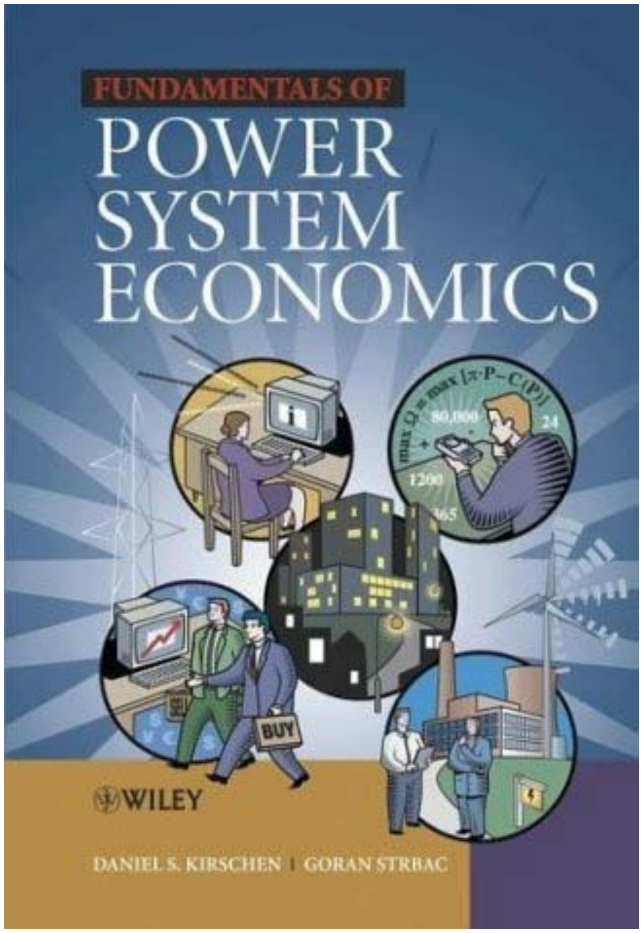
SOLUTIONS MANUAL



CHAPTER 3

MARKETS FOR ELECTRICAL ENERGY

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Chapter 3

3.2 The rules of the Syldavian electricity market stipulate that all participants must trade energy exclusively through the Power Pool. However, the Syldavia Aluminum Company (SALCo) and the Northern Syldavia Power Company (NSPCo) have signed a contract for difference for the delivery of 200 MW *on a continuous basis at a strike price of* 16 $/MWh.

a. Trace the flow of power and money between these companies when the pool price takes the following values: 16 $/MWh, 18 $/MWh *and* 13 $/MWh.

Pool price $/MWh	SALCo (Consumer)	NSPCo (Producer)
16 $/MWh	Pays into to the pool: $ 3200 Difference: $ 0	Gets paid from pool: $ 3200 Difference: $ 0
18 $/MWh	Pays into to the pool: $ 3600 Difference: 3200-3600 = $ -400 Gets paid by NSPCo: $ 400	Gets paid: $ 3600 Difference: 3600-3200 = $ 400 Pays to SALCo: $ 400
13 $/MWh	Pays into to the pool: $ 2600 Difference: 3200-2600 = $ 600 Pays to NSPCo: $ 600	Gets paid from pool: $ 2600 Difference: 2600-3200 = $ -600 Gets paid by SALCo: $ 600

b. What happens if during one hour the Northern Syldavia Power Company is only able to deliver 50 MWh *and the pool price is* 18 $/MWh*?*

SALCo pays $200 \times 18 = \$ 3600$ into the pool. Because the contract of difference specifies that it should pay only $200 \times 16 = \$ 3200$, it receives $200 \times (18 - 16) = \$400$ from NSPCo. On the other hand, NSPCo receives $50 \times 18 = \$900$ from the pool and pays $ 400 to SALCo.

c. What happens if during one hour the Syldavia Aluminum Company consumes only 100 MWh *and the pool price is* 13 $/MWh*?*

SALCo pays $100 \times 13 = \$1300$ into the pool. NSPCo receives $200 \times 13 = \$2600$ from the pool. Since the contract for difference is for 200 MWh at 16 $/MWh, SALCo pays to NSPCo $200 \times (16 - 13) = \$600$.

3.3 The following six companies participate, along with others, in the Southern Antarctica electrical energy market:

- ***Red:** a generating company owning a portfolio of plants with a maximum capacity of* 1000 MW.

- ***Green:*** *another generating company with a portfolio of plants with a maximum capacity of* 800 MW.
- ***Blue:*** *a retailer of electrical energy.*
- ***Yellow:*** *another retailer of electrical energy.*
- ***Magenta:*** *a trading company with no generating assets and no demand.*
- ***Purple:*** *another trading company with no physical assets.*

The following information pertains to the operation of this market for Monday 29 *February* 2016 *between* 1:00 *p.m. and* 2:00 *p.m.*

Load Forecasts
Blue and Yellow forecast that their customers will consume respectively 1200 MW *and* 900 MW *during that hour.*

Long-term contracts
June 2015*: Red signs a contract for the supply of* 600 MW *at* 15 $/MWh *for all hours between 1 January* 2015 *and* 31 *December* 2020
July 2015*: Blue signs a contract for the purchase of* 700 MW *for all hours between* 1 *February* 2016 *and* 31 *December* 2016*. The price is set at* 12 $/MWh *for off-peak hours and at* 15.50 $/MWh *for peak hours.*
August 2015*: Green signs a contract for the supply of* 500 MW *at* 16 $/MWh *for peak hours in February* 2016*.*
September 2015*: Yellow signs a contract for the purchase of electric energy. The contract specifies a profile of daily and weekly volumes and a profile for daily and weekly prices. In particular, on weekdays between* 1:00 *and* 2:00 *p.m., the volume purchased is* 550 MW *at* 16.25 $/ MWh*.*

Futures contracts
All contracts are for delivery on 29 *February* 2016 *between* 1:00 *p.m. and* 2:00 *p.m.*

Date	***Company***	***Type***	***Amount***	***Price***
10/9/15	*Magenta*	*Buy*	50	14.50
20/9/15	*Purple*	*Sell*	100	14.75
30/9/15	*Yellow*	*Buy*	200	15.00
10/10/15	*Magenta*	*Buy*	100	15.00
20/10/15	*Red*	*Sell*	200	14.75
30/10/15	*Green*	*Sell*	250	15.75
30/10/15	*Blue*	*Buy*	250	15.75
10/11/15	*Purple*	*Buy*	50	15.00
15/11/15	*Magenta*	*Sell*	100	15.25
20/11/15	*Yellow*	*Buy*	200	14.75
30/11/15	*Blue*	*Buy*	300	15.00
10/12/15	*Red*	*Sell*	200	16.00
15/12/15	*Red*	*Sell*	200	15.50
20/12/15	*Blue*	*Sell*	50	15.50
15/1/16	*Purple*	*Sell*	200	14.50
20/1/16	*Magenta*	*Buy*	50	14.25

10/2/16	*Yellow*	*Buy*	50	14.50
20/2/16	*Red*	*Buy*	200	16.00
25/2/16	*Magenta*	*Sell*	100	17.00
28/2/16	*Purple*	*Buy*	250	14.00
28/2/16	*Yellow*	*Sell*	100	14.00

Options Contracts
In November 2015*, Red bought a put option for* 200 MWh *at* 14.75 $/MWh*. The option fee was* $50*.*
In December 2015*, Yellow bought a call option for* 100 MWh *at* 15.50 $/MWh*. The option fee was* $25*.*

Outcome

- *The spot price on the Southern Antarctica electricity market was set at* 15.75 $/MWh *for* 29 *February* 2016 *between* 1:00 *and* 2:00 *p.m.*
- *Due to difficulties at one of its major plants, Red was only able to generate* 800 MW*. Its average cost of production was* 14.00 *$/MWh.*
- *Green generated* 770 MW *at an average cost of* 14.25 $/MWh*.*
- *Blue's demand turned out to be* 1250 MW*. Its average retail price was* 16.50 $/MWh*.*
- *Yellow demand turned out to be* 850 MW*. Its average retail price was* 16.40 $/MWh*.*

Assuming that all imbalances are settled at the spot market price, calculate the profit or loss made by each of these participants.

Red (Generator)

Type	Quantity sold	Quantity purchased	Price	Expenses	Revenues
Long term sale	600		15		9000
Future sale	200		14.75		2950
Put option fee				50	0
Future sale	200		16	0	3200
Future sale	200		15.5	0	3100
Future purchase		200	16	3200	0
Production cost		800	14	11200	0
Spot purchase		200	15.75	3150	0
Balance	1200	1200		17600	18250
Profit					**650**

Green (Generator)

Type	Quantity sold	Quantity purchased	Price	Expenses	Revenues
Long term sale	500		16	0	8000
Future sale	250		15.75	0	3937.5

Production cost		770	14.25	10972.5	0
Spot sale	20		15.75	0	315
Balance	770	770		10972.5	12252.5
Profit					**1280**

Blue (Supplier)

Type	Quantity sold	Quantity purchased	Price	Expenses	Revenues
Long term purchase		700	15.5	10850	0
Future purchase		250	15.75	3937.5	0
Future purchase		300	15	4500	0
Future sale	50		15.5	0	775
Retail sale	1250		16.5	0	20625
Spot purchase		50	15.75	787.5	0
Balance	1300	1300		20075	21400
Profit					**1325**

Yellow (Supplier)

Type	Quantity sold	Quantity purchased	Price	Expenses	Revenues
Long term purchase		550	16.25	8937.5	0
Future purchase		200	15	3000	0
Future purchase		200	14.75	2950	0
Option fee				25	
Future purchase		50	14.5	725	0
Future sale	100		14	0	1400
Retail sale	850		16.4	0	13940
Exercise option		100	15.5	1550	0
Spot sale	150		15.75	0	2362.5
Balance	1100	1100		17187.5	17702.5
Profit					**515**

Magenta (trader)

Type	Quantity sold	Quantity purchased	Price	Expenses	Revenues
Future purchase		50	14.5	725	0
Future purchase		100	15	1500	0
Future sale	100		15.25	0	1525
Future purchase		50	14.25	712.5	0
Future sale	100		17	0	1700
Balance	200	200		2937.5	3225
Profit					**287.5**

Purple (trader)

Type	Quantity	Quantity	Price	Expenses	Revenues

	sold	purchased			
Future sale	100		14.75	0	1475
Future purchase		50	15	750	0
Future sale	200		14.5	0	2900
Future purchase		250	14	3500	0
Balance	300	300		4250	4375
Profit					**125**

3.4 The operator of a centralized market for electrical energy has received the bids shown on the table below for the supply of electrical energy during a given period:

Company	*Amount (MWh)*	*Price ($/MWh)*
Red	100	12.5
Red	100	14.0
Red	50	18.0
Blue	200	10.5
Blue	200	13.0
Blue	100	15.0
Green	50	13.5
Green	50	14.5
Green	50	15.5

a. Build the supply curve

To build the supply curve we arrange the bids in increasing order of bid price and plot the resulting staircase function. The length of each step is equal to the amount of each bid. It is useful to label each step with the name of the company that submitted the bid.

Table P3.4-a: Supply curve for Problem 3.4

Company	Amount (MWh)	Cumulative amount (MWh)	Price ($/MWh)
Blue	200	200	10.5
Red	100	300	12.5
Blue	200	500	13.0
Green	50	550	13.5
Red	100	650	14.0
Green	50	700	14.5
Blue	100	800	15.0
Green	50	850	15.5
Red	50	900	18.0

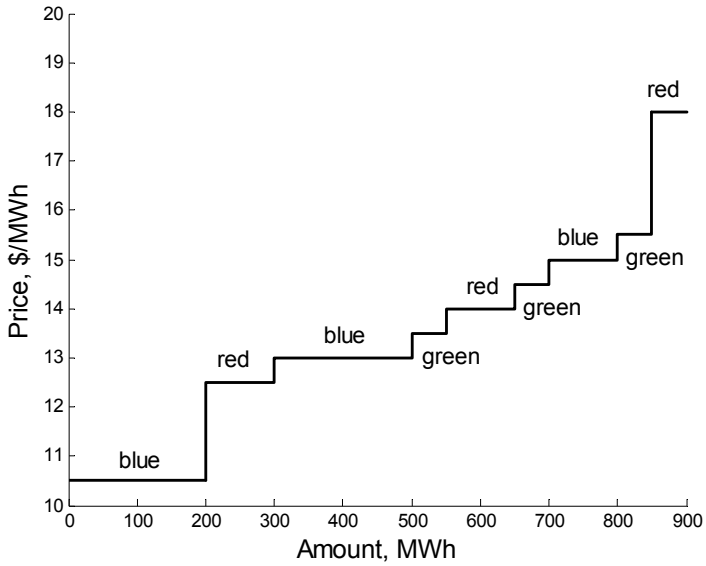


Figure P3.4-a: Supply curve for Problem 3.4

b. *Assume that this market operates unilaterally, i.e. that the demand does not bid and is represented by a forecast. Calculate the market price, the quantity produced by each company, and the revenue of each company for each of the following loads*: 400 MW, 600MW, 875 MW.

Using Table P3.4 or Figure P3.4-a, we see that for a load of 400 MW, the market price will be set at 13 $/MWh. We can also find the bids (or parts of bids that will be accepted. Using this information, we can calculate the revenues of each company: These results are shown in Table P3.4-b.

Table P3.4-b: Accepted bids and revenues for a load of 400 MW

Company	Amount (MW)	Revenue ($)
Blue	200	2600
Red	100	1300
Blue	100	1300

Therefore the total revenues are as follows:

Blue: $3900
Red = $1300
Green: $0.

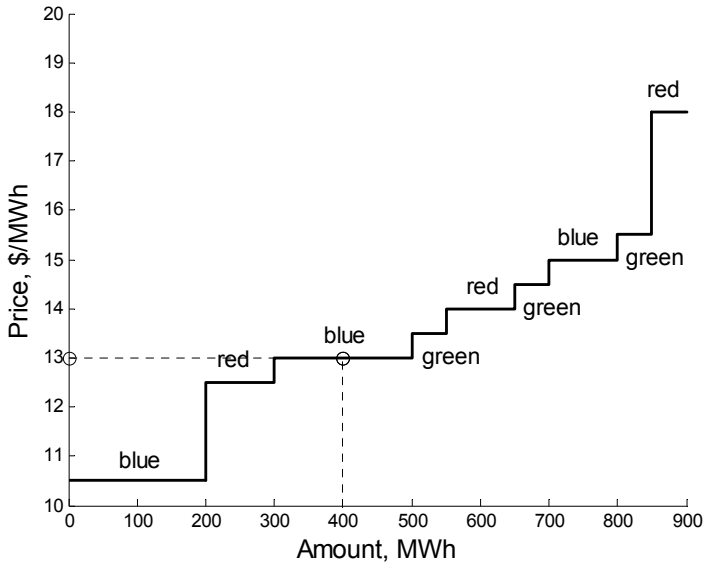


Figure P3.4-b: Market equilibrium for a load of 400 MW

Similarly, a demand of 600 MW the price is 14 $/MWh and the productions of each company are as shown in Table P3.4-c:

Table P3.4-c: Accepted bids and revenues for a load of 600 MW

Company	Amount (MW)	Revenue ($)
Blue	200	2800
Red	100	1400
Blue	200	2800
Green	50	700
Red	50	700

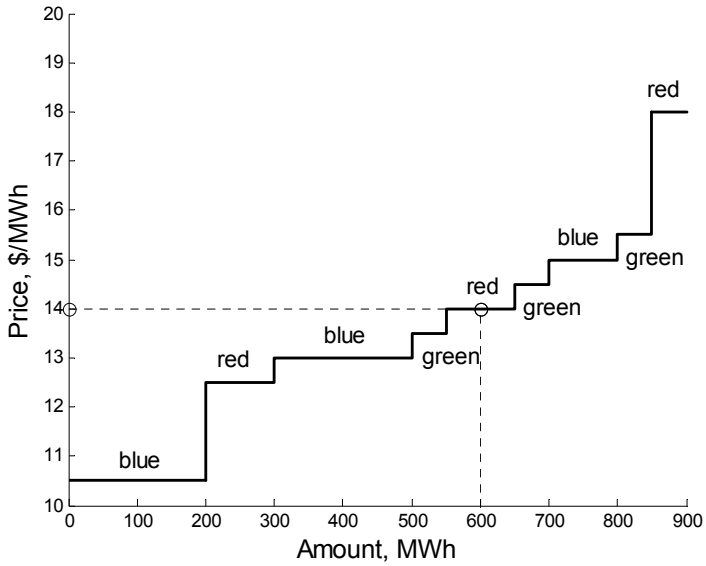


Figure P3.4-c: Market equilibrium for a load of 600 MW

The total revenues are as follows:

Blue = $5600
Red = $2100
Green = $700

Finally, if the demand is 875 MW, the price is 18 $/MWh, and the productions are as shown in Table P3.4-d.

Table P3.4-d: Accepted bids and revenues for a load of 900 MW

Company	Amount (MW)	Revenue ($)
Blue	200	3600
Red	100	1800
Blue	200	3600
Green	50	900
Red	100	1800
Green	50	900
Blue	100	1800
Green	50	900
Red	25	450

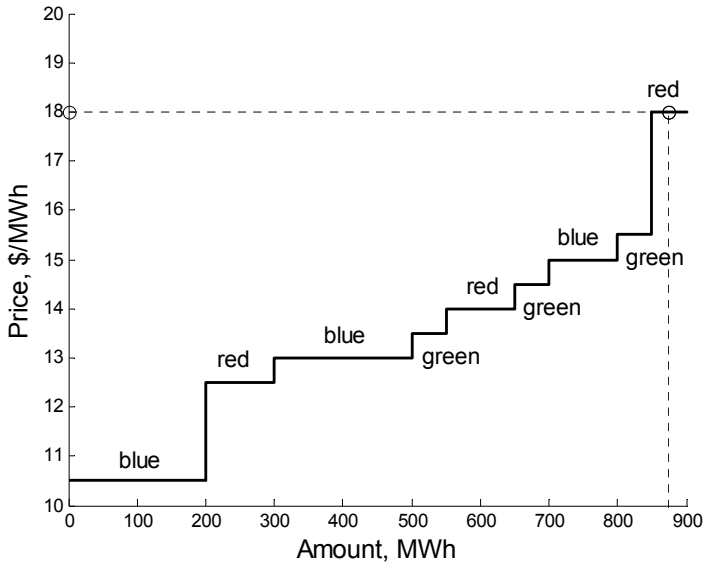


Figure P3.4-d: Market equilibrium for a load of 900 MW

The total revenues are as follows:

Blue = $ 9000
Red = $4050
Green = $2700.

c. *Suppose that instead of being treated as constant, the load is represented by its inverse demand curve, which is assumed to have the following form*:

$$D = L - 4.0 \cdot \pi$$

where D is the demand, L is the forecasted load and π *is the price. Calculate the effect that this price sensitivity of demand has on the market price and the quantity traded*.

For each of the forecasted load, the demand can be computed, and then the demand curve can be plotted with supply curve, and at the point in which crosses is where the market equilibrium lies, Figure P3.4-d.

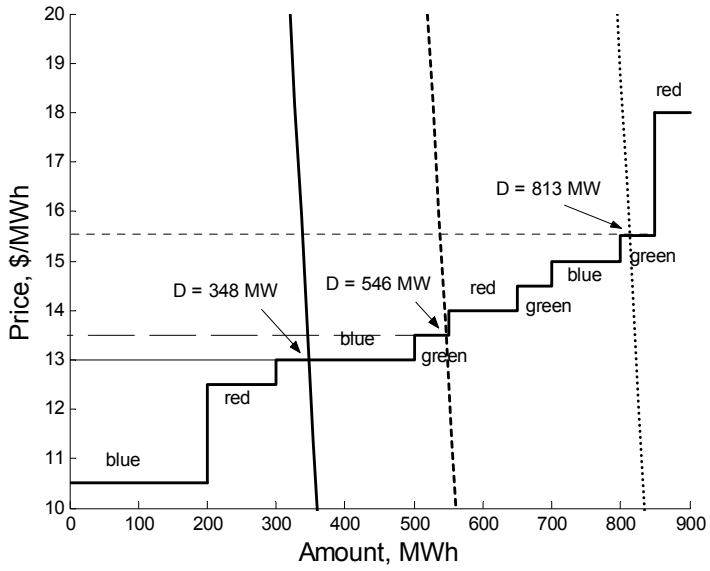


Figure P3.4-d: Market equilibrium for the three forecasted loads, 400, 600 and 875 MW

For a forecasted load of 400 MW, the price is 13 $/MWh, and the demand is 348 MW. Using the supply curve, we get the productions and revenues shown in Table P3.4-e.

Table P3.4-e: Accepted bids and revenues for a price-sensitive load of 400 MW:

Company	Amount (MW)	Revenue ($)
Blue	200	2600
Red	100	1300
Blue	48	624

The total revenues of each company are as follows:

Blue = $3224
Red = $1300
Green = $0

For a forecasted load of 600 MW, the price would be 14 $/MWh if the demand was price-insensitive. However, using the supply curve, we see that such a demand would result in a price of 13.5 $/MWh. At that price, the price-sensitive demand is:

$D = 600 - 4.0 \times 13.5 = 546$ MW.

Under these conditions, the productions and revenues are as shown in Table P3.4-f.

Table P3.4-f: Accepted bids and revenues for a price-sensitive load of 600 MW:

Company	Amount (MW)	Revenue ($)
Blue	200	2700
Red	100	1350

Blue	200	2700
Green	44	594

Then the revenues are as follows: Blue = $ 5400, Red = $ 1350 and Green = $ 549.

Finally is the forecasted load is 875 MW, then the demand would be 813 MW for a price of 15.5 $/MWh, as shown in Figure 5.4-d; and under these conditions the productions and revenues are shown in Table P3.4-g.

Table P3.4-f: Accepted bids and revenues for a price-sensitive load of 875 MW:

Company	Amount (MW)	Revenue ($)
Blue	200	3100
Red	100	1550
Blue	200	3100
Green	50	775
Red	100	1550
Green	50	775
Blue	100	1550
Green	13	201.5

Then the revenues are as follows: Blue = $ 7750, Red = $ 3100 and Green = $ 1751.5.

3.5 *The Syldavian Power and Light Company owns one generating plant and serves some load. It has been actively trading in the electricity market and has established the following position for* 11 *June between* 10:00 *and* 11:00 *a.m.:*

- *Long term contract for the purchase of* 600 MW *during peak hours at a price of* 20.00 $/MWh
- *Long term contract for the purchase of* 400 MW *during off peak hours at a price of* 16.00 $/MWh
- *Long term contract with a major industrial user for the sale of* 50 MW *at a flat rate of* 19.00 $/MWh
- *The remaining customers purchase their electricity at a tariff of* 21.75 $/MWh
- *Future contract for the sale of* 200 MWh *at* 21.00 $/MWh
- *Future contract for the purchase of 100 MWh at* 22.00 $/MWh
- *Call option for* 150 MWh *at an exercise price of* 20.50 $/MWh
- *Put option for* 200 MWh *at an exercise price of* 23.50 $/MWh
- *Call option for* 300 MWh *at an exercise price of* 24.00 $/MWh.

The option fee for all the options is 1.00 $/MWh. *The peak hours are defined as being the hours between* 8:00 *a.m. and* 8:00 *p.m.*

The outcome for 11 June between 10:00 *and* 11:00 *is as follows:*

- *The spot price is set at* 21.50 $/MWh.

- *The total load of the Syldavian Power and Light Company is* 1200 MW*, including the large industrial customer.*
- *The power plant produces* 300 MWh *at an average cost of* 21.25 $/MWh.

a. *Assuming that all imbalances are settled at the spot market price, calculate the profit or loss made by the company during that hour.*

b. *What value of the spot market would reduce the profit or loss of the company to zero? Would this change in spot price affect any of the option contracts?*

To compute the profit of loss that the company made during that hour it is necessary to compute the revenues and expenses made during such hour; this are shown in Table P3.5-a:

Table P3.5-a Expenses and Revenues for the different items

Item	Energy bought	Energy sold	Price	Expenses	Revenue
Industrial customer		50	19	0	950
Other customer		1150	21.75	0	25012.5
Future contract		200	21	0	4200
Put option		200	23.5	0	4700
Long-term contract	600		20	12000	0
Future contract	100		22	2200	0
Call option	150		20.5	3075	0
Generation	300		21.25	6375	0
Spot market purchase	450		21.5	9675	0
150 MW call option fee			1	150	0
200 MW put option fee			1	200	0
300 MW call option fee			1	300	0
Balance	1600	1600		33975	34862.5
Profit					887.5

To find the spot market price at which the profits reduce to zero, then the expenses have to be equal to the revenue, therefore it can be written:

$$24300+450\pi_{\text{spot}} = 34862.5$$

$$\pi_{spot} = \frac{(34862.5 - 24300)}{450}$$

Then the price would be: 23.4722 $/MWh, because with this price the 450 MWh purchase would offset the profit. The 20.5 $/MWh call option and the 23.50 $/MWh put option would still be in the money. The 24.00 $/MWh call option would still be out if the money.

3.6 A company called Borduria Energy owns a nuclear power plant and a gas-fired power plant. Its trading division has entered into the following contracts for 25 *January:*

T-1. A forward contract for the sale of 50 MW *at a price of* 21.00 $/MWh*. This contract applies to all hours.*

T-2. A long term contract for the sale of 300 MW *during off peak hours at a price of* 14.00 $/MWh

T-3. A long-term contract for the sale of 350 MW *at* 20 $/MWh *during peak hours.*

In addition, for the trading period from 2:00 *p.m. to* 3:00 *p.m. on that day, it has entered into the following transactions:*

T-4. A future contract for the purchase of 600 MWh *at* 20.00 $/MWh

T-5. A future contract for the sale of 100 MWh *at* 22.00 $/MWh

T-6. A put option for 250 MWh *at an exercise price of* 23.50 $/MWh

T-7. A call option for 200 MWh *at an exercise price of* 22.50 $/MWh

T-8. A put option for 100 MWh *at an exercise price of* 18.75 $/MWh

T-9. A bid in the spot market to produce 50 MW *using its gas-fired plant at* 19.00 $/MWh

T-10. A bid in the spot market to produce 100 MW *using its gas-fired plant at* 22.00 $/MWh

The option fee for all call and put options is $2.00 / MWh*. The peak hours are defined as being the hours between* 8:00 *a.m. and* 8:00 *p.m.*

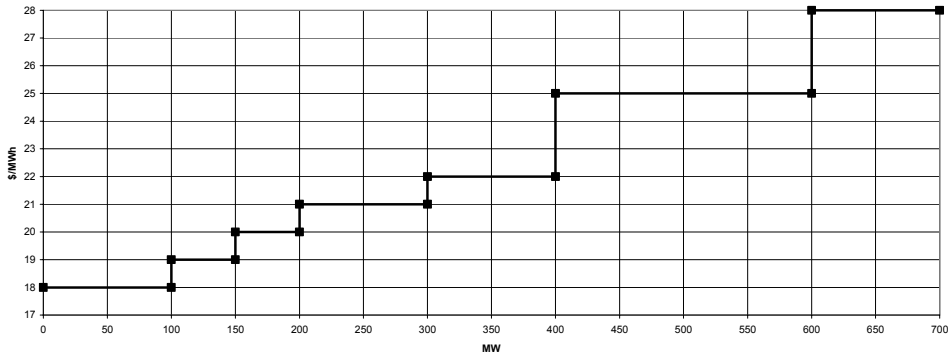
Borduria Energy also sells electrical energy directly to small consumers through its retail division. Residential customers pay a tariff of 25.50 $/MWh *and commercial consumers pay a tariff of* 25.00 $/MWh*. Borduria Energy does not sell electricity to industrial consumers.*

The graph on Figure 3.3 shows the stack of bids that the spot market operator has received for the trading period from 2:00 *p.m. to* 3:00 *p.m. on* 25 *January. In order to balance load and generation, it accepted bids for* 225 MW *in increasing order of price for that hour. The spot price was set at the price of the last accepted bid.*

During that hour, the residential customers served by Borduria Energy consumed 300 MW *while its commercial customers consumed* 200 MW*. The nuclear power plant produced* 400 MWh *at an average cost of* 16.00 $/MWh*. Its gas-fired plant produced* 200 MWh *at an average cost of* 18.00 $/MWh*. All imbalances are settled at the spot market price.*

a. Calculate the profit or loss made by Borduria Energy during that hour.

b. Calculate the effect that the sudden outage of the nuclear generating plant at 2:00 *p.m. on* 25 *January would have on the profit (or loss) of Borduria Energy for that hour.*



To compute the profit or loss that Borduria Energy made during that hour it is necessary to compute the expenses and revenues made for the different items, as shown in Table 3.6-a.

Table P3.5-a Expenses and Revenues for the different items

Item	Energy bought (MWh)	Energy sold (MWh)	Price ($/MWh)	Expenses ($)	Revenue ($)
Future T4	600		20	12000	0
Nuclear Unit	400		16	6400	0
Gas-fire unit	200		18	3600	0
Forward T1		50	21	0	1050
Long-term T3		350	20	0	7000
Future T5		100	22	0	2200
Exercise put option T6		250	23.5	0	5875
Spot sale T9		50	21	0	1050
Residential customer		300	25.5	0	7650
Commercial customer		200	25	0	5000
Balancing spot purchase	100		21	2100	0
Fee option T6			2	500	0
Fee option T7			2	400	0
Fee option T8			2	200	0
Balance	1300	1300		25200	29825
Profit					4625

If a sudden outage of the nuclear generating plant at 2:00 p.m. on 25 January occurs, then two scenarios could be considered:

a) Borduria Energy can exercise its option contracts and the expenses-revenues table would be as follows:

Table P3.5-b.a Expenses and Revenues exercising option contracts

Item	Energy bought (MWh)	Energy sold (MWh)	Price ($/MWh)	Expenses ($)	Revenue ($)
Future T4	600		20	12000	
Nuclear Unit	**0**		**16**	**0**	

Gas-fired unit	200		18	3600	
Forward T1		50	21		1050
Long-term T3		350	20		7000
Future T5		100	22		2200
Exercise call option T7	**200**		**22.5**	**4500**	
Spot sale T9		**50**	**28**		**1400**
Spot sale T10		**100**	**28**		**2800**
Residential customer		300	25.5		7650
Commercial customer		200	25		5000
Balancing spot purchase	**150**		**28**	**4200**	
Fee option T6			2	500	0
Fee option T7			2	400	0
Fee option T8			2	200	0
Balance	1150	1150		25400	27100
Profit					1700

b) Borduria Energy cannot exercise its option contracts and the expenses-revenues table would be as follows:

Table P3.5-b.b Expenses and Revenues not exercising option contracts

Item	Energy bought (MWh)	Energy sold (MWh)	Price ($/MWh)	Expenses ($)	Revenue ($)
Future T4	600		20	12000	
Nuclear Unit	**0**		**16**	**0**	
Gas-fired unit	200		18	3600	
Forward T1		50	21		1050
Long-term T3		350	20		7000
Future T5		100	22		2200
Exercise put option T6		250	23.5		5875
Spot sale T9		**50**	**28**		**1400**
Spot sale T10		**100**	**28**		**2800**
Residential customer		300	25.5		7650
Commercial customer		200	25		5000
Balancing spot purchase	**600**		**28**	**16800**	
Fee option T6				500	
Fee option T7				400	
Fee option T8				200	
Balance	1400	1400		33500	32975
Profit					-525