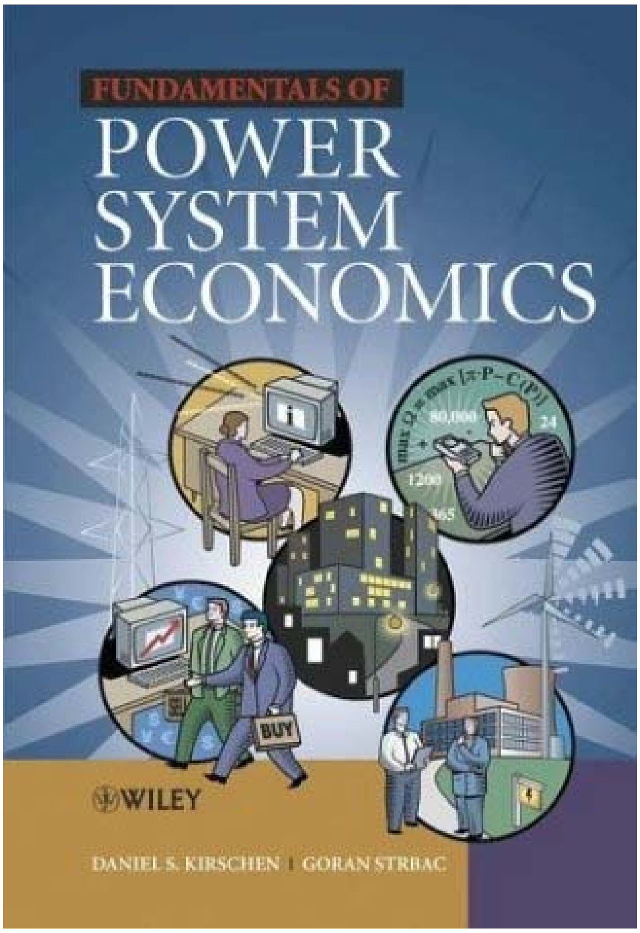
SOLUTIONS MANUAL



CHAPTER 8

INVESTING IN TRANSMISSION

MIGUEL A. ORTEGA-VAZQUEZ | DANIEL S. KIRSCHEN

Chapter 8

8.3 *Consider the two-bus power system shown in Figure* P8.1. *Assume that the demand is constant and insensitive to price and that energy is sold at its marginal cost of production and that there are no limits on the output of the generators. What is the maximum price that could be charged for transmission if the marginal costs of generation are as follows*?

$$MC_A = 25 \text{ \$/MWh}$$
$$MC_B = 17 \text{ \$/MWh}$$

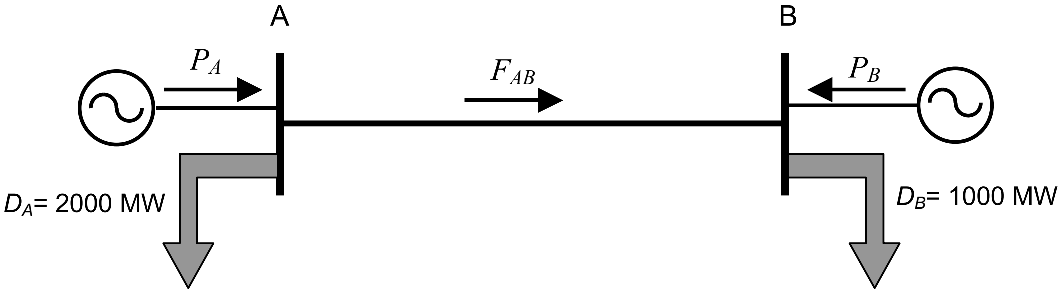


Figure P8.1: Two-bus power system for Problems 8.3, 8.4, 8.5, 8.6, 8.7, 8.8, and 8.9.

The maximum price that can be charged for transmission is the difference between the marginal cost of the energy at each of the buses; that is 25 – 17 = 8 \$/MWh. This is because it would not be in the best interest of the owner of the transmission line to charge more than 8 \$/MWh because such a charge would discourage consumers from making use of the transmission system.

8.4 *Consider the two-bus power system shown in Figure P8.1. Assume that the demand is constant and insensitive to price and that energy is sold at its marginal cost of production and that there are no limits on the output of the generators. The marginal cost of production of the generators connected to buses* A *and* B *are given respectively by the following expressions:*

$$MC_A = 20 + 0.03 P_A \text{ [\$/MWh]}$$
$$MC_B = 15 + 0.02 P_B \text{ [\$/MWh]}$$

Plot the marginal value of transmission as a function of the capacity of the transmission line connecting buses A *and* B.

The plot of the short run marginal value of transmission was obtained using the spreadsheet P8_4.xls.

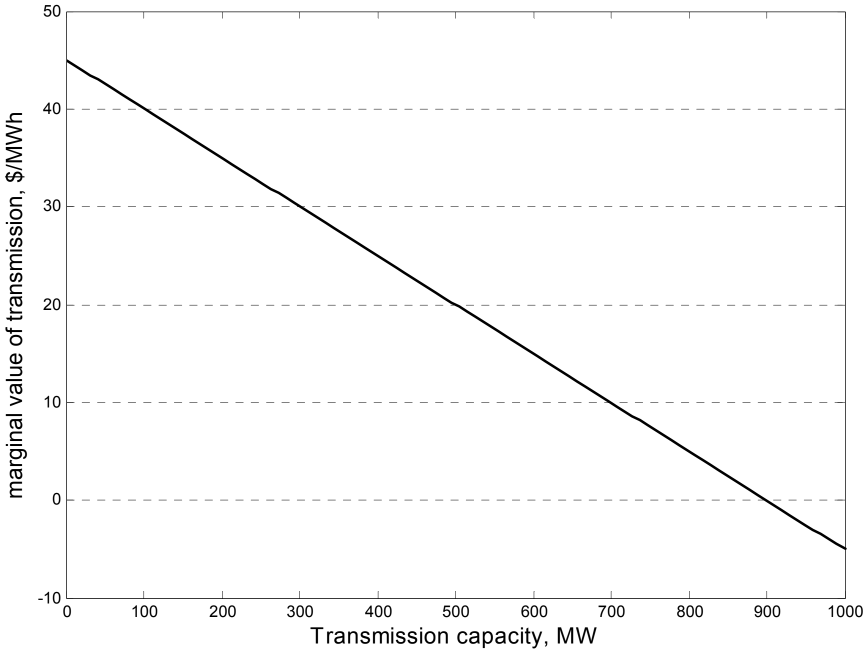


Fig. 8.2: Short run marginal value of transmission as a function of the transmission capacity for the case of Problem 8.4

8.5 Determine the transmission demand function for the system of Problem 8.4

In this particular example we can write the following two power flow equations:

$$F_{AB} = P_A - D_A$$
$$F_{BA} = -F_{AB} = P_B - D_B$$

Therefore:

$$P_A = F_{AB} + D_A$$
$$P_B = D_B - F_{AB}$$

Since the price that users of the transmission system are willing to pay is given by the difference between the prices for electrical energy at the two buses, we have:

$$\pi_T = 20 + 0.03P_A - 15 - 0.02P_B$$
$$\pi_T = 20 + 0.03F_{AB} + 60 - 15 + 0.02F_{AB} - 20$$
$$\pi_T = 45 + 0.05F_{AB}$$

Since the marginal cost at B is smaller than at the marginal cost at A the flow is positive from B to A, we should thus rewrite the equation in terms of F_{BA}:

$$\pi_T = 45 - 0.05 F_{BA}$$

This is the transmission demand function $\pi_T = 45 - 0.05 F_{BA}$.

8.6 *Calculate the hourly long range marginal cost of the transmission line of Problem 8.4 assuming that the line is* 500 km *long, that the amortized variable cost of building the line is* 210 \$/(MW x km x year).

The hourly long run marginal cost is given by:

$$c_T(T) = \frac{k\,l}{\tau_0} = \frac{210 \times 500}{8760} = 11.98 \approx 12.00 \text{ \$/MWh}$$

8.7 *Determine the optimal capacity of the transmission line of Problems 8.4, 8.5 and 8.6, assuming the loading conditions shown on Figure* P8.1.

The optimal transmission capacity is such that the supply and demand for transmission are in equilibrium, therefore:

$$\pi_T = c_T$$

Combining the transmission demand function ($\pi_T = 45 - 0.05 F_{BA}$) obtained in Problem *8.5* and the transmission supply function determined in Problem 8.6, we get:

$$12 = 45 - 0.05 F_{BA}$$

The optimal capacity for these conditions is thus 660 MW.

8.8 *Determine the optimal capacity of the transmission line of Problems 8.4, 8.5 and 8.6, for the three-part load duration curves summarized in the table below.*

Assume that the periods of high, medium and low load coincide at both buses.

Period	*Load at A [MW]*	*Load at B [MW]*	*Duration [hours]*
High	4000	2000	1000
Medium	2200	1100	5000
Low	1000	500	2760

> *Compare the amount of congestion revenue collected annually for this optimal transmission capacity with the annuitized cost of building the transmission line.*

We first need to find the unconstrained economic dispatch for the three load levels and the corresponding generating costs. The variables costs of each system are:

$$C_A = 20P_A + 0.03P_A^2 \ \text{\$/h}$$
$$C_B = 15P_B + 0.02P_B^2 \ \text{\$/h}$$

Therefore the Lagrangian function is:

$$\ell(P_A, P_B, \lambda) = 20P_A + 0.03P_A^2 + 15P_B + 0.02P_B^2 + \lambda(D - P_A - P_B)$$

And the optimality conditions are:

$$\frac{\partial \ell}{\partial P_A} = 20 + 0.03P_A - \lambda = 0$$
$$\frac{\partial \ell}{\partial P_B} = 15 + 0.02P_B - \lambda = 0$$
$$\frac{\partial \ell}{\partial \lambda} = L - P_A - P_B = 0$$

Writing these conditions as a linear system of equations of the form **Ax** = **b**, we get:

$$\begin{bmatrix} 0.03 & 0 & -1 \\ 0 & 0.02 & -1 \\ -1 & -1 & 0 \end{bmatrix} \begin{bmatrix} P_A \\ P_B \\ \lambda \end{bmatrix} = \begin{bmatrix} -20 \\ -15 \\ -L \end{bmatrix}$$

Solving these equations for the three load levels, we get the results shown in the table below:

Load (MW)	P_A (MW)	P_B (MW)	Total hourly generation cost (\$/h)
1500	500	1000	52,500
3300	1220	2080	186,780
6000	2300	3700	534,000

Since solving the rest of this problem by hand is somewhat tedious, it is best to use a spreadsheet such as the one found in file P8_8.xls.

This spreadsheet consists of four parts: one for each load level and one for the annual cost.

In each of the first three parts, the top lines show the load at each bus, the total load and the number of hours during which this load condition occurs.

The next lines show the unconstrained dispatch, i.e. if the transmission had an infinite capacity and the two parts of the system could be treated as a single bus. This part includes the output of each unit, the cost of each unit and the total cost.

The bottom part shows the optimal constrained dispatch for different values of the transmission capacity. In particular, it shows for each load level the output of each unit, the cost of each unit and the cost of the constraint that might be imposed by the transmission capacity. The cost of the constraint is calculated by taking the difference between the sum of the generation costs for the constrained conditions and the total cost for the unconstrained economic dispatch.

The first column of the fourth (rightmost) part of the spreadsheet shows the annual cost of the constraint. It is obtained by weighting the cost of constraint for each load level by the number of hours during which this level is assumed to happen. The next column shows the cost of building a line of the corresponding capacity. The last column is the total annual transmission cost, i.e. the sum of the annual constraint and investment costs. By inspecting this column we conclude that the optimal capacity for this system is 750MW because this capacity produces the minimum total cost.

For the revenue recovery, see the solution to Problem 8.9.

8.9 *Calculate the amount of congestion revenue collected annually for a transmission capacity* 33.3% *higher and* 33.3% *lower than the optimal transmission capacity calculated in Problem* 8.8. *Compare these values to the annuitized cost of building the transmission line*.

First, for the base case, which is for a transmission line of 750 MW capacity, the revenue recovery for each load period and for the whole year is shown on the table below. The dispatch of units A and B are obtained from the spreadsheet used for problem 8.8.

F	Load A	Load B	P_A	P_B	π_A	π_B	Hours	Total
750	4000	2000	3250	2750	117.5	70	1000	35625000
750	2200	1100	1450	1850	63.5	52	5000	43125000
750	1000	500	250	1250	27.5	40	2760	0
								78,750,000

The annuitized cost of building a line with a 750MW capacity is:

$$210 \times 500 \times 750 = \$\ 78,750,000$$

For a line with a capacity 33.33 % higher the numbers are as in the following table:

F	Load A	Load B	P_A	P_B	π_A	π_B	Hours	Total
1000	4000	2000	3000	3000	110	75	1000	35000000
1000	2200	1100	1200	2100	56	57	5000	0
1000	1000	500	0	1500	20	45	2760	0
								35,000,000

For comparison, the annuitized cost of building a line with a 1000 MW capacity is:

$$210\times500\times1000 = \$\ 105,000,000$$

Finally, for a line with a capacity 33.33 % lower, we have:

F	Load A	Load B	P_A	P_B	π_A	π_B	Hours	Total
500	4000	2000	3500	2500	125	65	1000	30000000
500	2200	1100	1700	1600	71	47	5000	60000000
500	1000	500	500	1000	35	35	2760	0
								90,000,000

For comparison, the annuitized cost of building a line with a 500 MW capacity is:

$$210\times500\times500 = \$\ 52,500,000$$

The congestion revenue is thus exactly equal to the annuitized cost of building the line when the capacity is optimal. If the capacity exceeds the optimum, the congestion revenue is below the annuitized cost. On the other hand, if the capacity is below the optimum, the congestion revenue exceeds the annuitized cost.