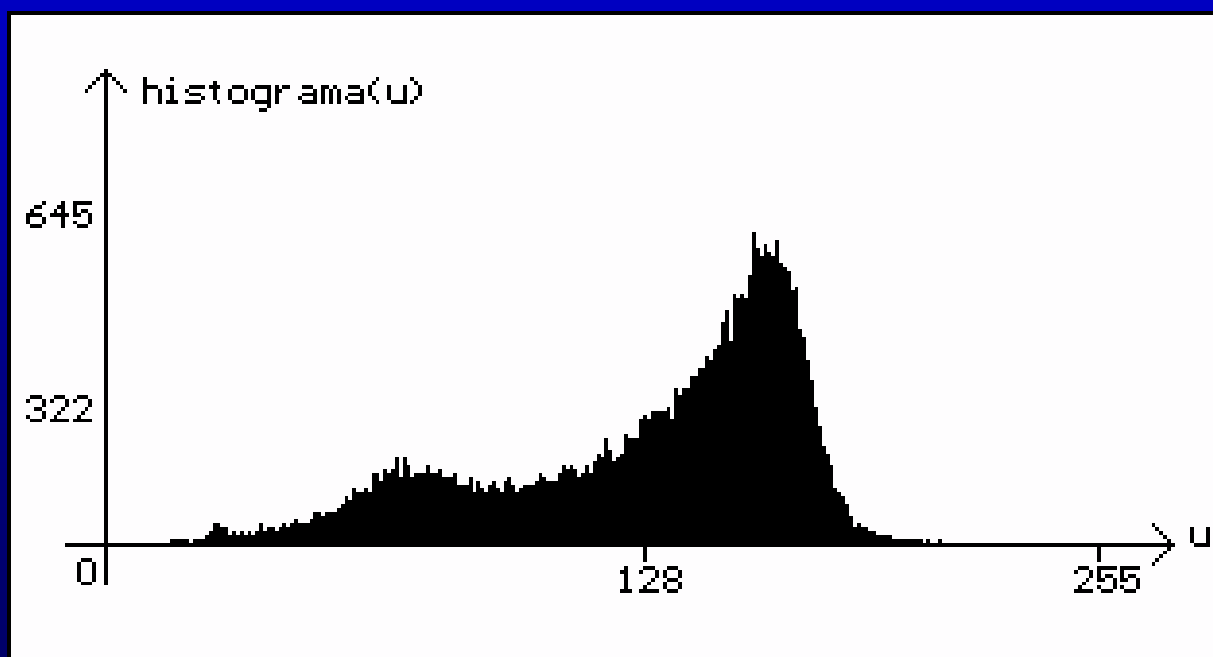
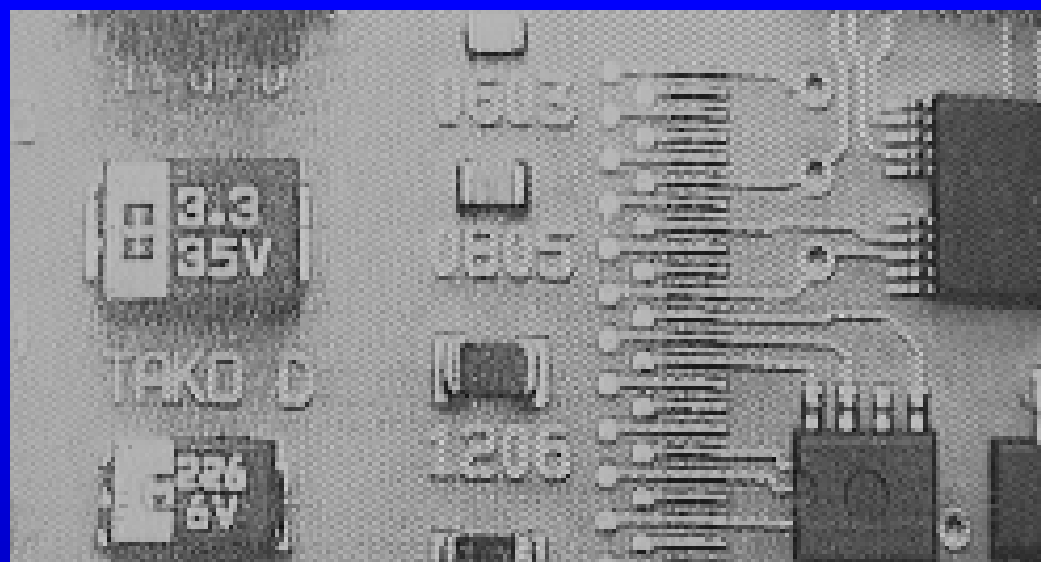
(b)



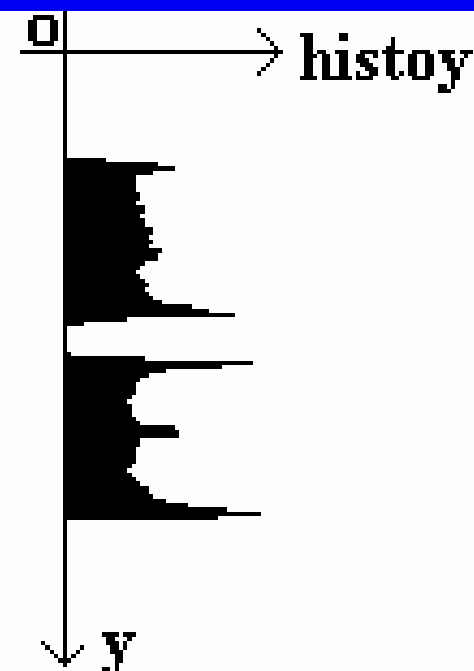
(c)



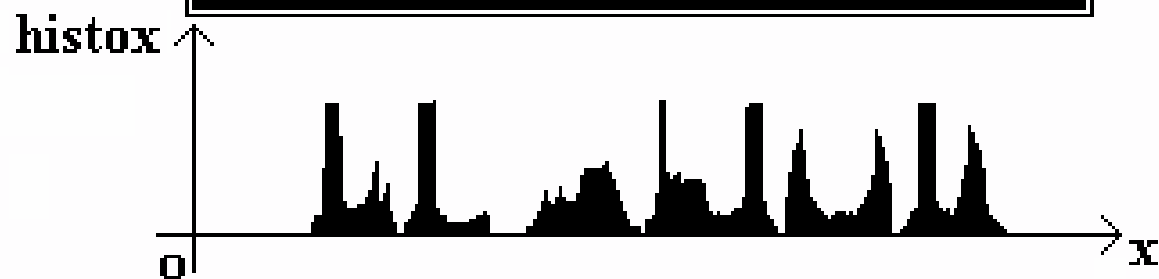
(d)

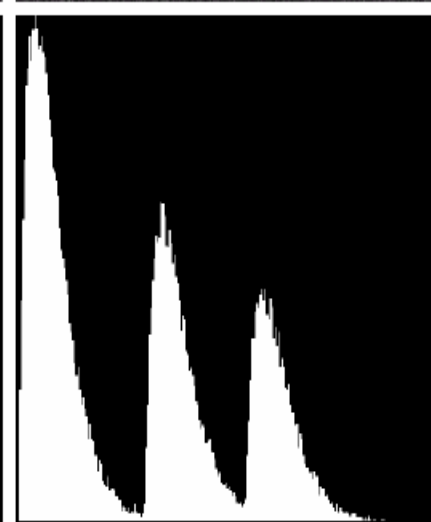
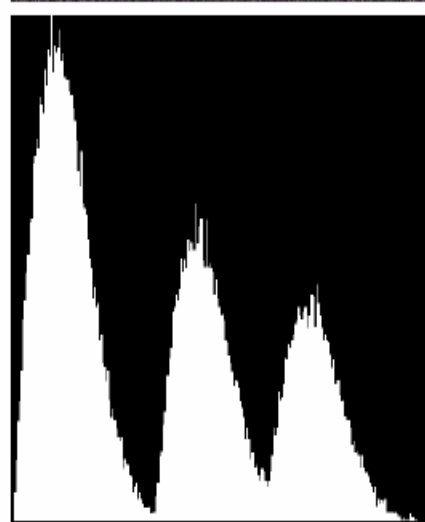
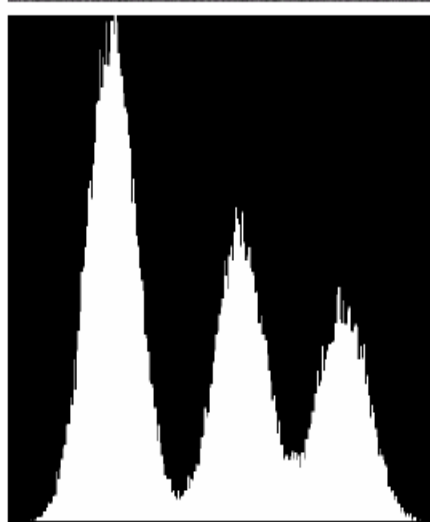
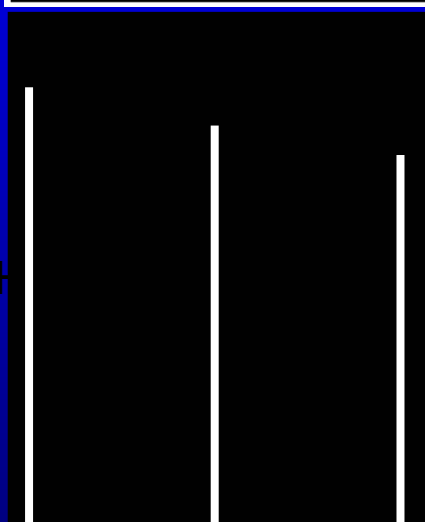
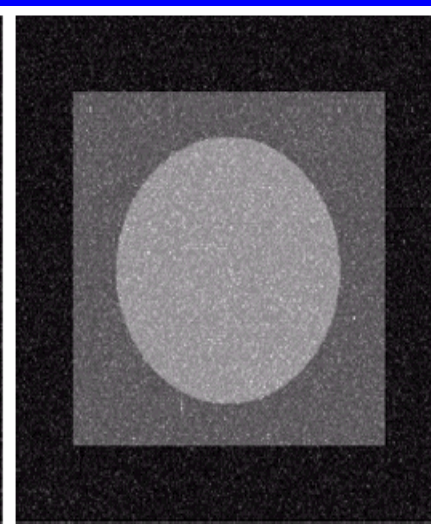
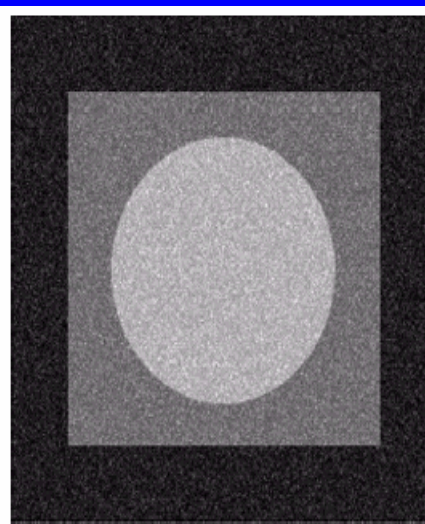
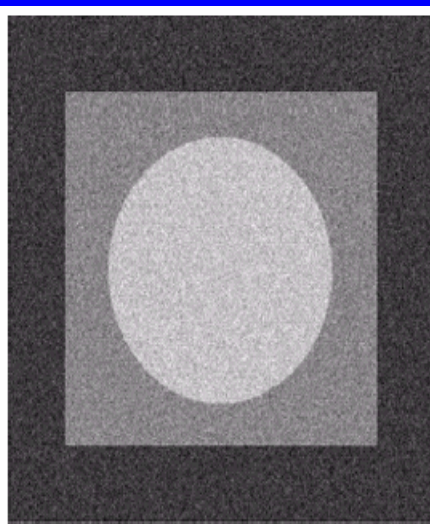
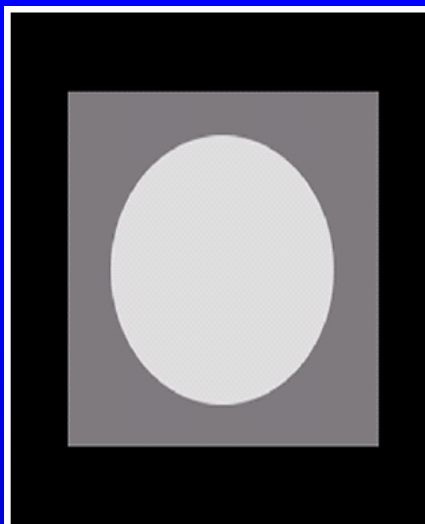


EL AMOR DE CHILE



EL AMOR





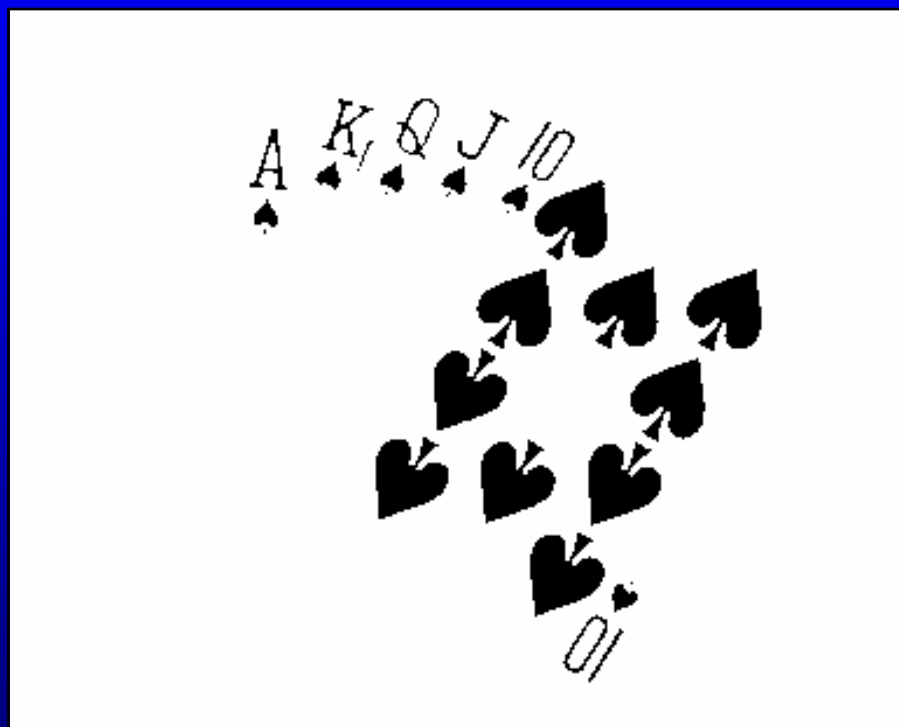
Gaussian

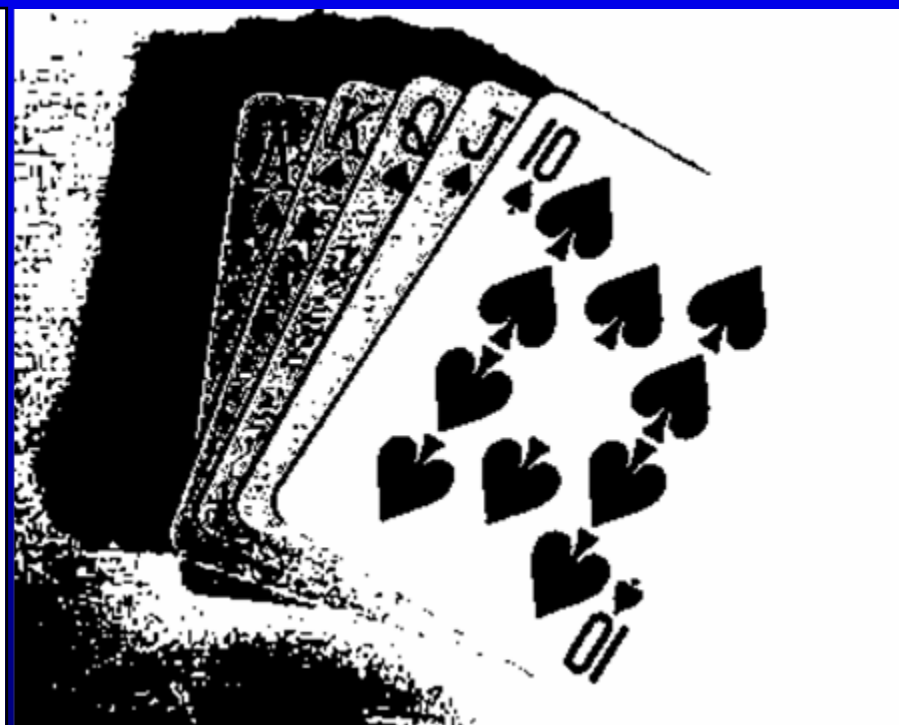
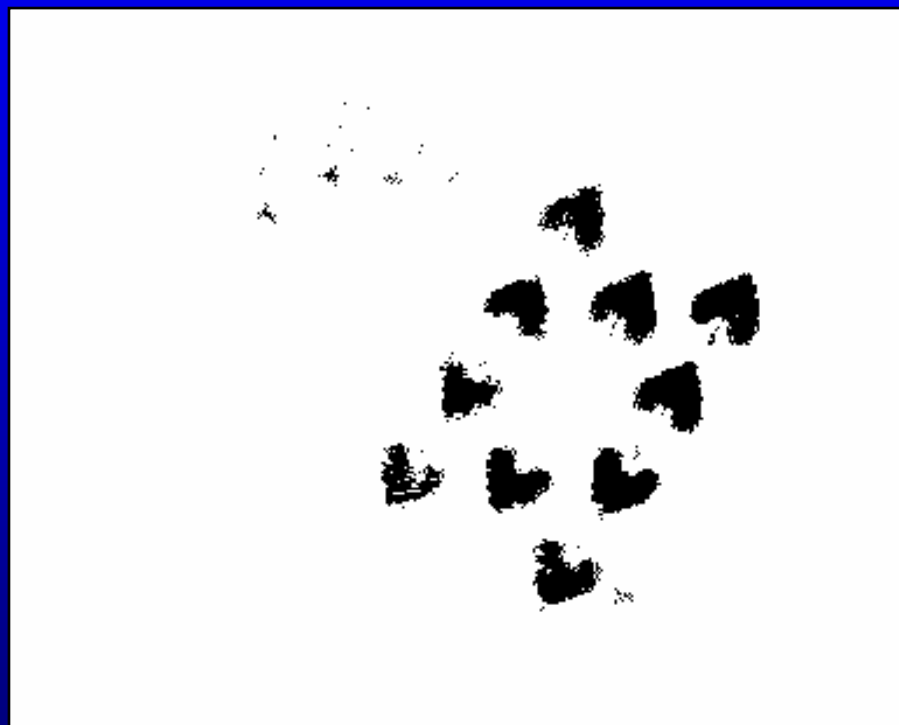
Rayleigh

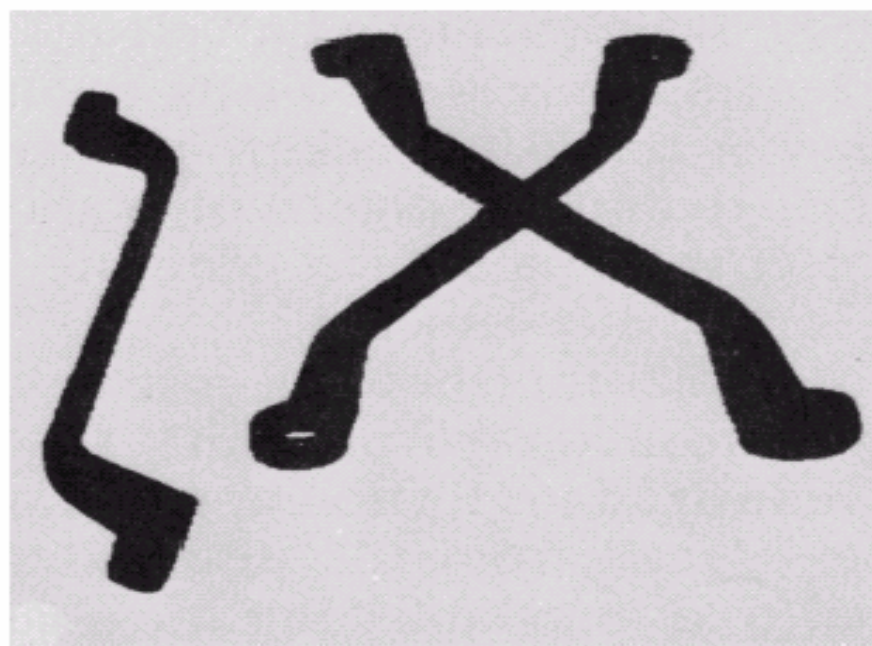
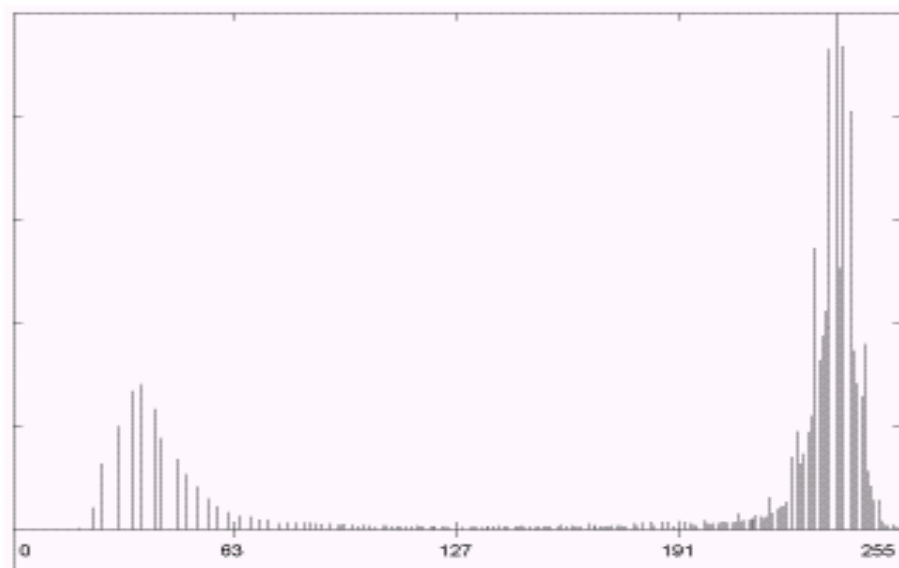
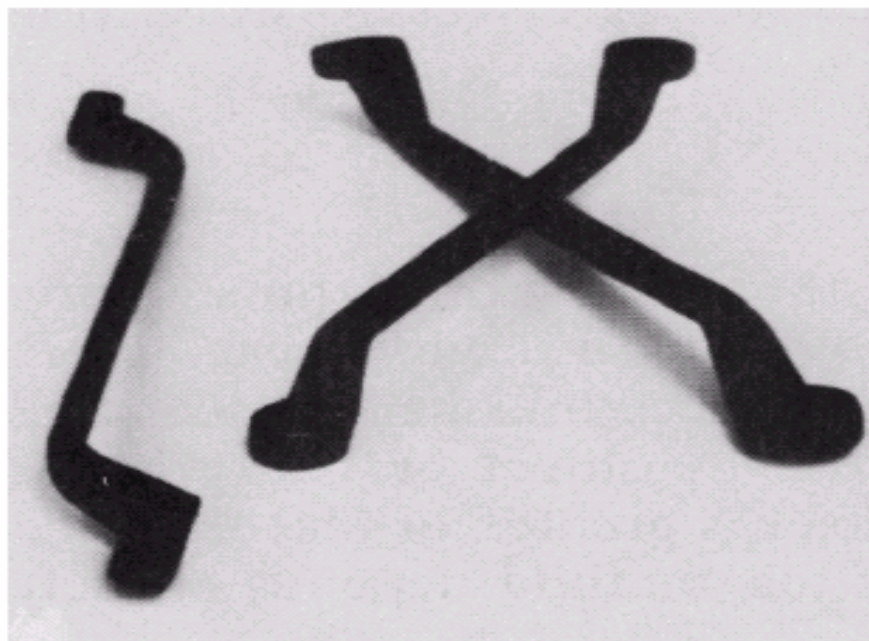
Gamma

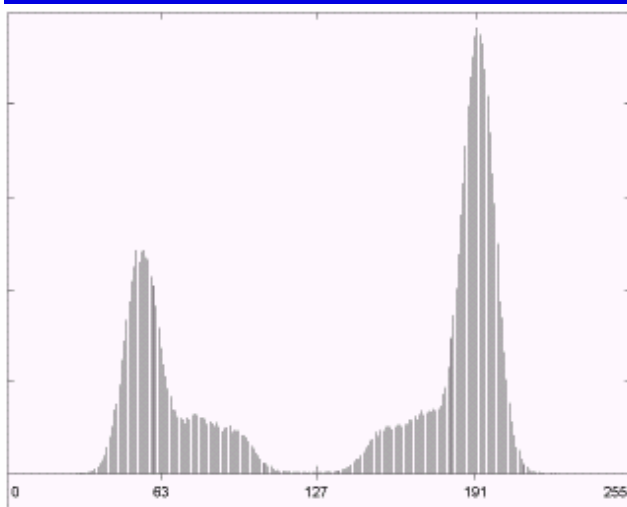
Uso de Umbrales

$$g(x, y) = \begin{cases} 1 & \text{if } f(x, y) > T \\ 0 & \text{if } f(x, y) \leq T \end{cases}$$

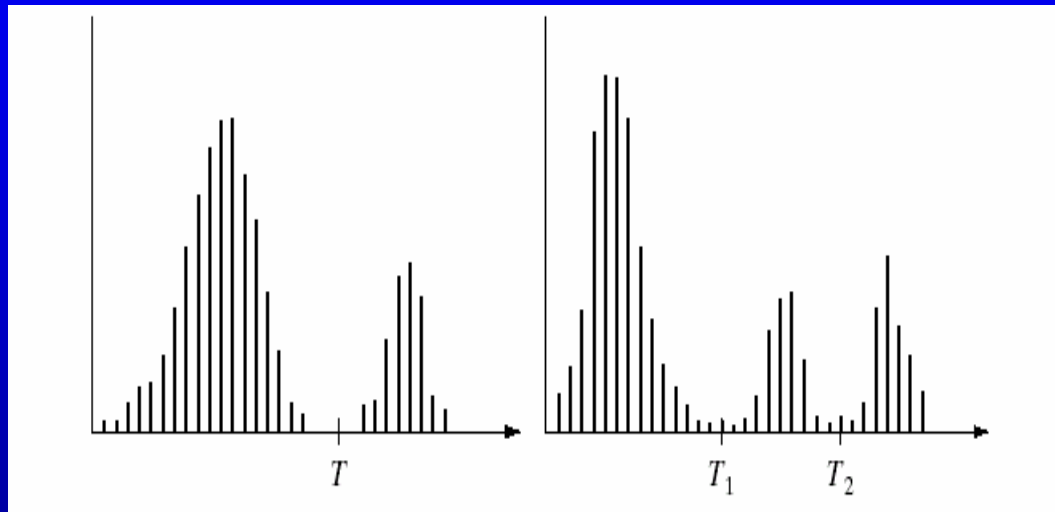




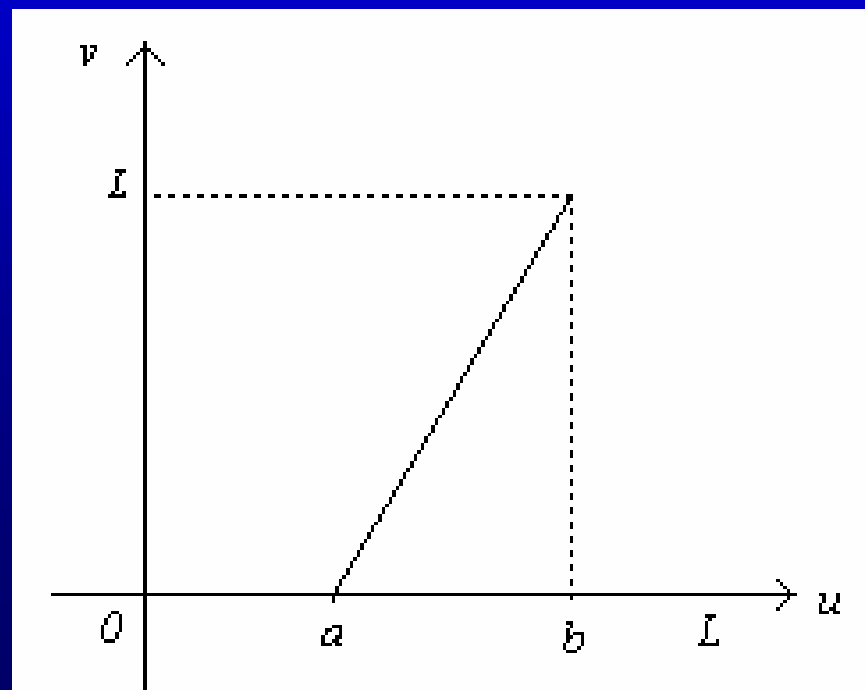
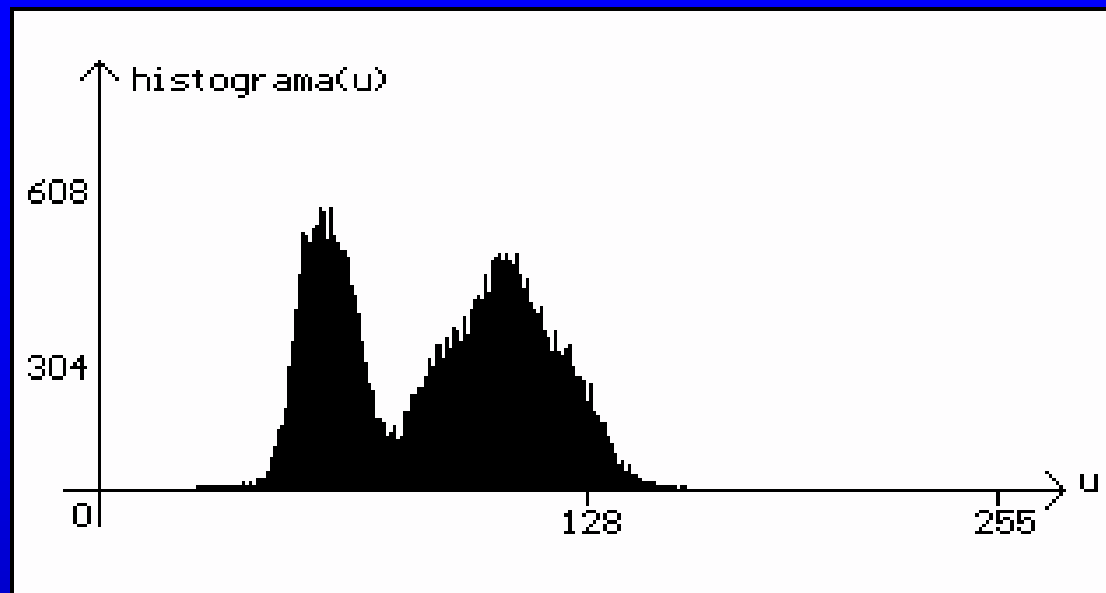


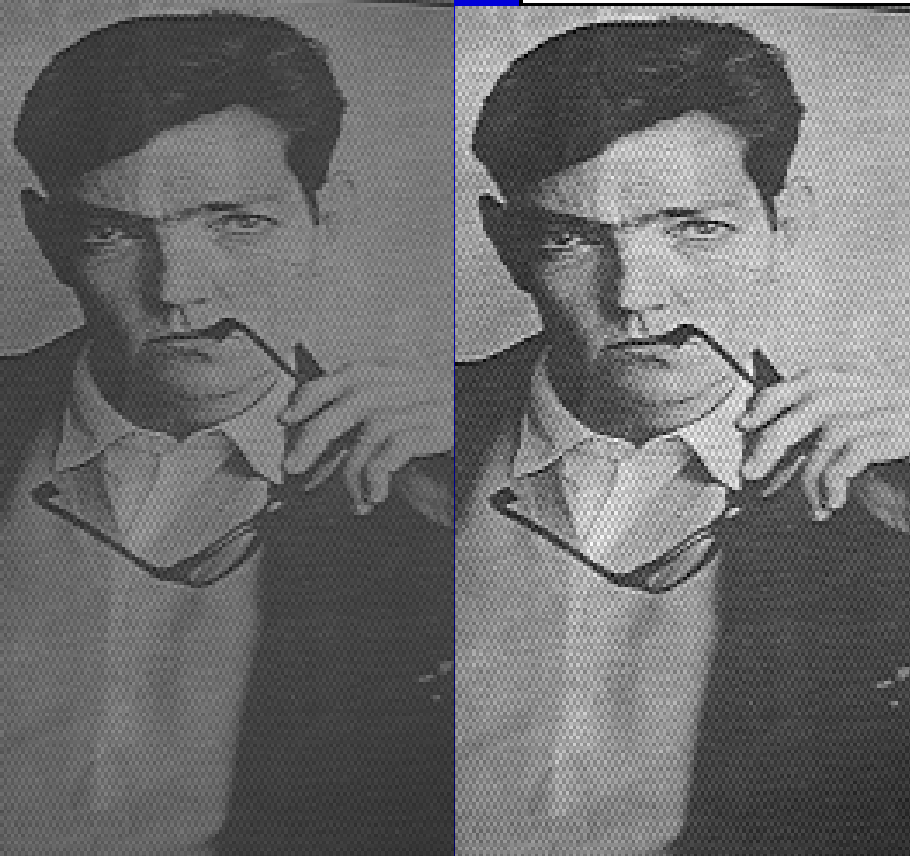
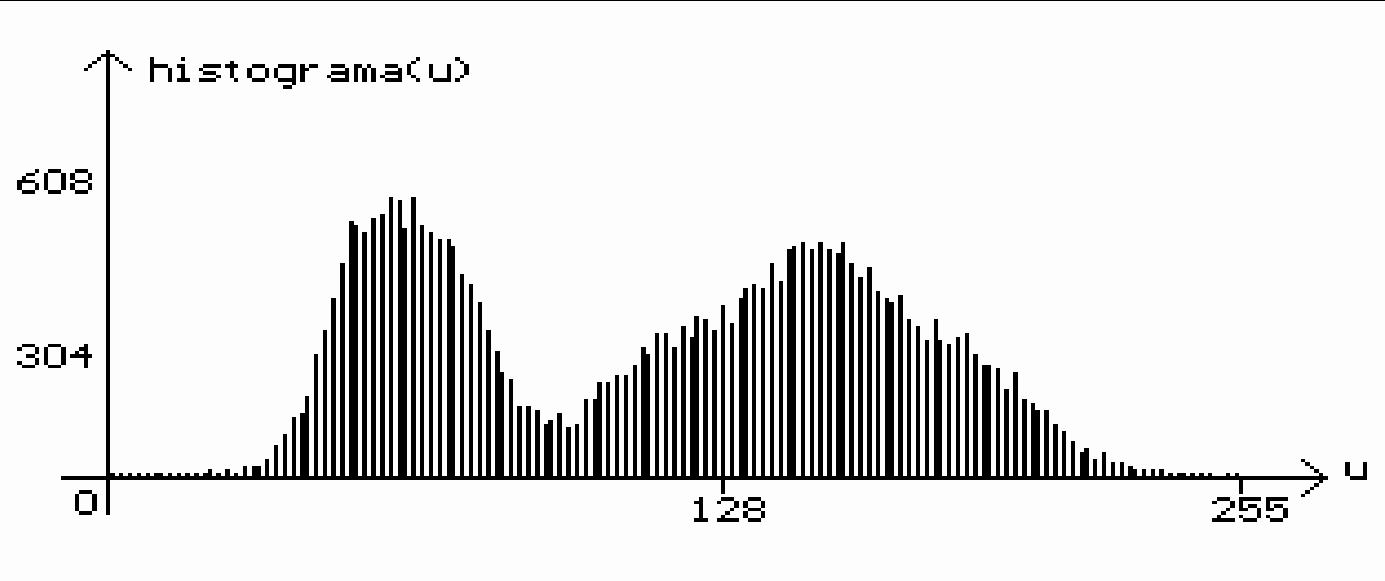


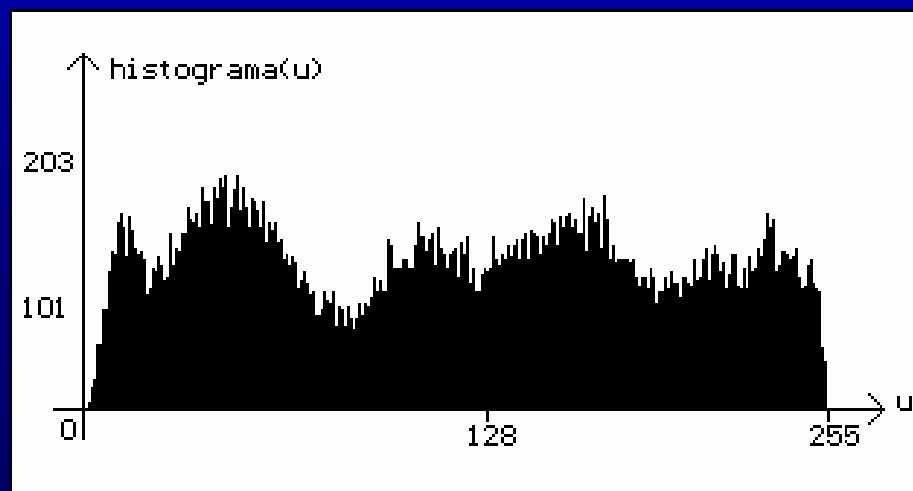
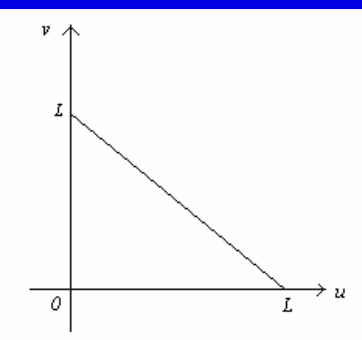
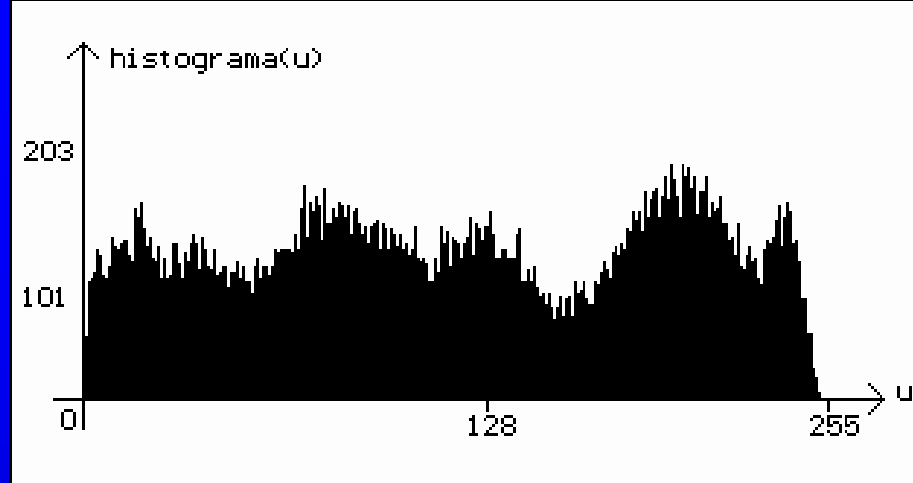




Varios umbrales

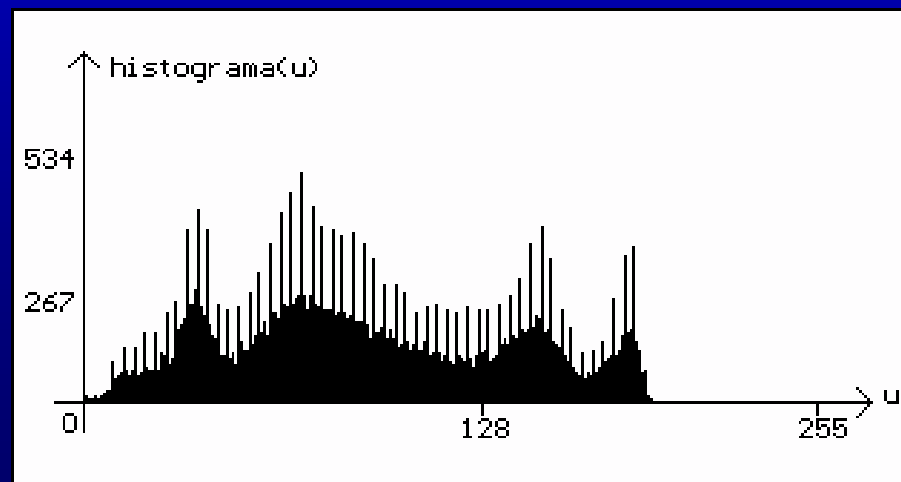
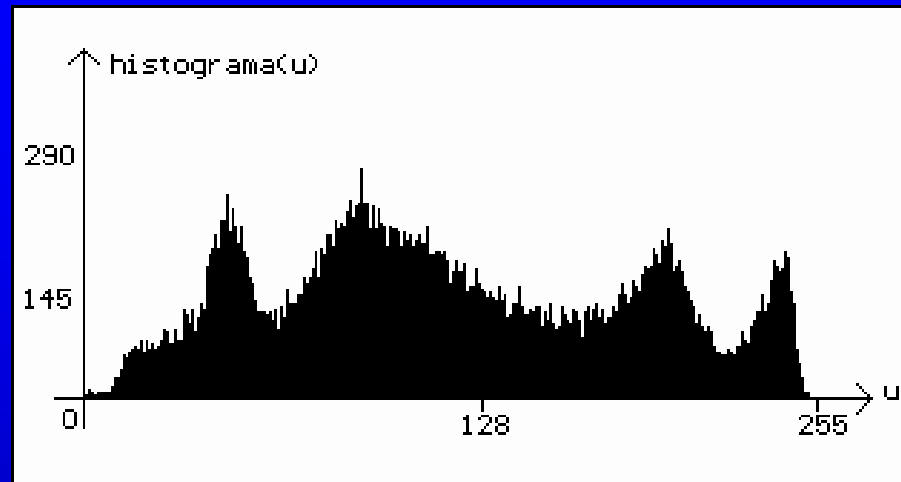




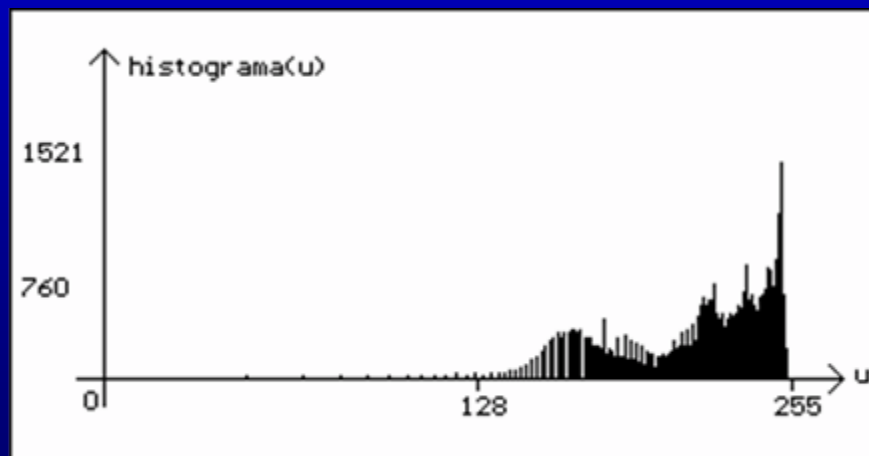
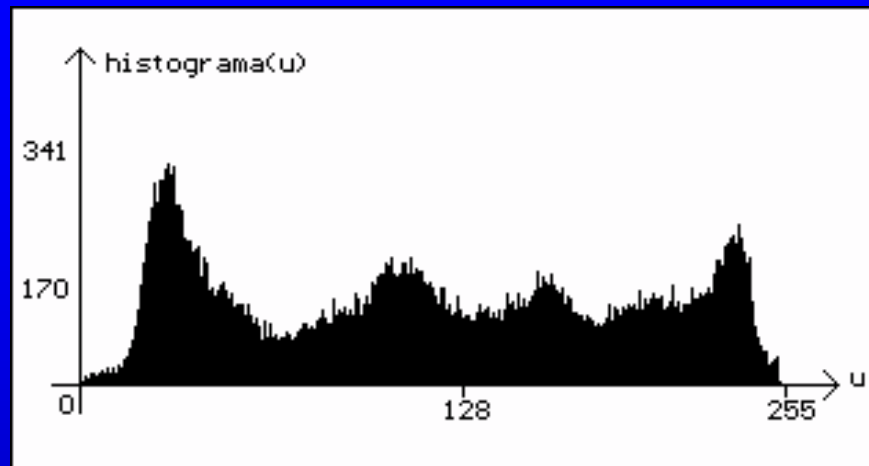




Compresión lineal de rango



Compresión logarítmica de rango

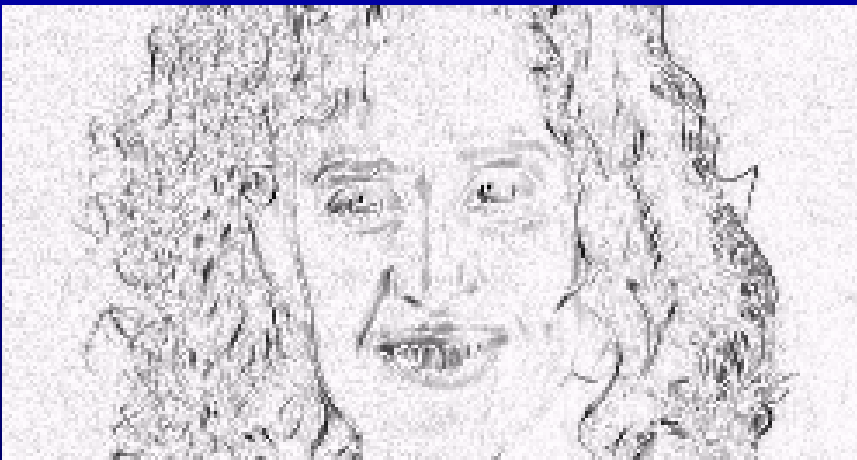


- Procesamiento de múltiples imágenes
 - operaciones lógicas
 - matemáticas $+ - * /$ (razón espectral i.r.)
 - reducción de ruido
- Composición de Imágenes
 - imágenes no relacionadas

Substracción de Imágenes



- Detección de fallas – líneas de producción
- Monitoreo de seguridad
- Substracción de fondo – vasos sanguíneos
- Detección de movimientos
- Registro de imágenes
- Procesamiento de múltiples imágenes
 - operaciones lógicas
 - matemáticas $+-*/$
 - reducción de ruido



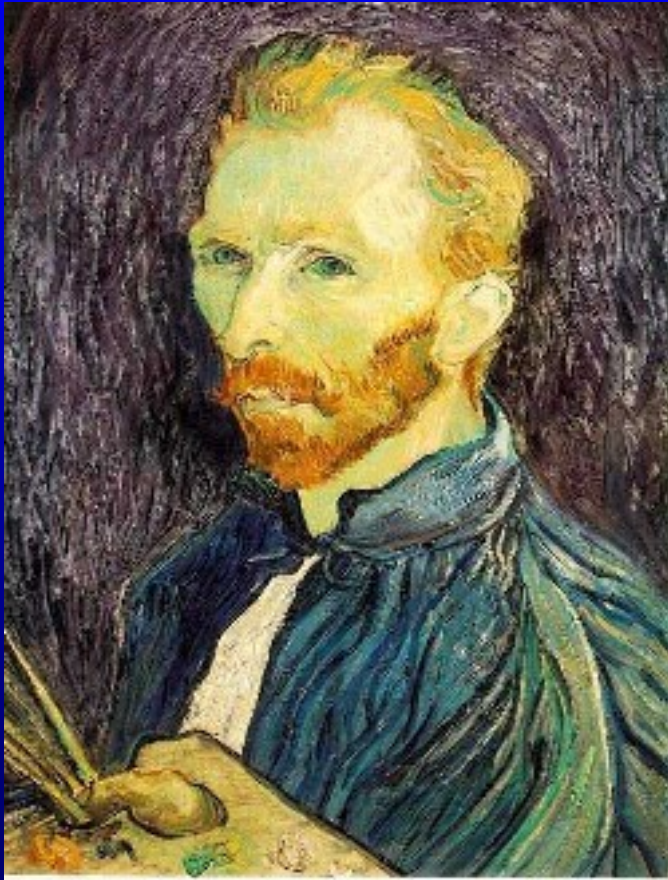
Magnificación por replicación



Magnificación por interpolación lineal



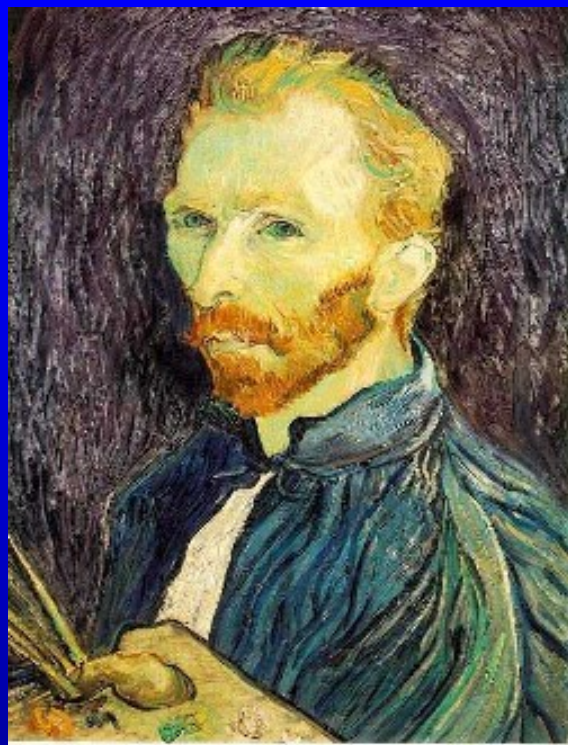
Submuestreo



$1/4$



$1/8$



$1/2$



$1/4$ (2x zoom)

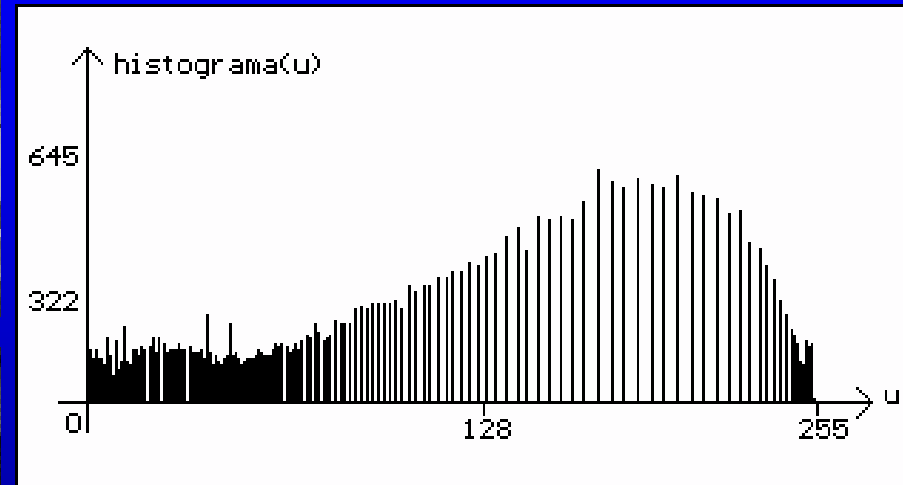
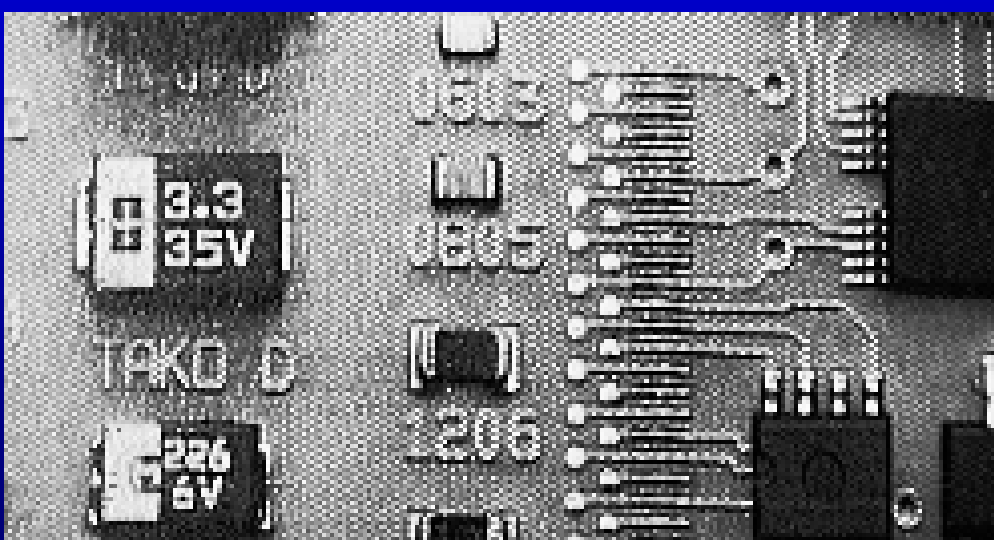
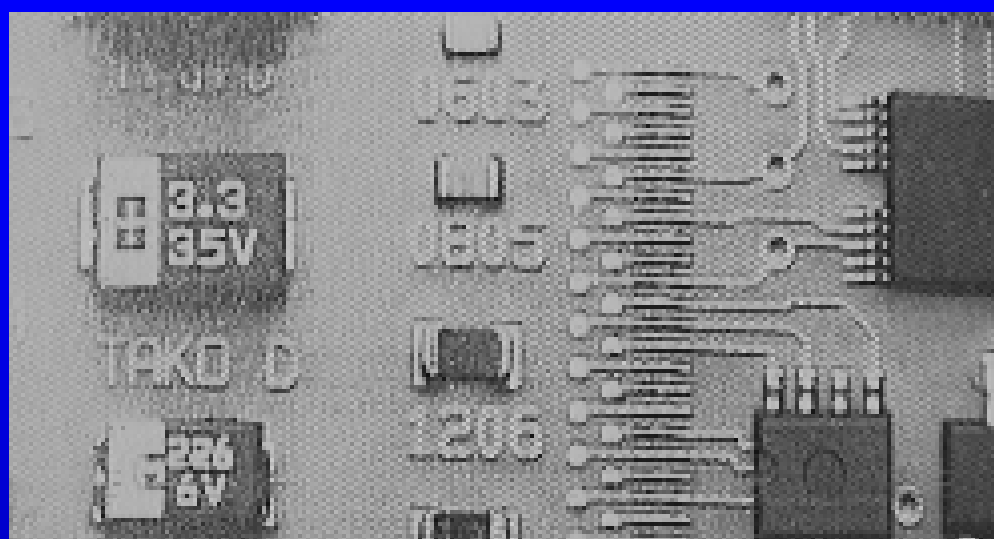


$1/8$ (4x zoom)

Medios-tonos



Ecualización de histogramas



Operaciones Espaciales: Promediación

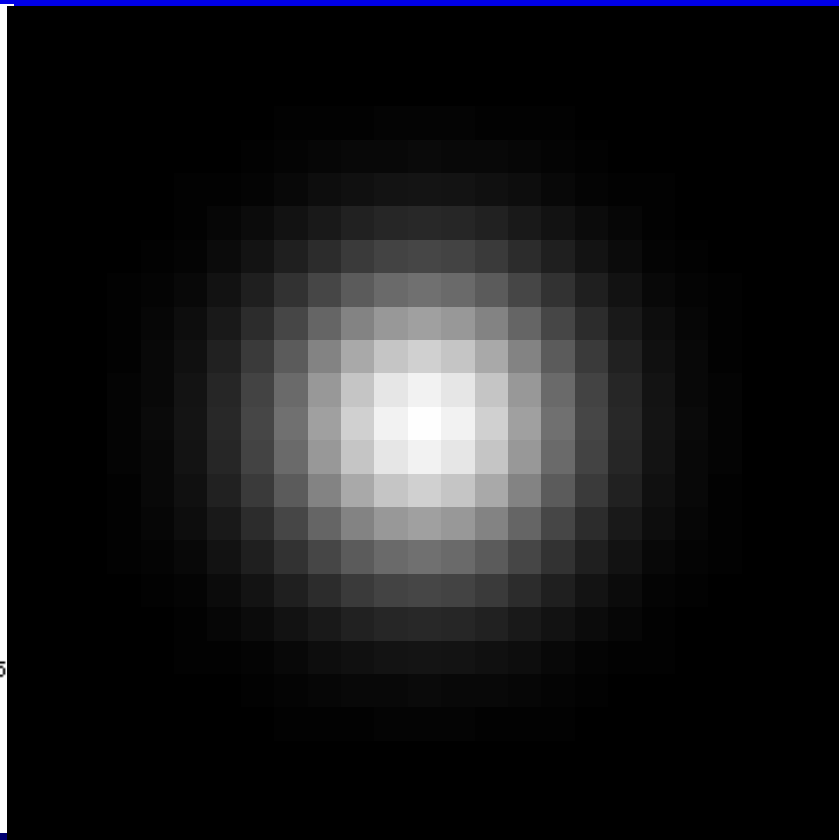
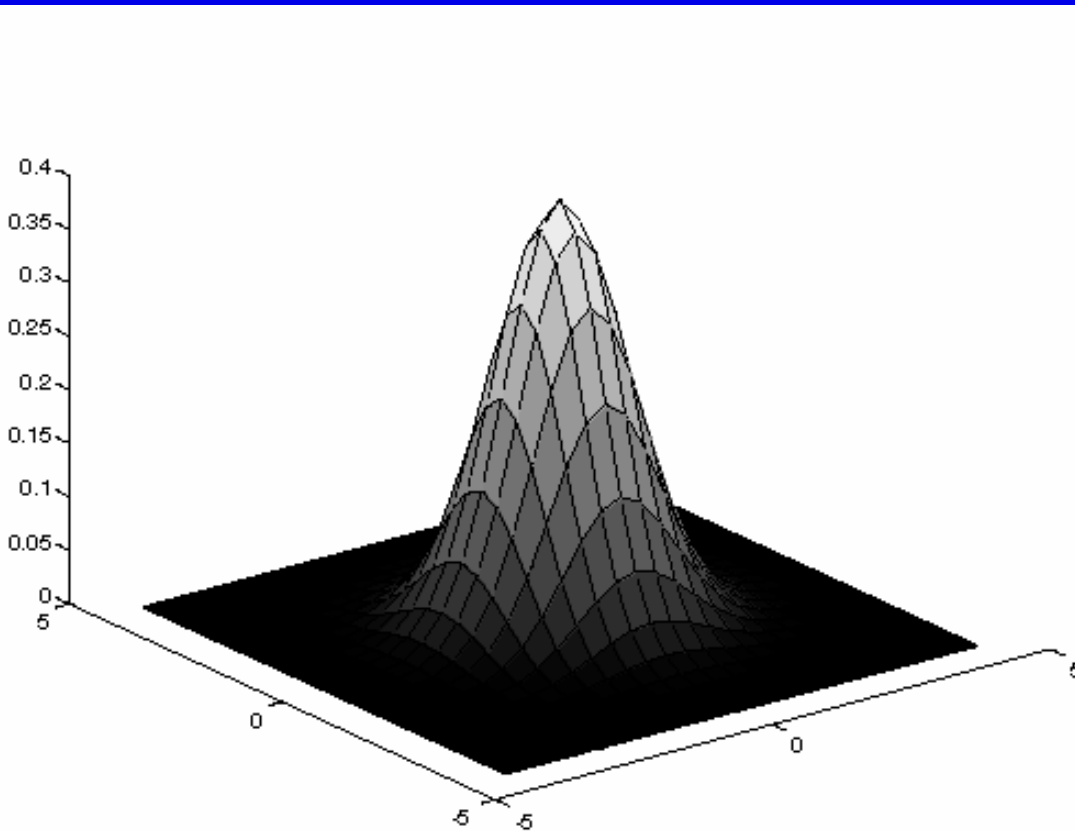


Suavizado por promediación



Suavizado con Gaussiana

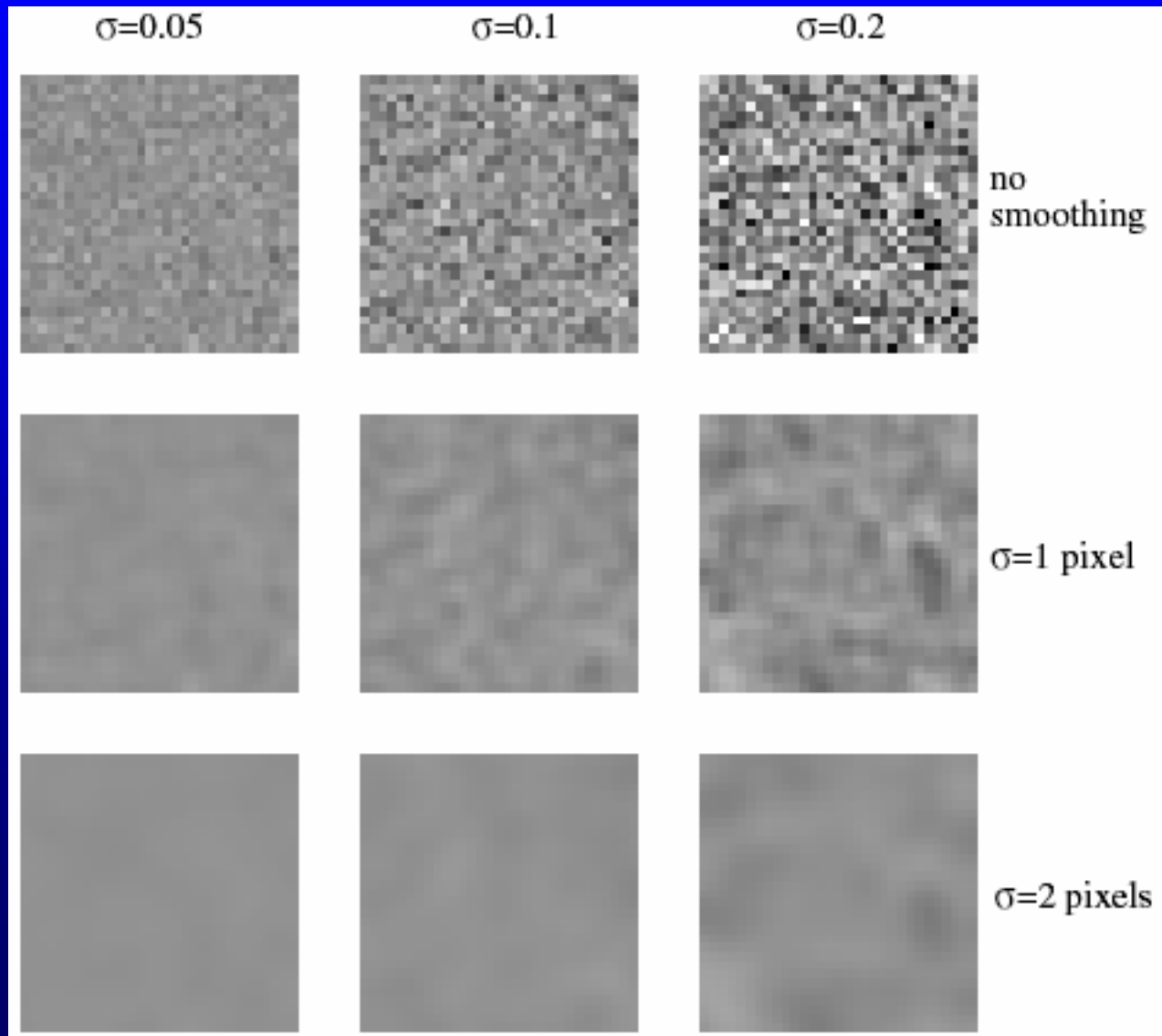
$$\exp\left(-\left(\frac{x^2 + y^2}{2\sigma^2}\right)\right)$$



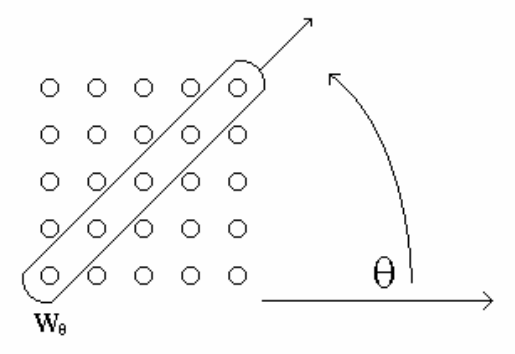
Filtrado Gaussiano



Ruido Gaussiano (columnas) suavizado por un filtro Gaussiano (filas)



Promediación Direccional



Filtrado Mediano

- Eliminación de artefactos
- Ruido binario





Filtrado

Temporal



Diferencias

$$\frac{\partial f}{\partial x} = \lim_{\varepsilon \rightarrow 0} \left(\frac{f(x + \varepsilon, y) - f(x, y)}{\varepsilon} \right)$$

$$\frac{\partial f}{\partial x} \approx \frac{f(x_{n+1}, y) - f(x_n, y)}{\Delta x}$$

$$\frac{\partial f}{\partial x} = f(x + 1) - f(x)$$

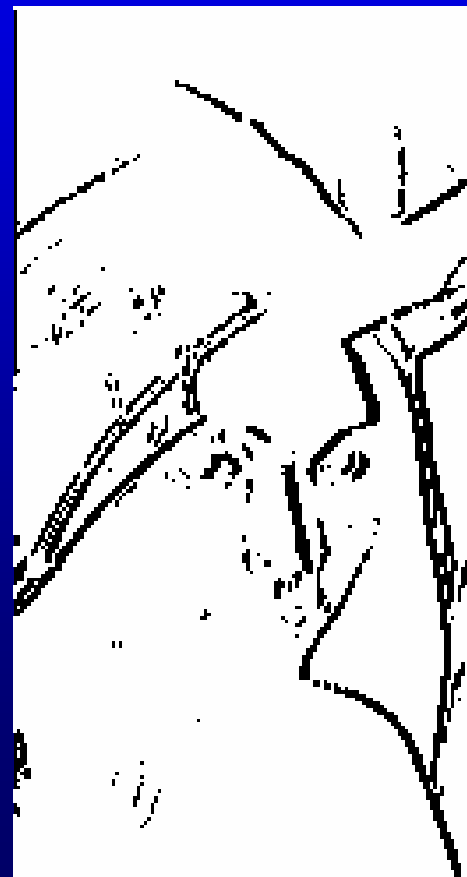
Derivada Hacia Adelante



Diferencia Central



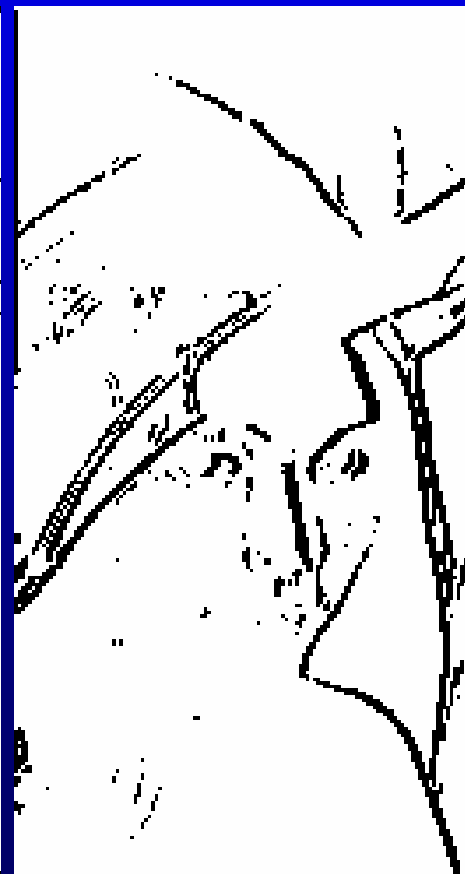
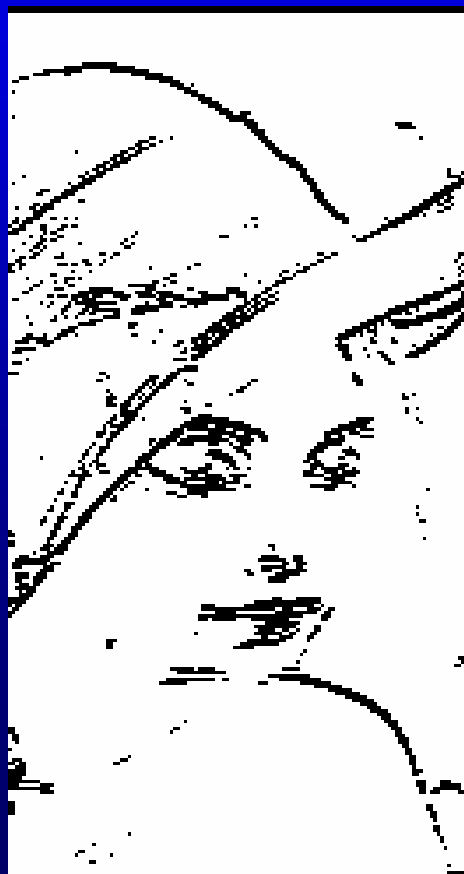
Operador de Prewitt



Operador de Sobel



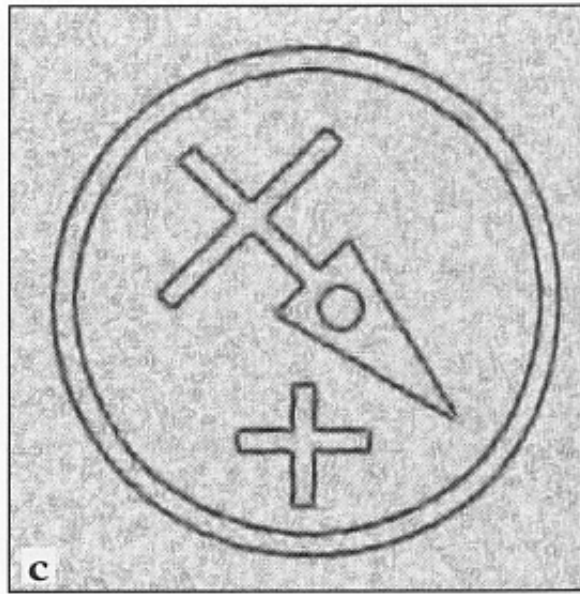
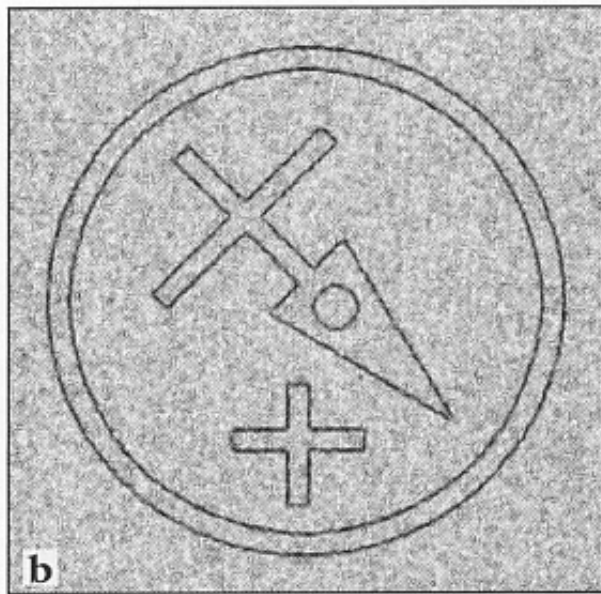
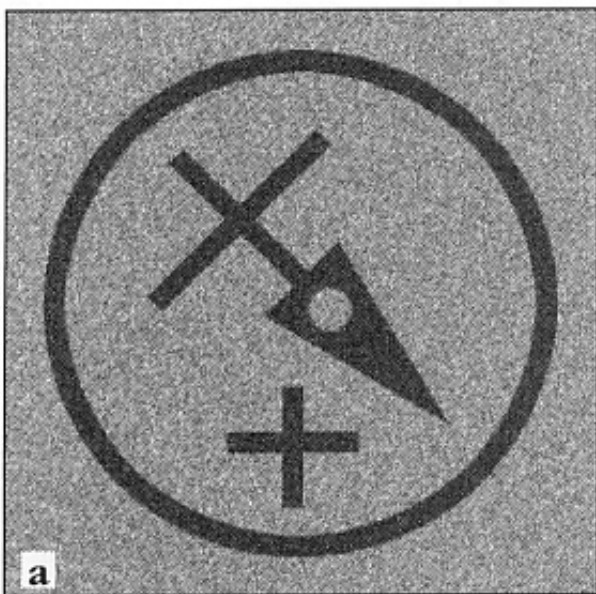
Operador Isotrópico



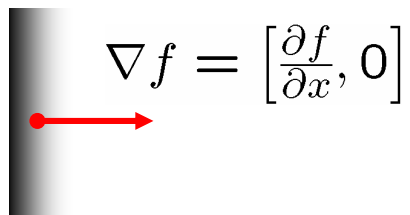
Operator Roberts



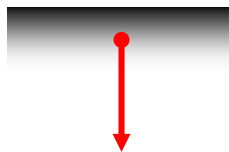
Sobel 3x3 y 5x5



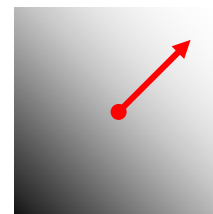
$$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right]$$



$$\nabla f = \left[\frac{\partial f}{\partial x}, 0 \right]$$



$$\nabla f = \left[0, \frac{\partial f}{\partial y} \right]$$



$$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right]$$

$$\theta = \tan^{-1} \left(\frac{\partial f}{\partial y} / \frac{\partial f}{\partial x} \right)$$

$$\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x} \right)^2 + \left(\frac{\partial f}{\partial y} \right)^2}$$

Horizontal Gradients Kernels $v_x: g_x = u * v_x$

0	1
-1	0

-1	0	1
-1	0	1
-1	0	1

-1	0	1
-2	0	2
-1	0	1

-1	0	1
$-\sqrt{2}$	0	$\sqrt{2}$
-1	0	1

Vertical Gradients Kernels $v_y: g_y = u * v_y$

1	0
0	-1

-1	-1	-1
0	0	0
1	1	1

-1	-2	-1
0	0	0
1	2	1

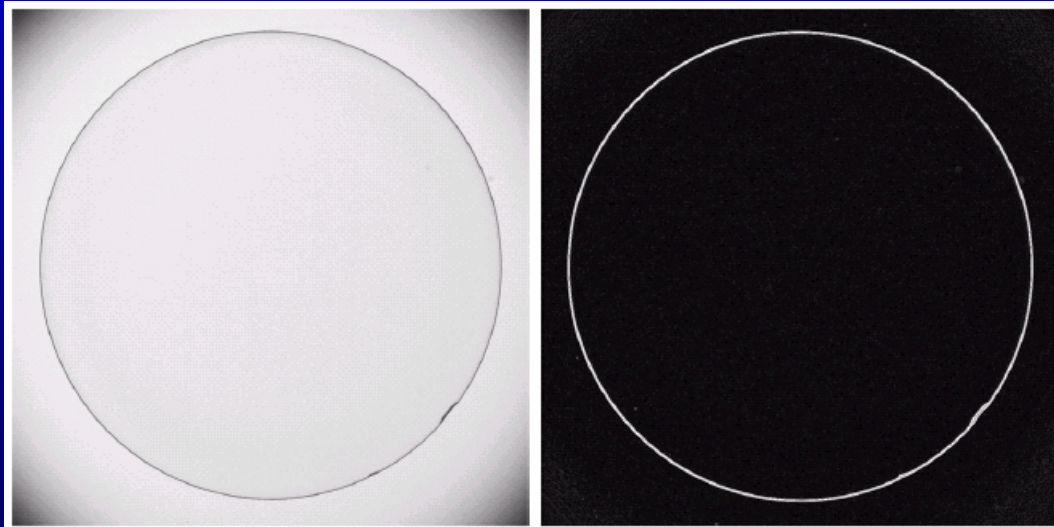
-1	$-\sqrt{2}$	-1
0	0	0
1	$\sqrt{2}$	1

Roberts

Prewitt

Sobel

Isotropic



Sobel

Gradient Operators (threshold)

$$g(i, j) = \sqrt{g_x^2(i, j) + g_y^2(i, j)} \quad \text{or} \quad g(i, j) = |g_x(i, j)| + |g_y(i, j)|$$

$$\theta_g(i, j) = \tan^{-1} \frac{g_y(i, j)}{g_x(i, j)}$$

$$\nabla f = \text{mag}(\nabla f)$$

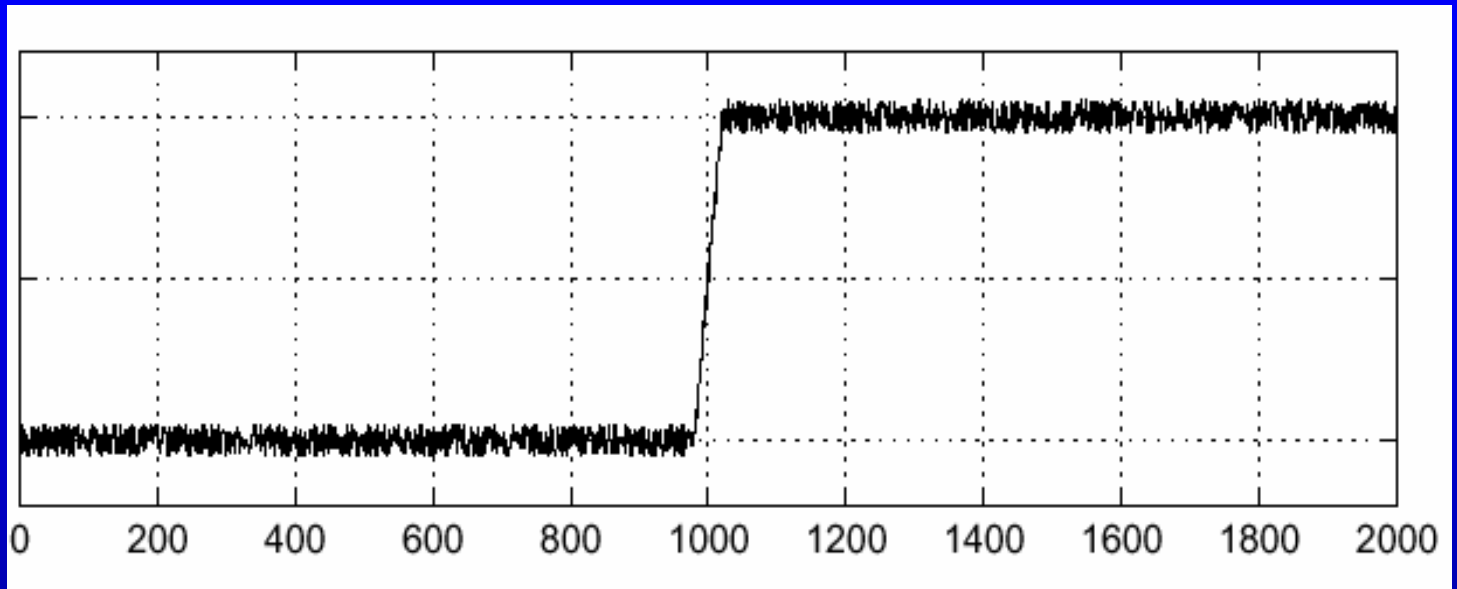
$$= [G_x^2 + G_y^2]^{1/2}$$

$$= \left[\left(\frac{\partial f}{\partial x} \right)^2 + \left(\frac{\partial f}{\partial y} \right)^2 \right]^{1/2}$$

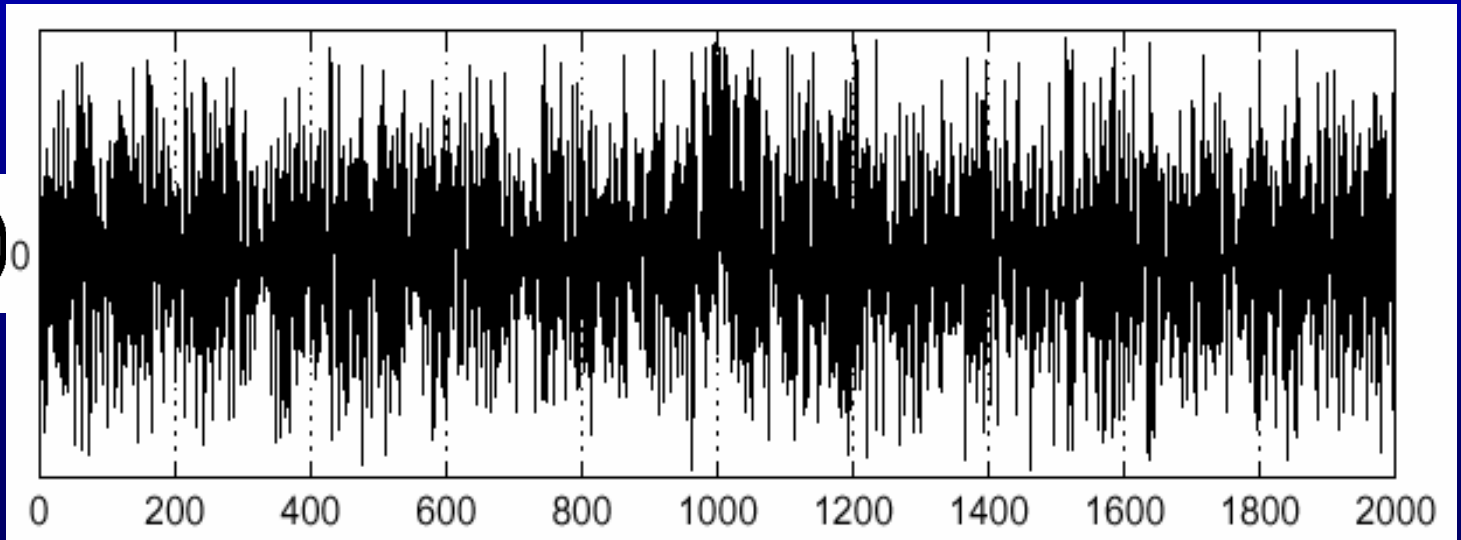
$$\nabla f \approx |G_x| + |G_y|$$

Efecto del ruido en la detección de bordes

$$f(x)$$

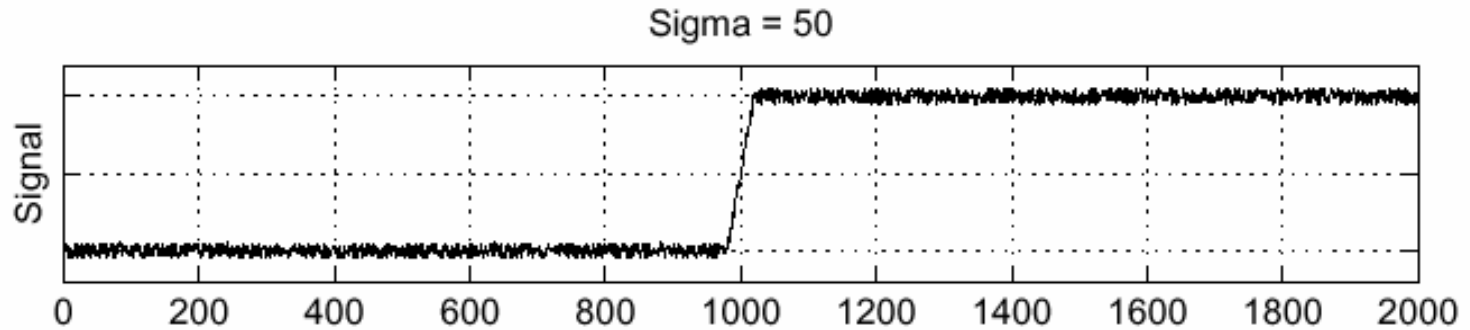


$$\frac{d}{dx}f(x)_0$$

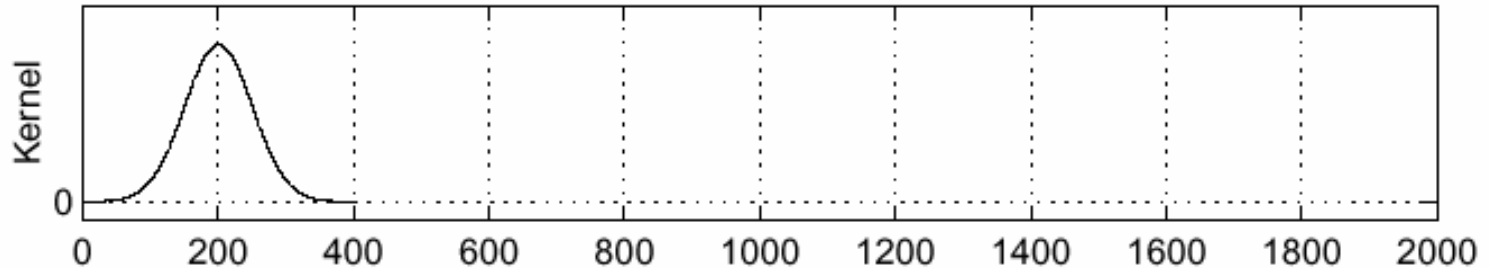


Solución: suavizar y luego buscar máximo en $\frac{\partial}{\partial x}(h \star f)$

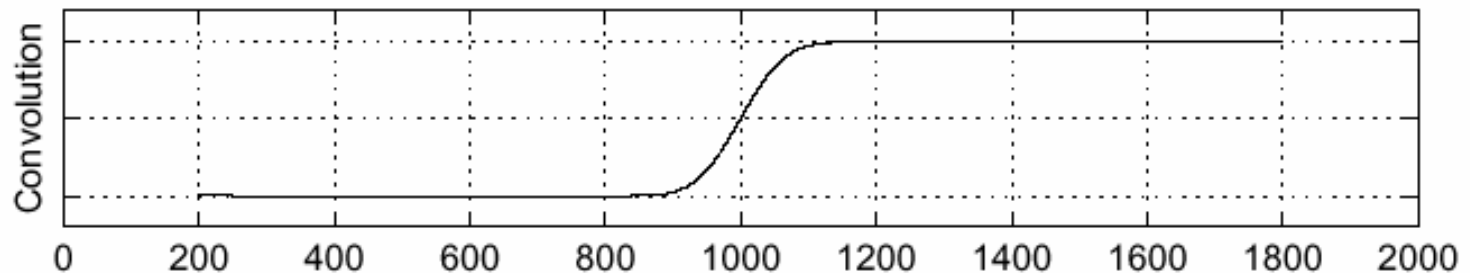
f



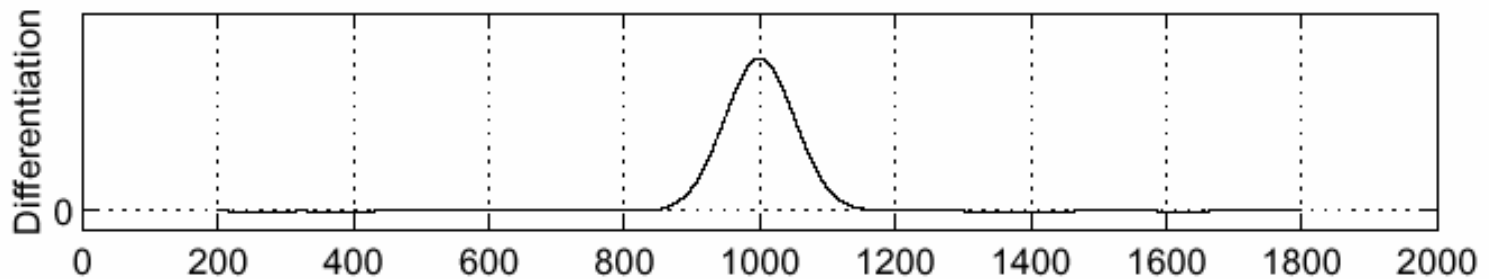
h



$h \star f$



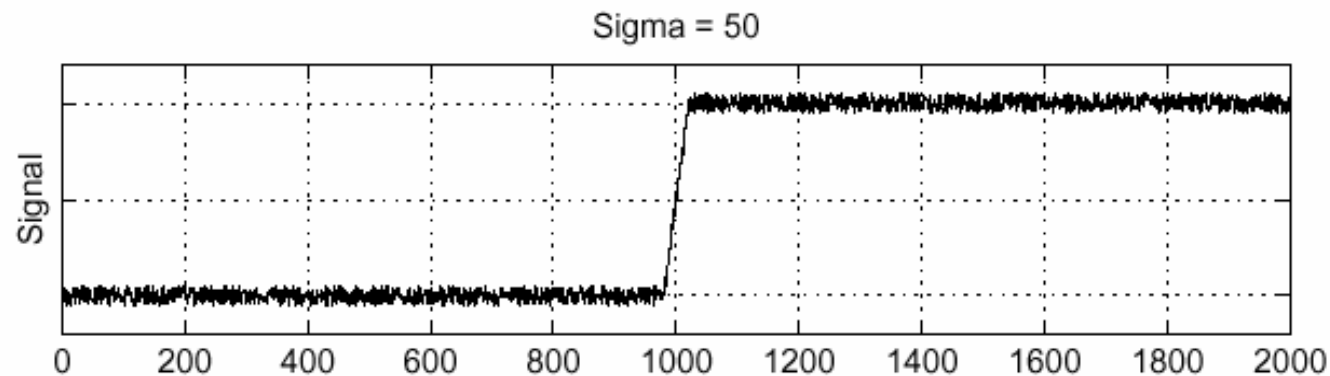
$\frac{\partial}{\partial x}(h \star f)$



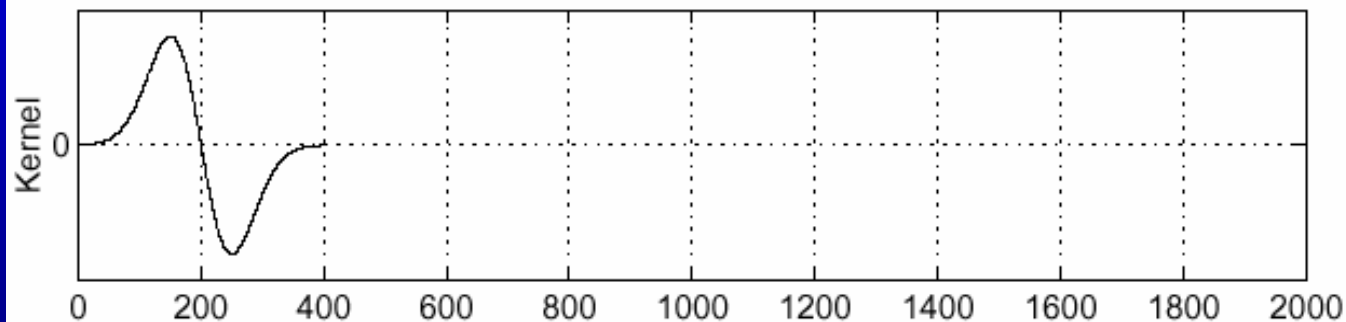
Teorema de la derivada de la convolución:

$$\frac{\partial}{\partial x}(h \star f) = \left(\frac{\partial}{\partial x}h\right) \star f$$

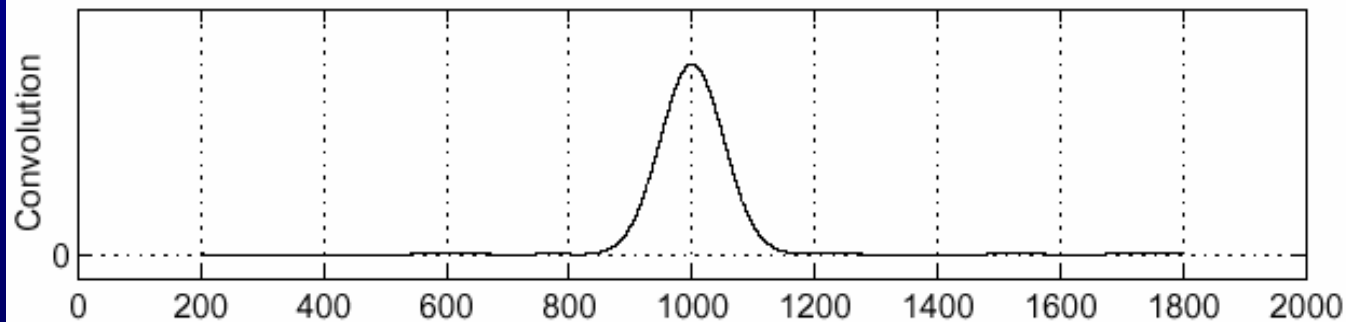
f



$\frac{\partial}{\partial x}h$



$\left(\frac{\partial}{\partial x}h\right) \star f$



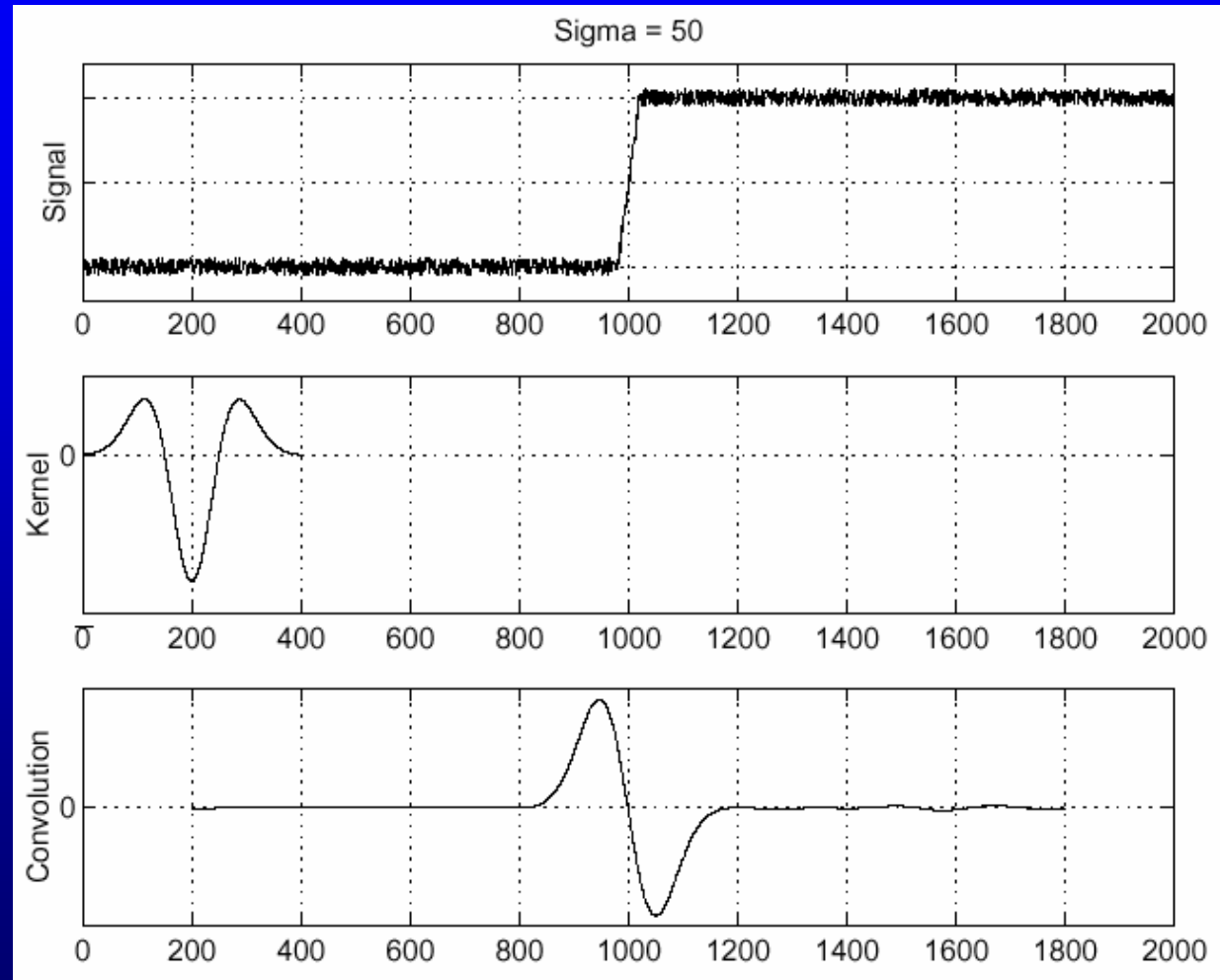
Operador LoG

$$\frac{\partial^2}{\partial x^2}(h \star f)$$

f

$$\frac{\partial^2}{\partial x^2}h$$

$$\left(\frac{\partial^2}{\partial x^2}h\right) \star f$$



$$\nabla^2 f = \frac{\partial^2 f}{\partial^2 x} + \frac{\partial^2 f}{\partial^2 y}$$

$$\frac{\partial^2 f}{\partial^2 x} = f(x+1, y) + f(x-1, y) - 2f(x, y)$$

$$\frac{\partial^2 f}{\partial^2 y} = f(x, y+1) + f(x, y-1) - 2f(x, y)$$

$$\nabla^2 f = [f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1)] - 4f(x, y)$$

0	1	0
1	-4	1
0	1	0

Laplace Operators (zero crossing)

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

0	-1	0
-1	4	-1
0	-1	0

-1	-1	-1
-1	8	-1
-1	-1	-1

1	-2	1
-2	4	-2
1	-2	1

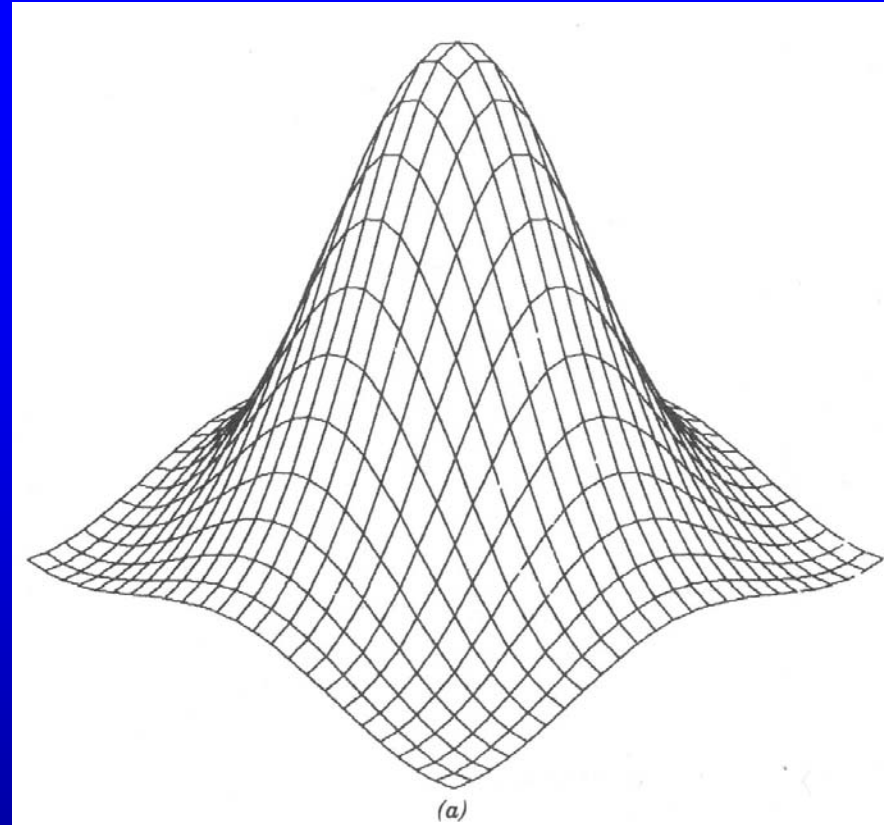
Filtro Gaussiano

7 × 7 Gaussian mask

1	1	2	2	2	1	1
1	2	2	4	2	2	1
2	2	4	8	4	2	2
2	4	8	16	8	4	2
2	2	4	8	4	2	2
1	2	2	4	2	2	1
1	1	2	2	2	1	1

15 × 15 Gaussian mask

2	2	3	4	5	5	6	6	6	5	5	4	3	2	2
2	3	4	5	7	7	8	8	8	7	7	5	4	3	2
3	4	6	7	9	10	10	11	10	10	9	7	6	4	3
4	5	7	9	10	12	13	13	13	12	10	9	7	5	4
5	7	9	11	13	14	15	16	15	14	13	11	9	7	5
5	7	10	12	14	16	17	18	17	16	14	12	10	7	5
6	8	10	13	15	17	19	19	19	17	15	13	10	8	6
6	8	11	13	16	18	19	20	19	18	16	13	11	8	6
6	8	10	13	15	17	19	19	19	17	15	13	10	8	6
5	7	10	12	14	16	17	18	17	16	14	12	10	7	5
5	7	9	11	13	14	15	16	15	14	13	11	9	7	5
4	5	7	9	10	12	13	13	13	12	10	9	7	5	4
3	4	6	7	9	10	10	11	10	10	9	7	6	4	3
2	3	4	5	7	7	8	8	8	7	7	5	4	3	2
2	2	3	4	5	5	6	6	6	5	5	4	3	2	2



LoG

5×5 Laplacian kernel

0	0	-1	0	0
0	-1	-2	-1	0
-1	-2	16	-2	-1
0	-1	-2	-1	0
0	0	-1	0	0

7×7 Mexican hat kernel

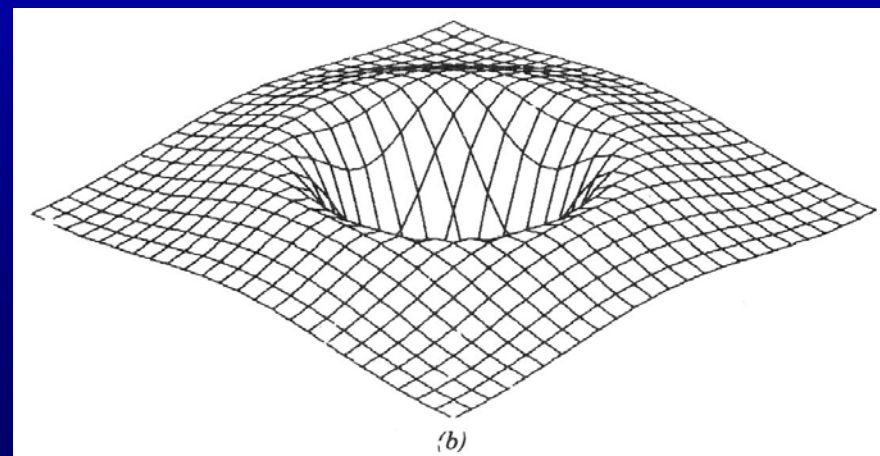
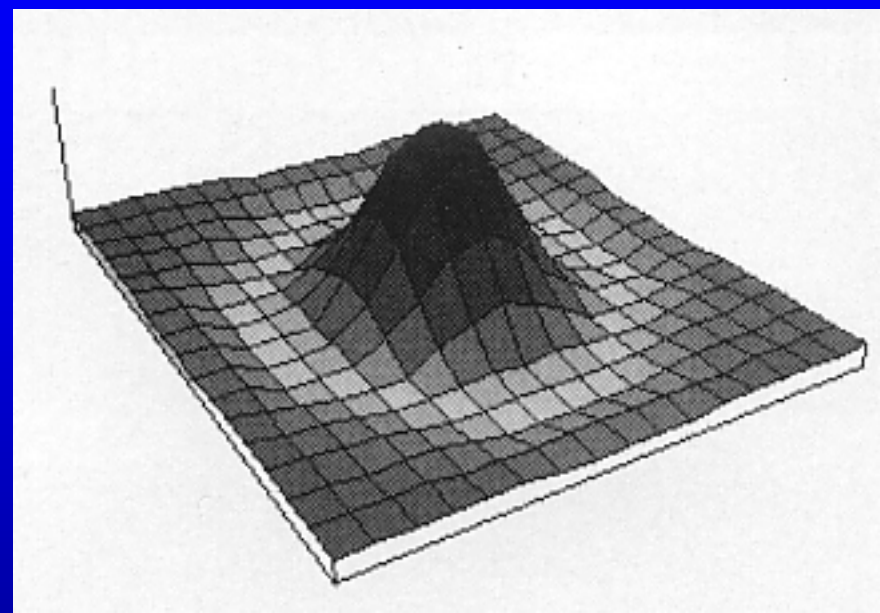
0	0	-1	-1	-1	0	0
0	-1	-3	-3	-3	-1	0
-1	-3	0	7	0	-3	-1
-1	-3	7	24	7	-3	-1
-1	-3	0	7	0	-3	-1
0	-1	-3	-3	-3	-1	0
0	0	-1	-1	-1	0	0

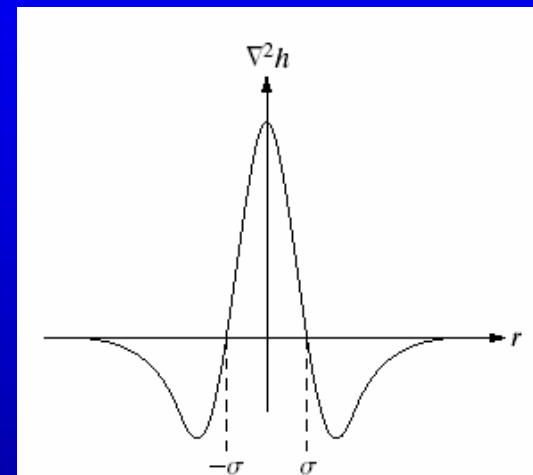
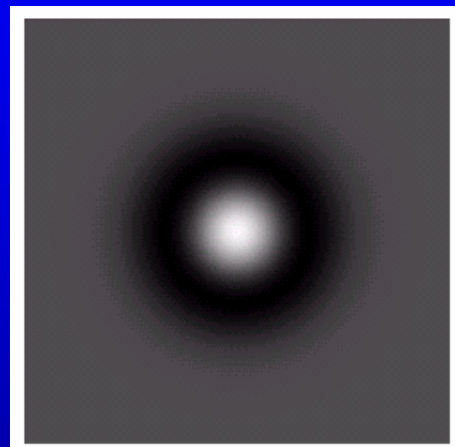
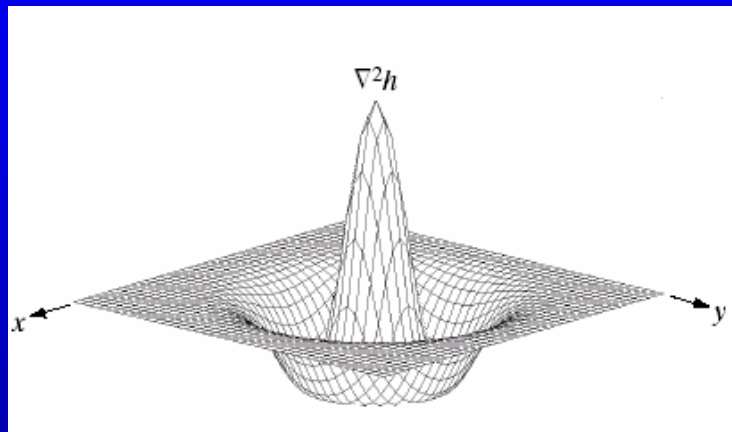
13×13 Mexican hat kernel

0	0	0	0	0	-1	-1	-1	0	0	0	0	0
0	0	0	-1	-1	-2	-2	-2	-1	-1	0	0	0
0	0	-2	-2	-3	-3	-4	-3	-3	-2	-2	0	0
0	-1	-2	-3	-3	-3	-2	-3	-3	-3	-2	-1	0
0	-1	-3	-3	-1	4	6	4	-1	-3	-3	-1	0
-1	-2	-3	-3	4	14	19	14	4	-3	-3	-2	-1
-1	-2	-4	-2	6	19	24	19	6	-2	-4	-2	-1
-1	-2	-3	-3	4	14	19	14	4	-3	-3	-2	-1
0	-1	-3	-3	-1	4	6	4	-1	-3	-3	-1	0
0	-1	-2	-3	-3	-3	-2	-3	-3	-3	-2	-1	0
0	0	-2	-2	-3	-3	-4	-3	-3	-2	-2	0	0
0	0	0	-1	-1	-2	-2	-2	-1	-1	0	0	0
0	0	0	0	0	-1	-1	-1	0	0	0	0	0

17×17 Mexican hat kernel

0	0	0	0	0	0	-1	-1	-1	-1	-1	0	0	0	0	0	0
0	0	0	0	-1	-1	-1	-1	-1	-1	-1	-1	-1	0	0	0	0
0	0	-1	-1	-1	-2	-3	-3	-3	-3	-3	-2	-1	-1	-1	0	0
0	0	-1	-1	-2	-3	-3	-3	-3	-3	-3	-3	-2	-1	-1	0	0
0	-1	-1	-2	-3	-3	-3	-2	-3	-2	-3	-3	-3	-2	-1	-1	0
0	-1	-2	-3	-3	-3	0	2	4	2	0	-3	-3	-3	-2	-1	0
-1	-1	-3	-3	-3	0	4	10	12	10	4	0	-3	-3	-3	-1	-1
-1	-1	-3	-3	-2	2	10	18	21	18	10	2	-2	-3	-3	-1	-1
-1	-1	-3	-3	-3	4	12	21	24	21	12	4	-3	-3	-3	-1	-1
-1	-1	-3	-3	-2	2	10	18	21	18	10	2	-2	-3	-3	-1	-1
-1	-1	-3	-3	-3	0	4	10	12	10	4	0	-3	-3	-3	-1	-1
0	-1	-2	-3	-3	-3	0	2	4	2	0	-3	-3	-3	-2	-1	0
0	-1	-1	-2	-3	-3	-3	-2	-3	-2	-3	-3	-3	-2	-1	-1	0
0	0	-1	-1	-2	-3	-3	-3	-3	-3	-3	-2	-1	-1	-1	0	0
0	0	-1	-1	-1	-2	-3	-3	-3	-3	-3	-2	-1	-1	-1	0	0
0	0	0	0	-1	-1	-1	-1	-1	-1	-1	-1	0	0	0	0	0
0	0	0	0	0	0	-1	-1	-1	-1	-1	0	0	0	0	0	0



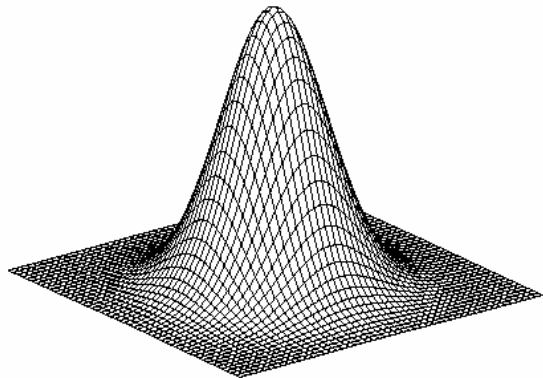


0	0	-1	0	0
0	-1	-2	-1	0
-1	-2	16	-2	-1
0	-1	-2	-1	0
0	0	-1	0	0

∇^2

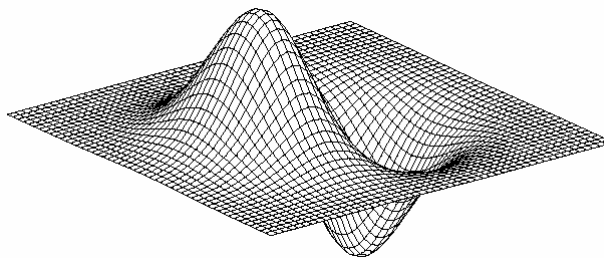
Operador Laplaciano

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$



Gaussiana

$$h_{\sigma}(u, v) = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{2\sigma^2}}$$

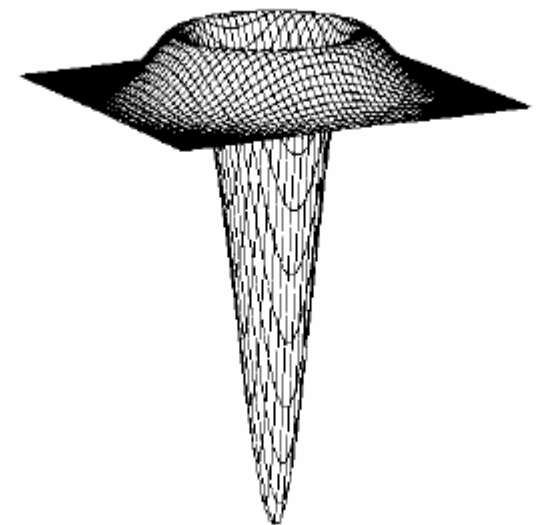


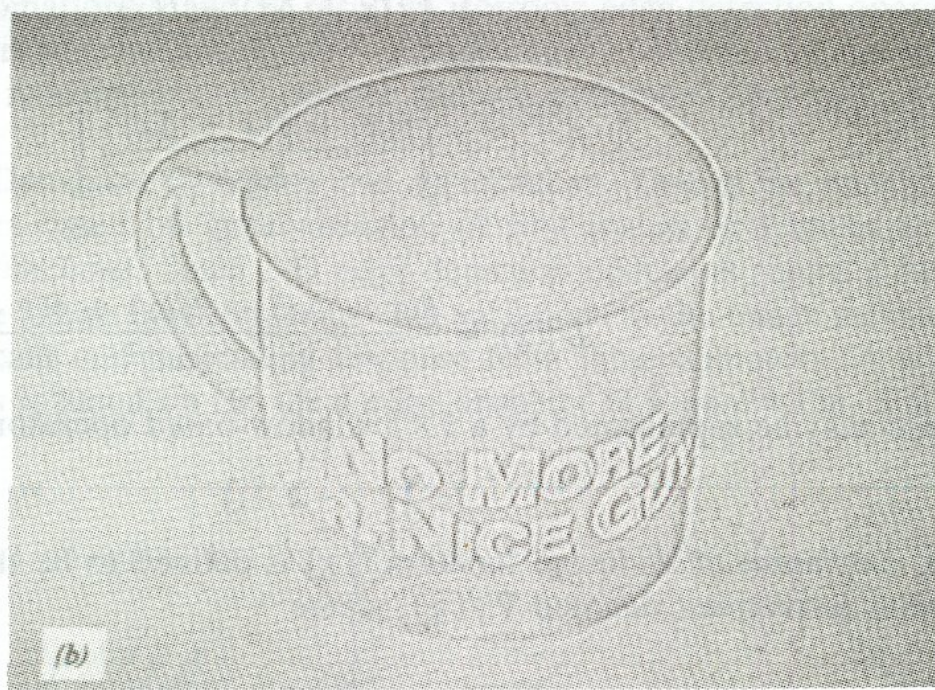
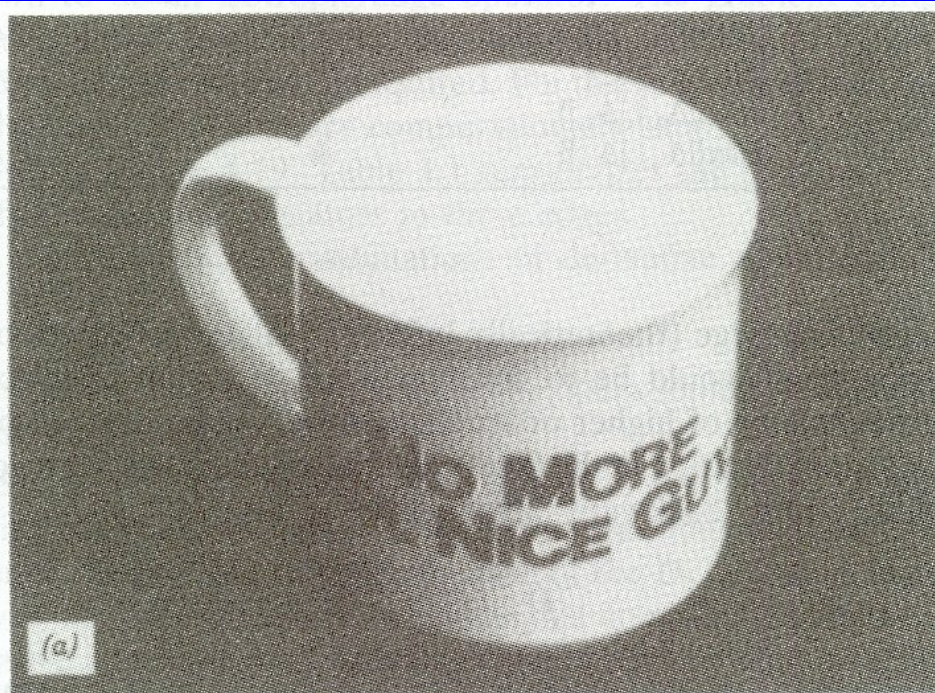
Derivada de la Gaussiana

$$\frac{\partial}{\partial x} h_{\sigma}(u, v)$$

 ∇

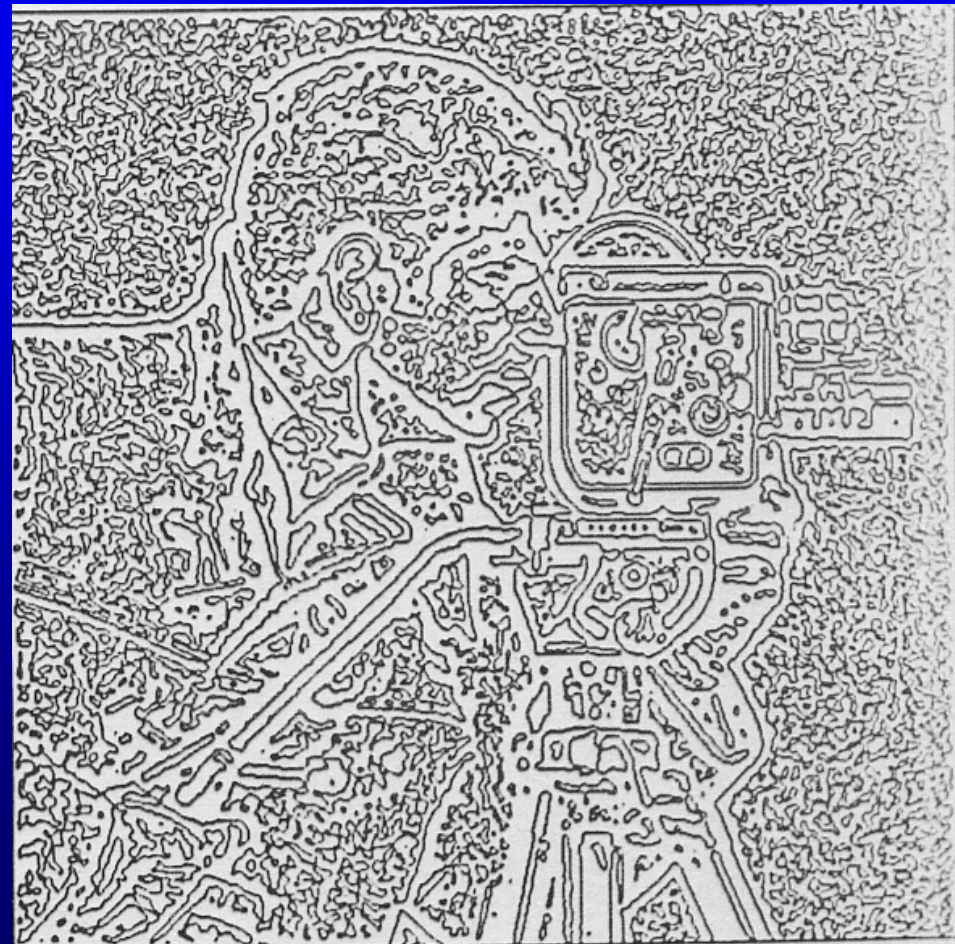
LoG





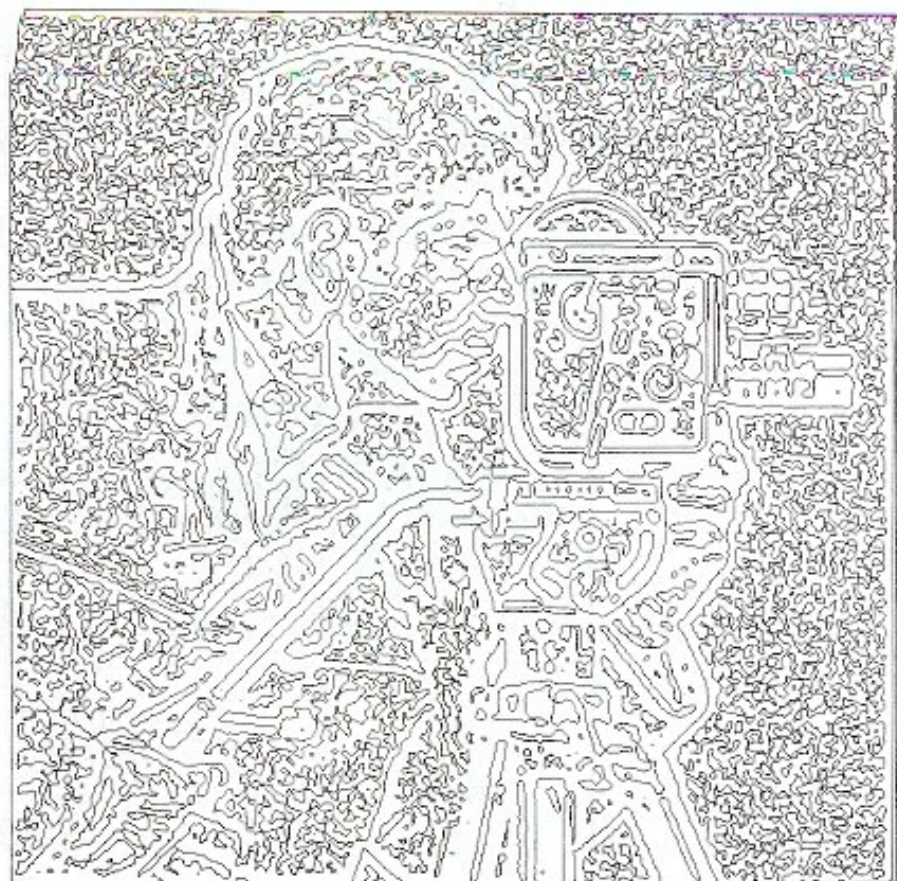
Operador Laplaciano

Laplaciano y cruces por cero





(a)



(b)

Bordes obtenidos con un detector Laplaciano y cruces por cero



(a)



(b)

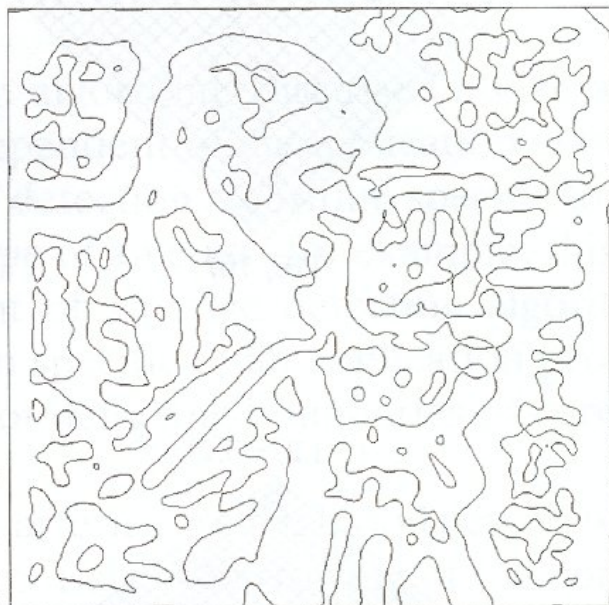


(c)

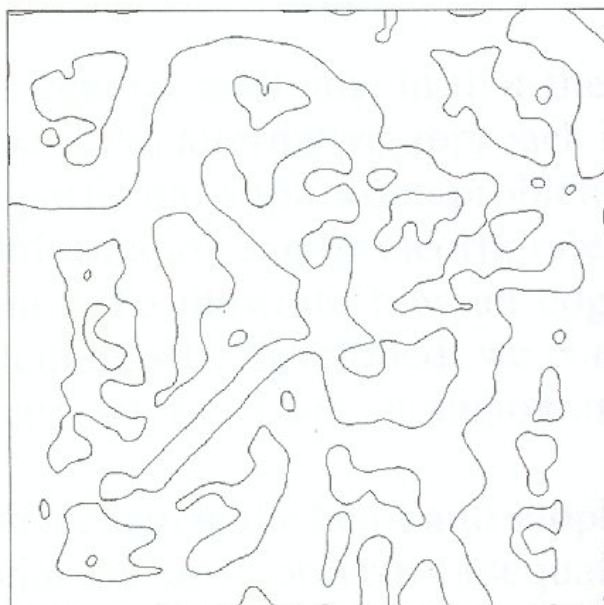
LoG: $\sigma=2$, $\sigma=4$, y $\sigma=6$



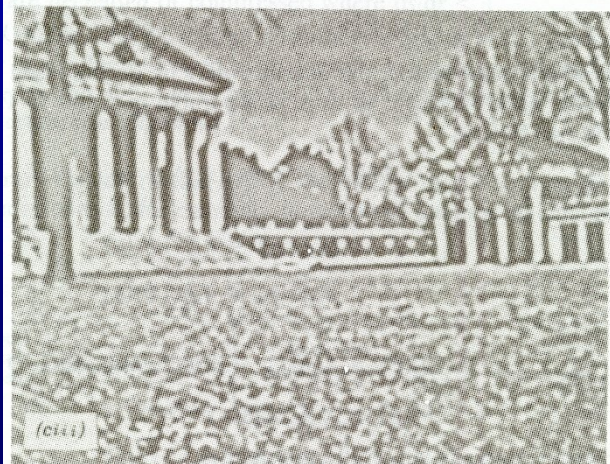
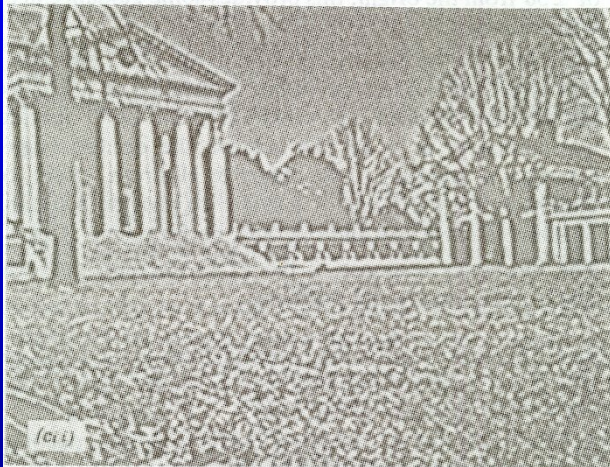
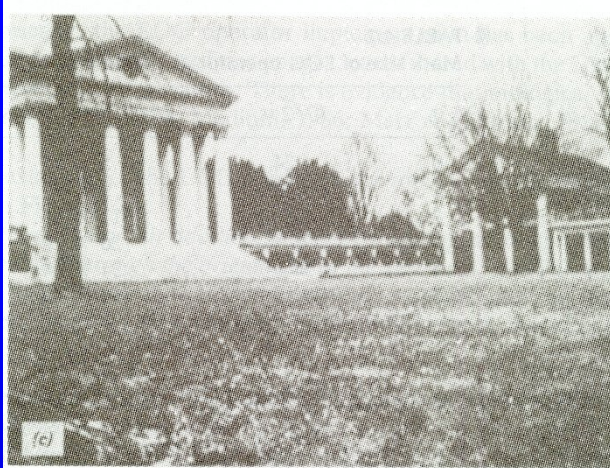
(d)



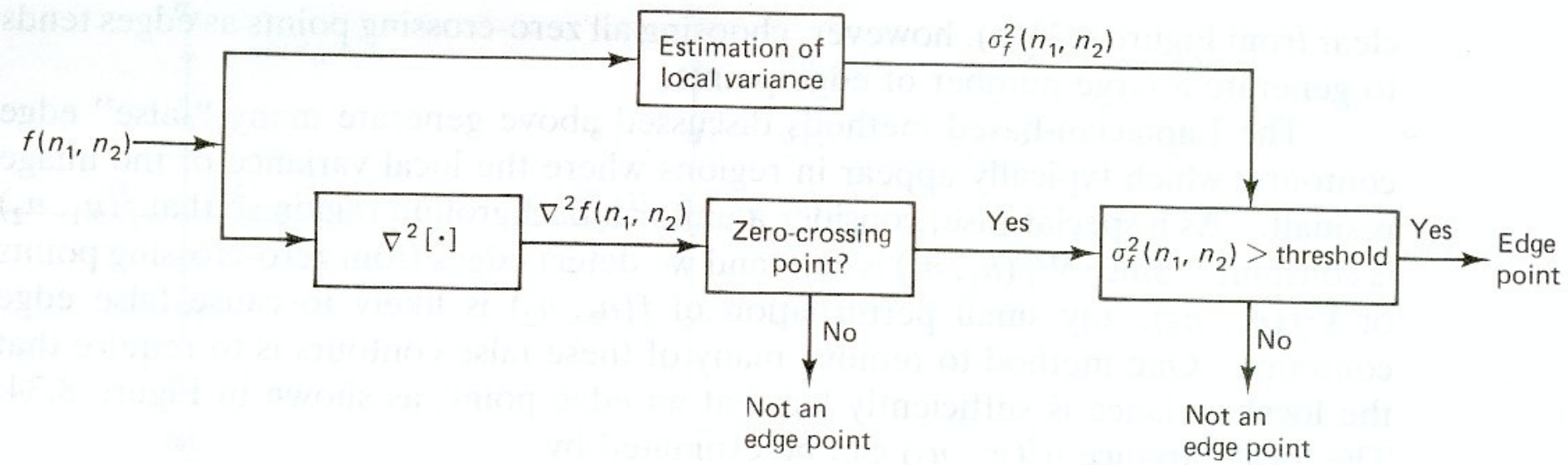
(e)



(f)

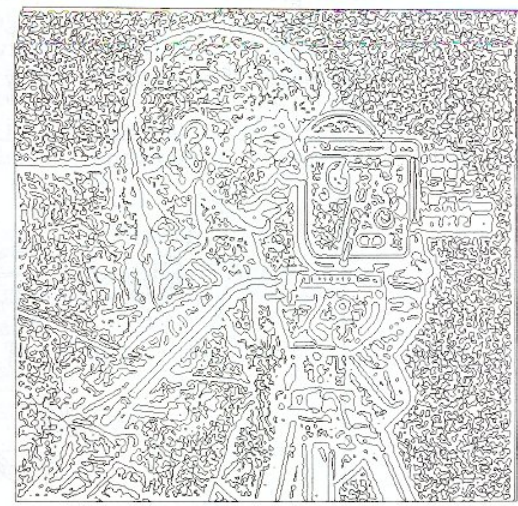


Operator LoG:
original
 $\sigma=0.5$
 $\sigma=1.0$





(a)

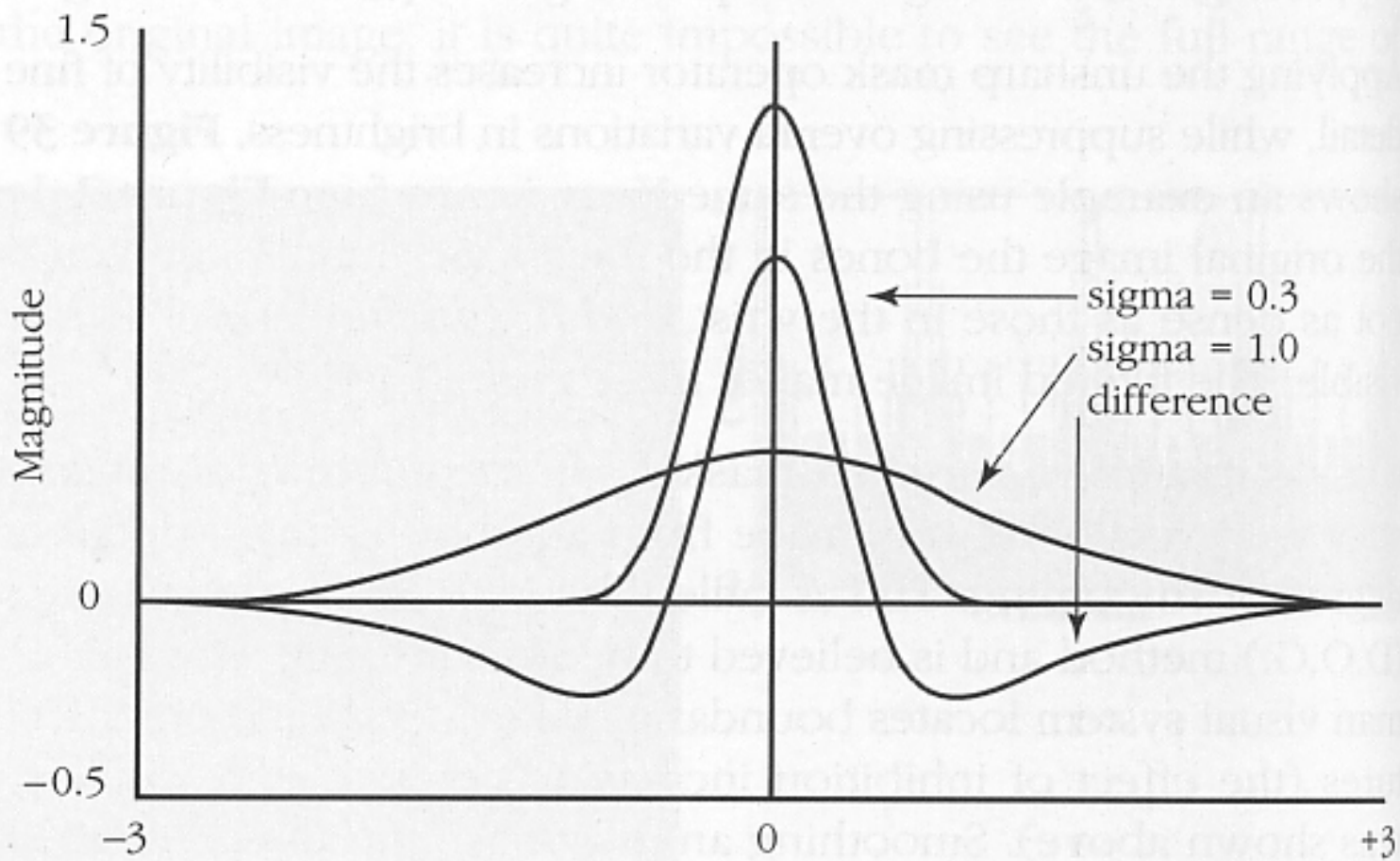


(b)



**Bordes con Laplaciano
cruces por cero y umbral
en base a varianza local**

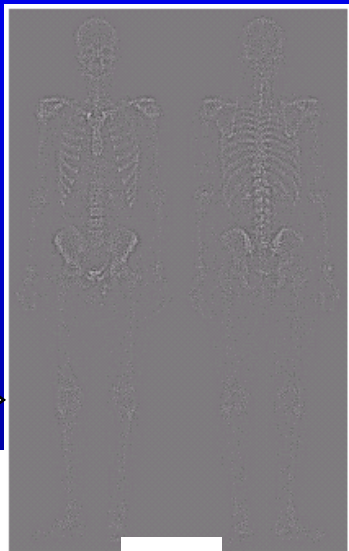
DoG





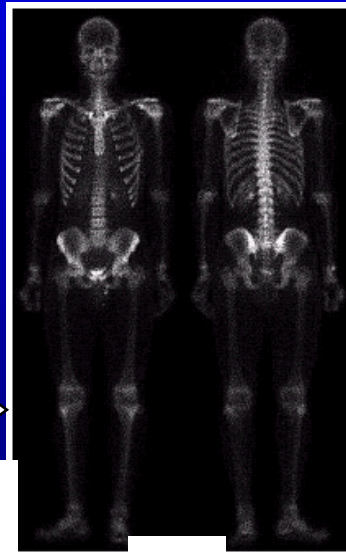
(a)

Laplacian filter of
bone scan (a)



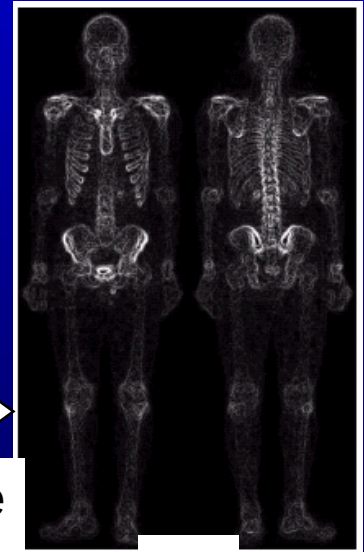
(b)

Sharpened version of
bone scan achieved
by subtracting (a)
and (b)



(c)

Sobel filter of bone
scan (a)



(d)

The product of (c) and (e) which will be used as a mask

(e)

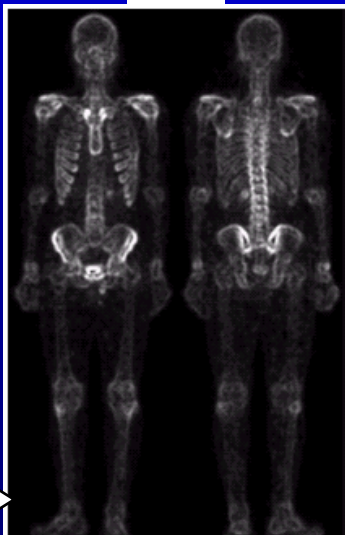
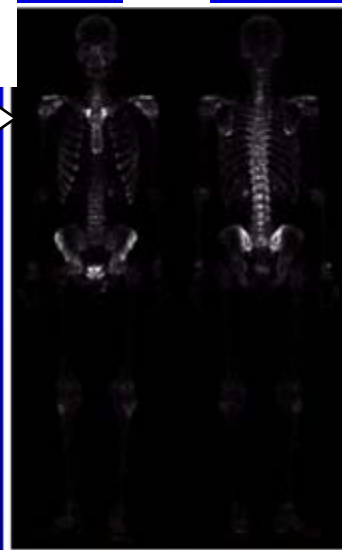


Image (d) smoothed with a 5*5 averaging filter

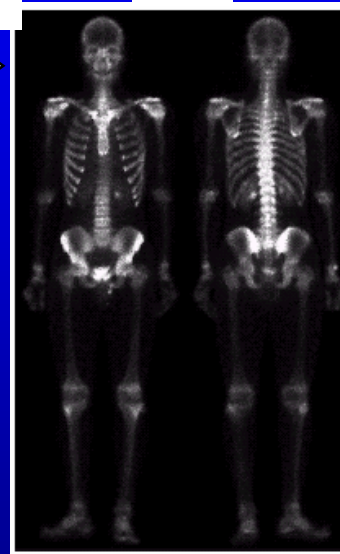
Sharpened image which is sum of (a) and (f)

(f)



Result of applying a power-law trans. to (g)

(g)



(h)

