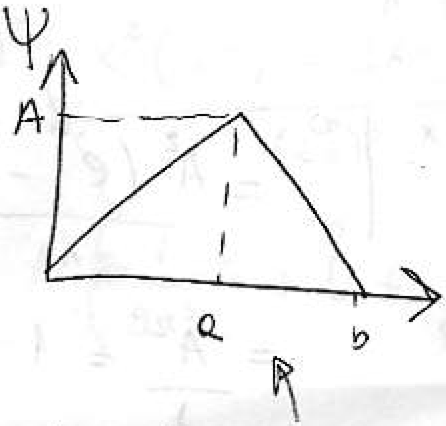
(P.1.7)

a) $$\int_{-\infty}^{\infty} |\psi|^2 dx = 1 \Rightarrow A = \sqrt{\frac{3}{b}}$$

b)



c) in $x = a$

d) $$\int_0^a |\psi(x,0)|^2 dx = \frac{A^2 \cdot a}{3} = \frac{a}{b}$$

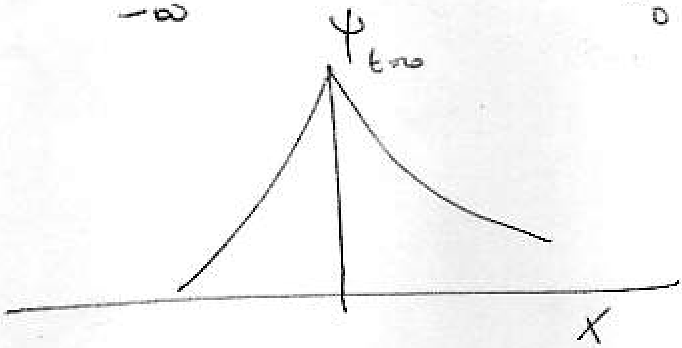
in $b = a \Rightarrow \frac{a}{a} = 1$

in $b = 2a \Rightarrow \frac{a}{2a} = \frac{1}{2}$

e) $$\int_0^b x\psi\, dx = \frac{\left(a \cdot \left(\sqrt{b} - 3\sqrt{\frac{1}{b}}\right) + 3\sqrt{\frac{1}{b}} \cdot b\right)\sqrt{3}}{6}$$

(P1,8) $\Psi(x,t) = A\, e^{-\lambda|x|}\, e^{-i\omega t}$

a) $$\int_{-\infty}^{\infty} |\Psi|^2 dx = A^2 \cdot 2 \int_0^{\infty} e^{-2\lambda x}\, dx = 1$$



$$= \cancel{2}A^2 \frac{1}{-\cancel{2}\lambda} e^{-2\lambda x}\Big|_0^{\infty} = \frac{A^2\,(e^{-\infty} - 1)}{-\lambda}$$

$$= \frac{A^2}{\lambda} = 1$$

$$\Rightarrow A = \sqrt{\lambda}$$

b) $\circ$ $$\int_{-\infty}^{\infty} x|\Psi|^2 dx = \int_{-\infty}^{\infty} x A^2 e^{-2\lambda|x|} dx = A^2 \int x e^{-2\lambda|x|} dx$$

$$= A^2 \int_{-\infty}^{0} x e^{(2\lambda x)} dx + A^2 \int_0^{+\infty} x e^{-2\lambda x} dx$$

$$\boxed{\text{con} \int x e^{yx} dx = \left(\frac{x}{y} - \frac{1}{y^2}\right) e^{xy}}$$

$$= A^2 \left(\frac{x}{2\lambda} - \frac{1}{4\lambda^2}\right) e^{2\lambda x}\Big|_{-\infty}^{0} + A^2 \left(\frac{x}{2\lambda} - \frac{1}{4\lambda^2}\right) e^{-2\lambda x}\Big|_0^{\infty}$$

$$= A^2 \left(\frac{-1}{4\lambda^2} - 0 + 0 + \frac{1}{4\lambda^2}\right) = A^2(0) = 0$$

$\circ$ $$\int_{-\infty}^{\infty} x^2|\Psi|^2 dx \Rightarrow \int x^2 |\Psi|^2 dx \Rightarrow \frac{(x^2y^2 - 2x\cdot y + 2)\, e^{xy}}{y^3}$$

$\int x^2 e^{xy} dx$

8)

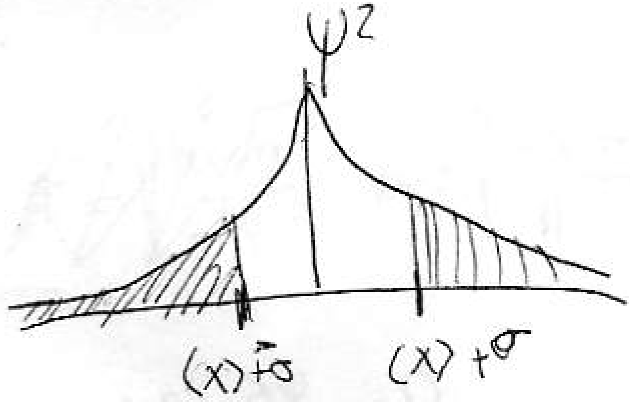
b) $\Rightarrow A^2\int_{-\infty}^{0} x^2 e^{2\lambda x}\,dx + A^2\int_{0}^{\infty} x^2 e^{-2\lambda x}\,dx$

$$= A^2\left.\frac{\left(x^2(2\lambda)^2 - 2x\cdot 2\lambda + 2\right)e^{x2\lambda}}{(2\lambda)^3}\right|_{-\infty}^{0} + A^2\frac{\left(x^2(-2\lambda)^2 + 2x2\lambda + 2\right)e^{-2\lambda x}}{(-2\lambda)^3}$$

$$= A^2\left(\frac{1}{8\lambda^3}\cdot 0 + 0 + \frac{1}{8\lambda^3} + 0\right) = A^2\left(\frac{2}{4\lambda^3}\right) = \frac{A^2}{2\lambda^3}$$

c) $\sigma^2 = \langle x^2\rangle - \underbrace{\langle x\rangle}_{0}{}^2$

$$\sigma = \sqrt{\frac{A^2}{2\lambda^3}} = \frac{A}{\sqrt{2\lambda^3}}$$



fuera del rango:

$$\mathbb{P} = \int_{-\infty}^{-\sigma} A^2 e^{2\lambda x}\,dx + \int_{\lambda}^{\infty} A^2 e^{-2\lambda x}\,dx = \left.\frac{A^2}{2\lambda}e^{2\lambda x}\right|_{-\infty}^{-\sigma} + \left.\frac{A^2}{-2\lambda}e^{-2\lambda x}\right|_{+\sigma}^{\infty}$$

$$= \frac{A^2}{2\lambda}e^{-2\lambda\sigma} + \frac{A^2}{-2\lambda}\cdot 0 - \frac{A^2}{-2\lambda}e^{-2\lambda\sigma} = \frac{2\cdot A^2}{2\lambda}e^{-2\lambda\sigma} = \frac{A^2 e^{-2\lambda\sigma}}{\lambda}$$

con $\sigma = \frac{A}{\sqrt{2\lambda^3}}$

(P1,14) $\Psi(x,t) = A e^{-a\left(\left(\frac{mx^2}{\hbar}\right)+it\right)}$ $\Psi\Psi^* = |\Psi|^2$

a) $\int_{-\infty}^{\infty} |\Psi|^2 dx = 1 = \int_{-\infty}^{\infty} A^2 e^{-2a\left(\frac{mx^2}{\hbar}+it\right)} dx$

$\boxed{\int_0^{+\infty} e^{-a^2x^2} dx = \frac{\sqrt{\pi}}{2a} \quad \underline{a>0}}$

$= \int_{-\infty}^{\infty} A^2 e^{-\frac{2amx^2}{\hbar}} dx = 1$

$\int_{-\infty}^{+\infty} e^{-ax^2} dx = \sqrt{\frac{\pi}{a}}$

$\Rightarrow A^2 \frac{\sqrt{\pi}\,\hbar}{\sqrt{2am}} = 1 \Rightarrow \cancel{A = \sqrt{\frac{2am}{\hbar\sqrt{\pi}}}}$ $A = \sqrt[4]{\frac{2am}{\pi\hbar}}$

b) S.E dp.t. $\frac{\partial^2\Psi}{\partial x^2} - \frac{2m}{\hbar^2}\Psi V + j\frac{2m}{\hbar}\frac{\partial\Psi}{\partial t} = 0$

$\frac{\partial\Psi}{\partial x} = A e^{-ait} e^{-a\left(\frac{mx^2}{\hbar}\right)} \cdot 2x \cdot \frac{-am}{\hbar}$

$\frac{\partial\Psi}{\partial t} = A e^{-\frac{amx^2}{\hbar}} \cdot e^{-ati} \cdot -ai$

$= \Psi \cdot -aj$

$\frac{\partial\Psi^2}{\partial x^2} = -\Psi' \cdot 2\frac{x\,am}{\hbar} \mp \Psi\, 2\cdot\frac{am}{\hbar}$

$= \Psi\left(\frac{2xam}{\hbar}\right)^2 - \Psi\frac{2am}{\hbar} = \Psi\left(\left(\frac{2xam}{\hbar}\right)^2 - \frac{2am}{\hbar}\right)$

$\Rightarrow \Psi\left(\left(\frac{2xam}{\hbar}\right)^2 - \frac{2am}{\hbar}\right) - \frac{2m}{\hbar^2}\Psi V + j\,\frac{2m}{\hbar}\Psi\cdot -aj = 0$

(1,14) $\left(\frac{2amx}{\hbar}\right)^2 - \cancel{\frac{2am}{\hbar}} - \frac{2m}{\hbar^2}V + \cancel{\frac{2am}{\hbar}} = 0$

$\cancel{\left(\frac{2amx}{\hbar}\right)^2 = \frac{2mV}{\hbar^2}}$ $\quad \left(\frac{2amx}{\hbar}\right)^2 = \frac{2mV}{\hbar^2}$

$$\boxed{2a^2mx^2 = V(x)}$$

c) $\langle x \rangle = 0$

$\langle x^2 \rangle = \cancel{A^2} \frac{\hbar}{4am} \cancel{\underbrace{\sqrt{\frac{\pi\hbar}{2am}}}_{A}} = \frac{\hbar}{4am}$

d) $\sigma_x = \sqrt{\langle x^2 \rangle - \cancel{\langle x \rangle^2}} = \sqrt{\frac{\hbar}{4am}}$

$\langle x \rangle^2 \to 0$