Physics of Electronics:

7. Junction Diodes

July – December 2008

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- IV characteristics of a junction diode
- Electron and hole efficiencies.
- Pn junction with finite dimensions.
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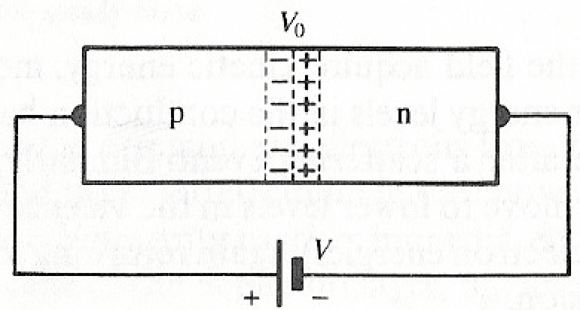
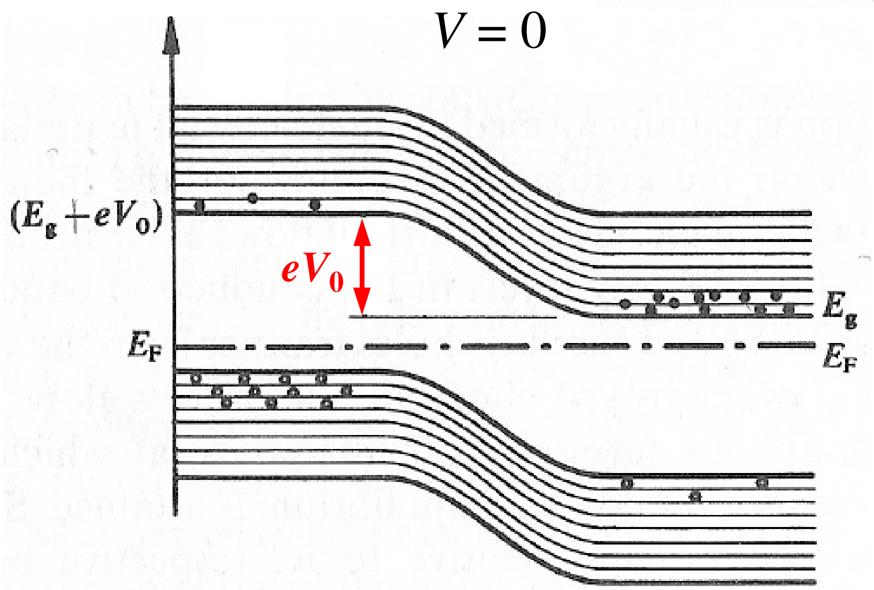
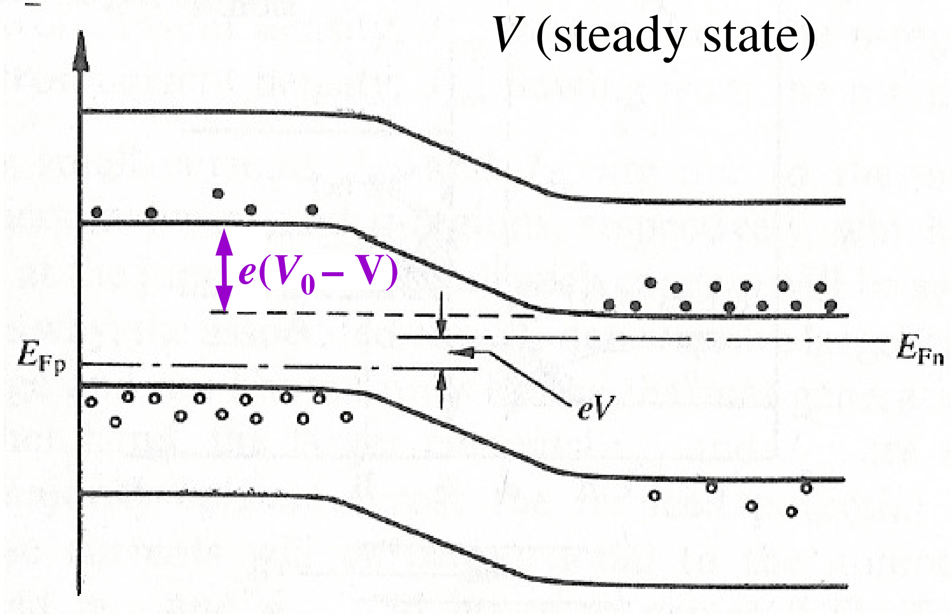
Pn junction with forward bias

- Consider a junction biased with a voltage V:





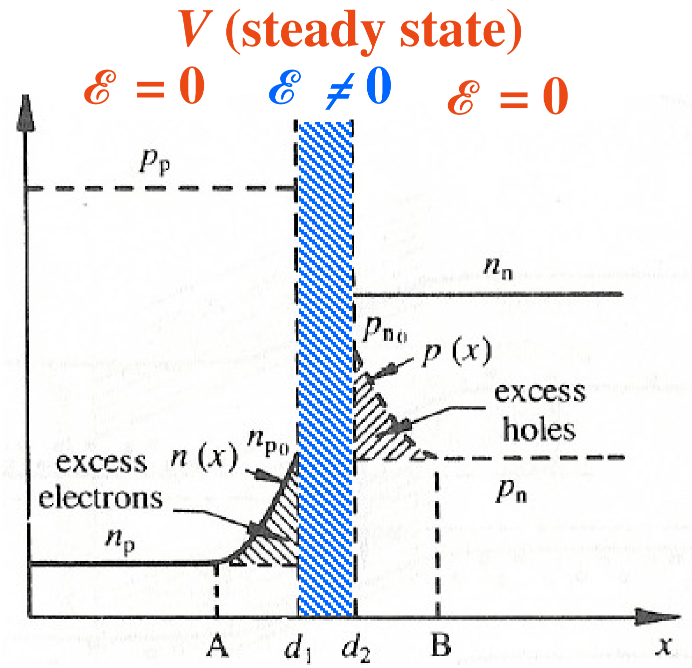


Same Fermi energy at both sides

Fermi energies separated by eV

Pn junction with forward bias

- Diode equation:
  - Continuity eq. for h^+ at $x \geq d_2$



$$\frac{\partial(\cancelto{0}{\delta p})}{\partial t} = -\frac{\delta p}{\tau_{Lh}} + \cancelto{0}{\mu_h \mathscr{E}_x} \frac{\partial(\delta p)}{\partial x} + D_h \frac{\partial^2(\delta p)}{\partial x^2}$$

$L_h{}^2 = \tau_{Lh} D_h$

$$\Rightarrow \delta p = C_1 \exp(-x/L_h) + \cancelto{0}{C_2} \exp(+x/L_h)$$

$$\Rightarrow p(x) = (p_{n0} - p_n)\exp(-x/L_h) + p_n$$

- Since $\mathscr{E} = 0$, there is only diffusion:

$$J_{Dh} = -eD_h \, dp/dx$$

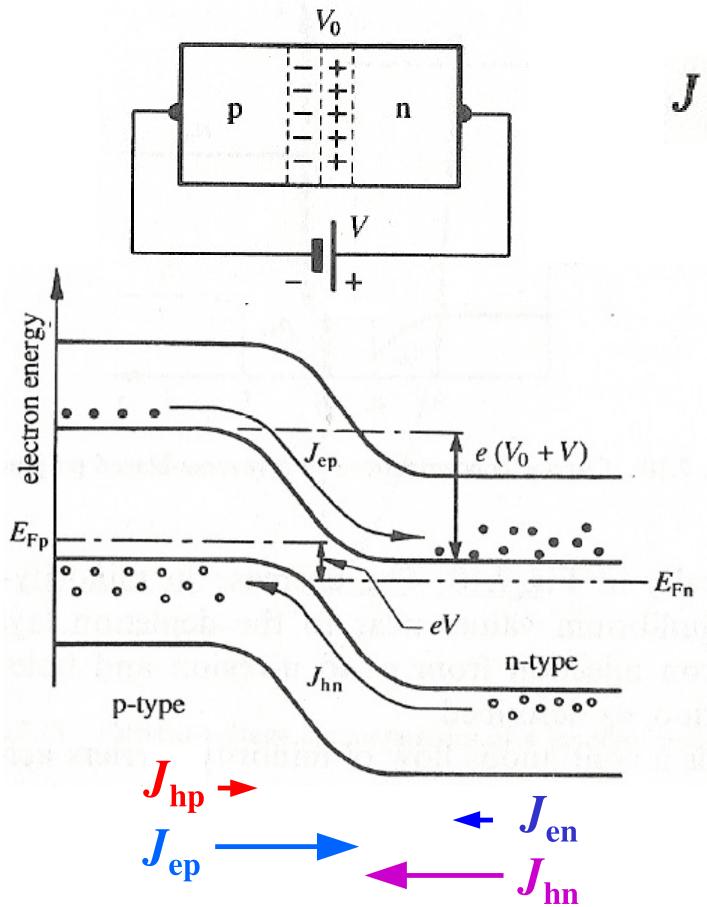
$$\Rightarrow J_h|_{d_2} = (eD_h/L_h)p_n \, [\exp(eV/kT) - 1]$$

  - Continuity eq. for e^- at $x \leq d_1$: $J_e|_{d_1} = (eD_e n_p/L_e) \, [\exp(eV/kT) - 1]$

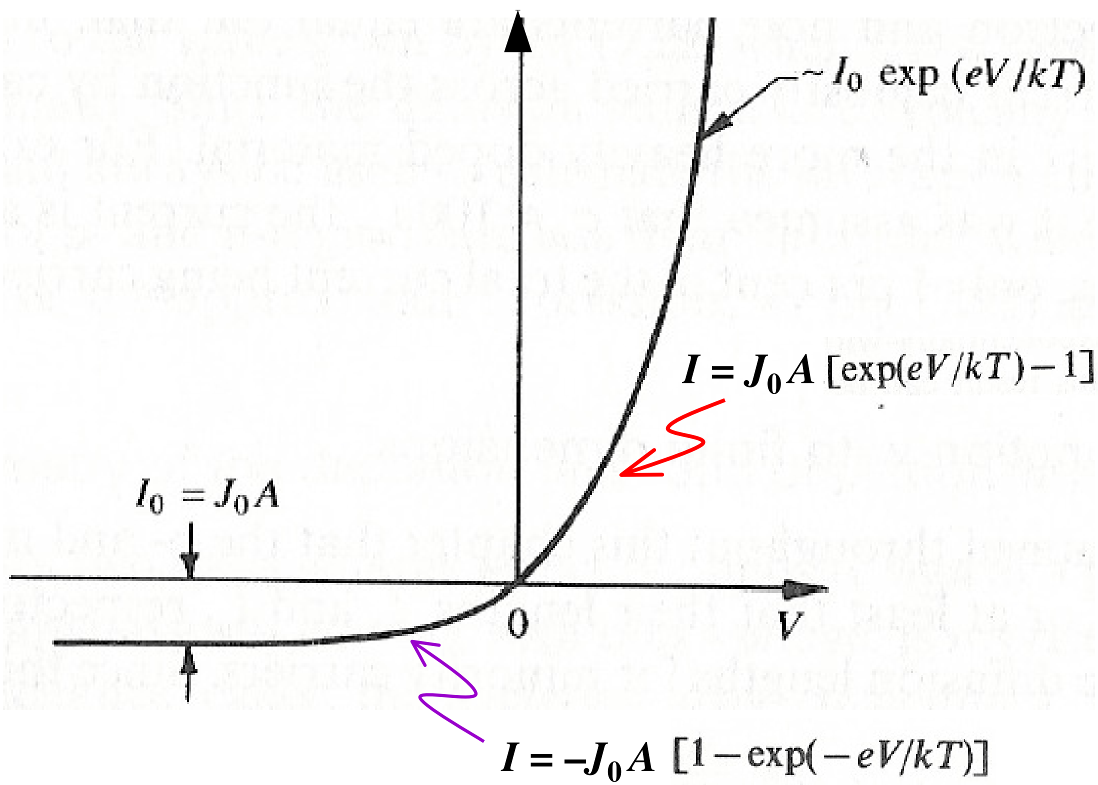
$$\boxed{J = J_h + J_e = e\left(\frac{D_h p_n}{L_h} + \frac{D_e n_p}{L_e}\right)[\exp(eV/kT) - 1]}$$

Pn junction with reversed bias

- Currents through the junction :
  - Applying diode equation (with $-V$):



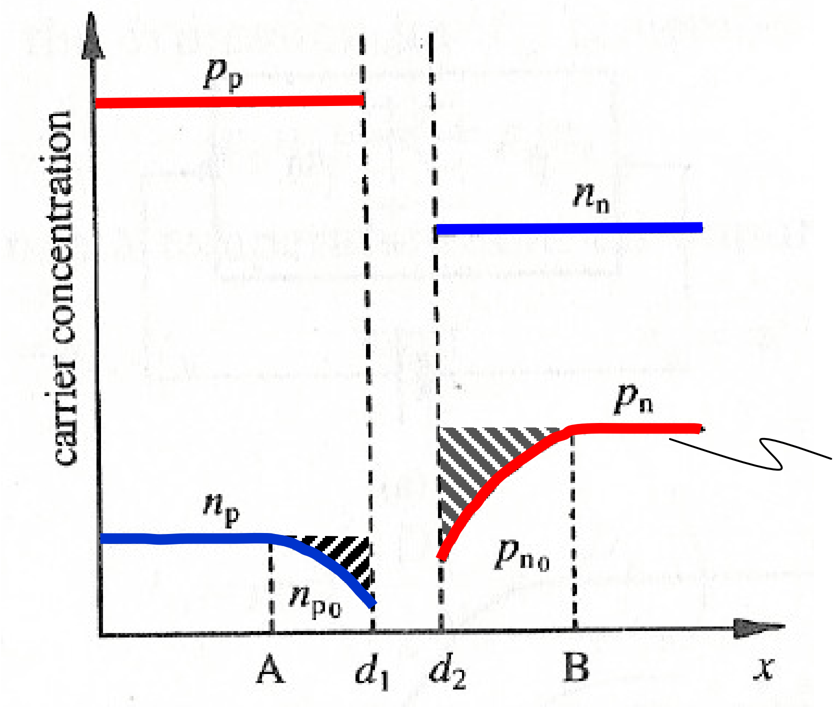
$$J = J_0[\exp(-eV/kT)-1] = -J_0[1-\exp(-eV/kT)]$$



$$J = \cancel{J_{hp}}^{0} - J_{hn} + \cancel{J_{en}}^{0} - J_{ep} \approx -J_0$$

Pn junction with reversed bias

- Decrease of minority carriers:
  - As before (with $-V$):



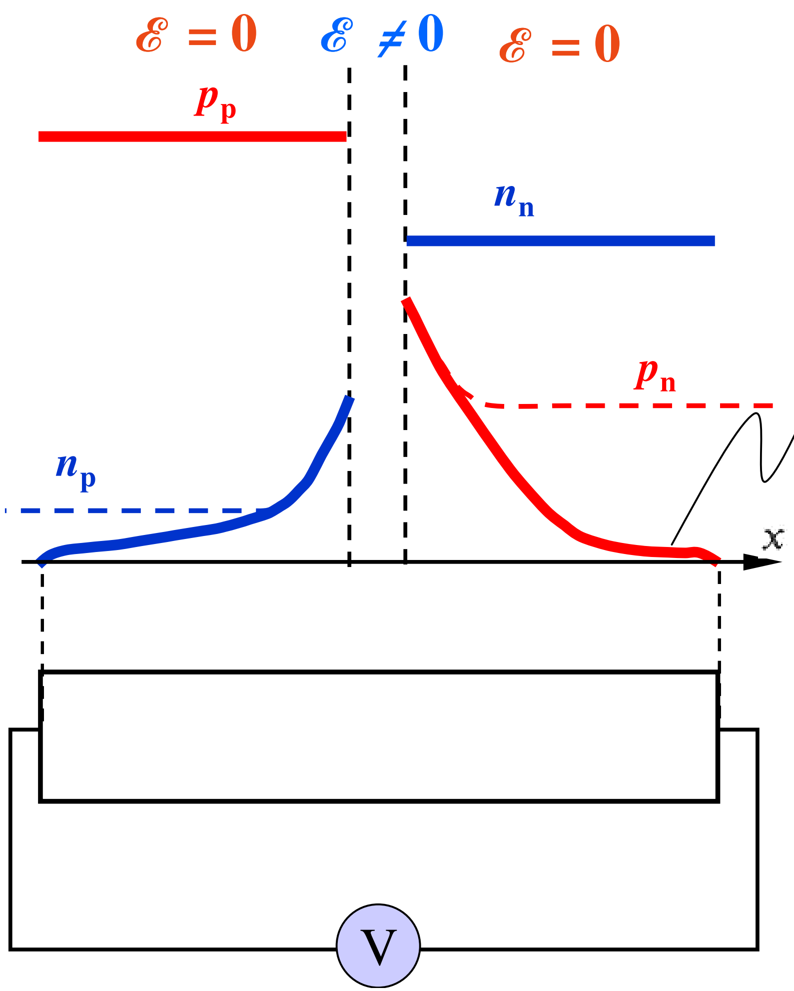
$$n_{p0} = n_p \exp(-eV/kT)$$

$$p_{n0} = p_n \exp(-eV/kT)$$

$$p(x) = p_n\{[\exp(-eV/kT) - 1]\exp(-x/L_h) + 1\}$$

Pn junction with finite dimensions

- Current in a finite junction:
  - From the continuity eq. we obtained



$$\delta p = C_1 \exp(-x/L_h) + C_2 \exp(+x/L_h)$$

$$p = p_{n0} = p_n \exp(eV/kT) \qquad \text{when } x = 0$$

$$p = 0 \qquad \text{when } x = l_n$$

$$p(x) = \left(\frac{p_n \exp(-l_n/L_h) + p_{n0} - p_n}{1 - \exp(-2l_n/L_h)}\right)\exp(-x/L_h)$$

$$-\left(\frac{p_n \exp(-l_n/L_h) + p_{n0} - p_n}{1 - \exp(-2l_n/L_h)} - (p_{n0} - p_n)\right)\exp(+x/L_h) + p_n$$

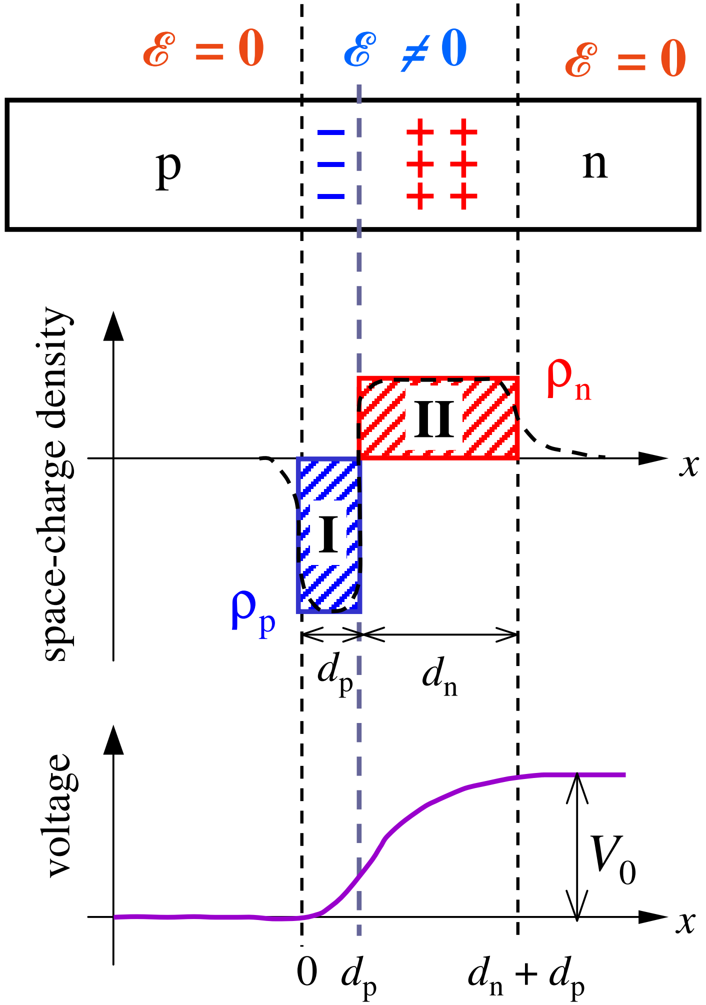
$$J_h = -eD_h\left(\frac{dp(x)}{dx}\right)\Bigg|_{x=0} = \frac{eD_h}{L_h} p_n \tanh\left(\frac{l_n}{L_h}\right)[\exp(eV/kT) - 1]$$

Idem for the electrons. Therefore the saturation current is:

$$J_0 = e\left[\frac{D_h p_n}{L_h}\tanh\left(\frac{l_n}{L_h}\right) + \frac{D_e n_p}{L_e}\tanh\left(\frac{l_p}{L_e}\right)\right]$$

Depletion-layer capacitance

- The depletion layer forms a capacitance (important for high frequency applications)



Using Poisson relation ($\partial^2 V/\partial x^2 = -\rho/\epsilon$) and continuity for voltage and its derivative:

I: $V_1(x) = eN_a x^2/2\epsilon$

II: $V_2 = -eN_d x^2/2\epsilon + C_1 x + C_2$

$$C_1 = ed_p(N_a + N_d)/\epsilon \qquad C_2 = -\frac{ed_p^2(N_a + N_d)}{2\epsilon}$$

Applying $\mathscr{E} = 0$ at $x \geq d_n + d_p$: $\boxed{d_p/d_n = N_d/N_a}$

Applying $V = V_0$ at $x \geq d_n + d_p$ & using previous relation:

$$\boxed{d_p = \left(\frac{2\epsilon V_0 N_d}{eN_a(N_a + N_d)}\right)^{1/2}} \qquad \boxed{d_n = \left(\frac{2\epsilon V_0 N_a}{eN_d(N_a + N_d)}\right)^{1/2}}$$

Depletion-layer capacitance

- Capacitance of the depletion layer:
  - Charge per unit area accumulated at the depletion layer

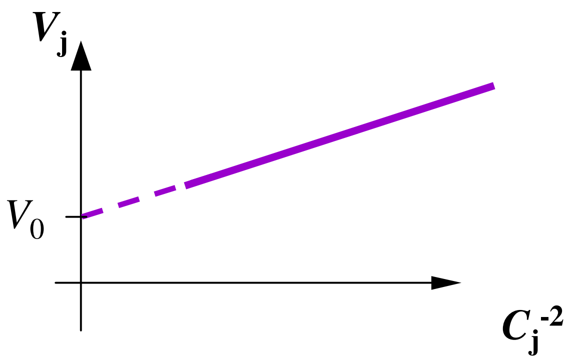
$$Q_j = eN_d d_n = eN_a d_p \xrightarrow{V_j = V_0 \pm V} Q_j = \left(\frac{2\epsilon e V_j N_a N_d}{N_a + N_d}\right)^{1/2}$$

  - The capacitance per unit area ($C_j = dQ_j/dV_j$) is then:

$$C_j = \left(\frac{\epsilon e N_a N_d}{2(N_a + N_d)}\right)^{1/2} \frac{1}{V_j^{1/2}} \Rightarrow \boxed{C_j = \epsilon/(d_p + d_n)}$$

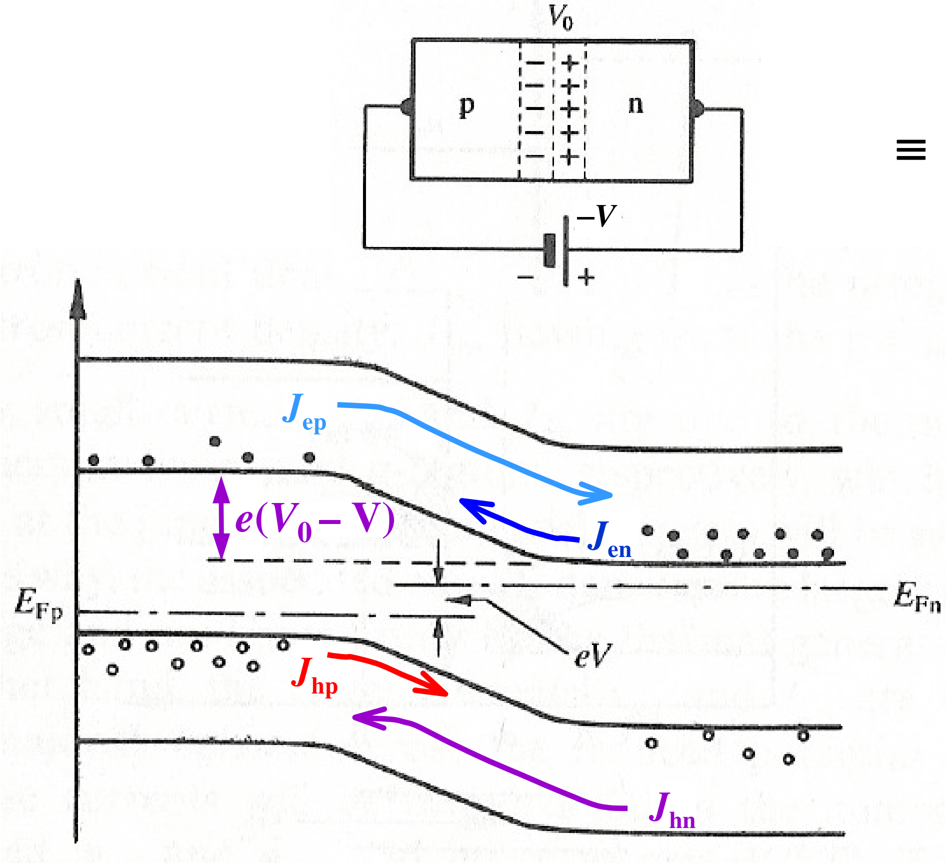
$$\Big\downarrow N_a \gg N_d:$$

$$V_j = V_0 + V \simeq \frac{1}{C_j^2}\left(\frac{\epsilon e N_d}{2}\right)^{1/2}$$



Small-AC-signal equivalent circuit

- Reversed biased junction



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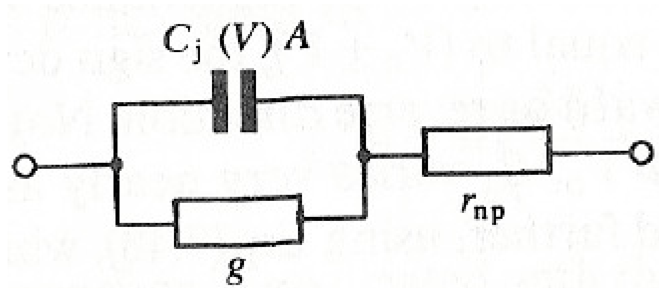


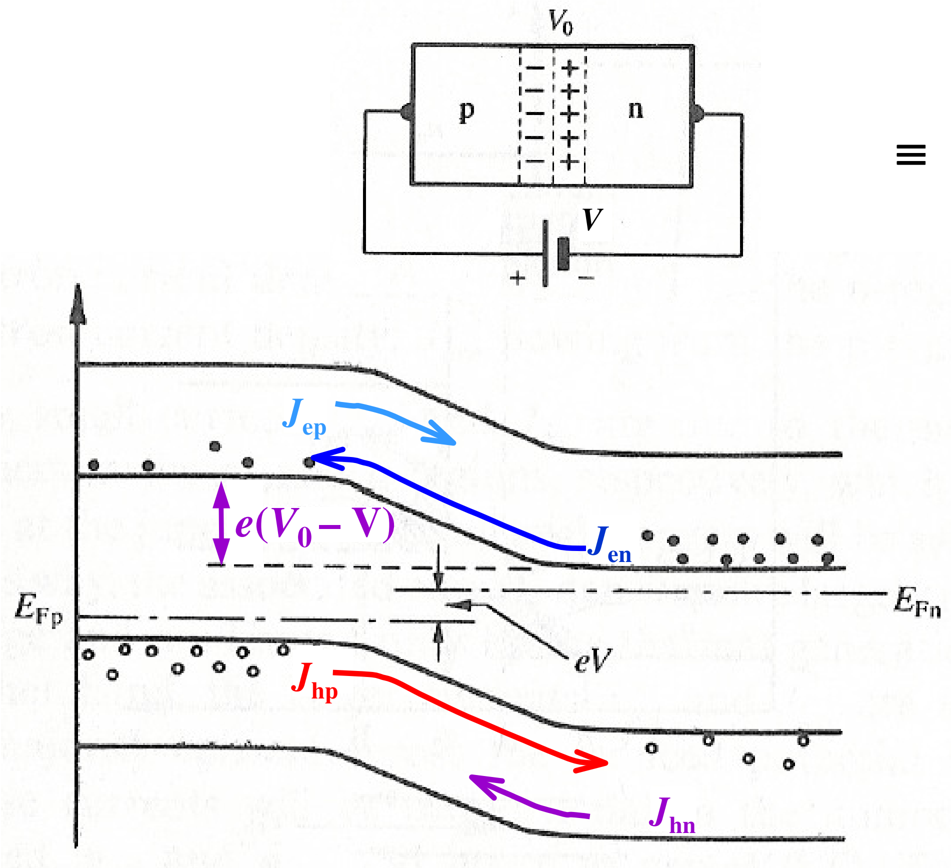
r_{np}: Resistance of the junction @ V=0

$C_j(V)$: Depletion layer capacitance

g: Conductance of the reverse flow

Small-AC-signal equivalent circuit

- Forward biased junction



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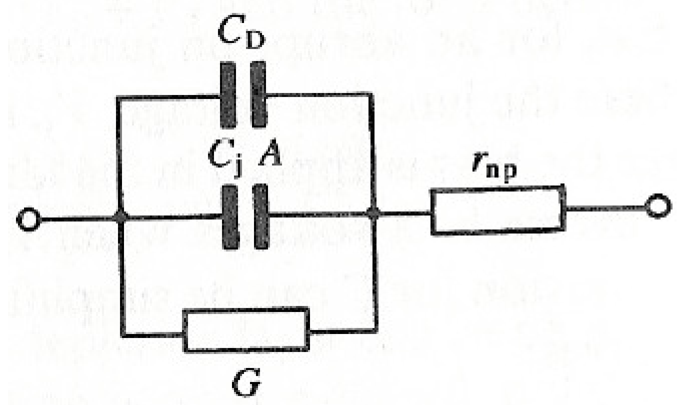


r_{np}: Resistance of the junction @ V=0

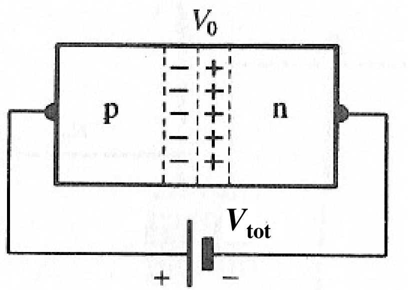
$C_j(V)$: Depletion layer capacitance

G: Conductance of the forward flow

C_D: Diffusion capacitance

Small-AC-signal equivalent circuit

- Forward biased junction



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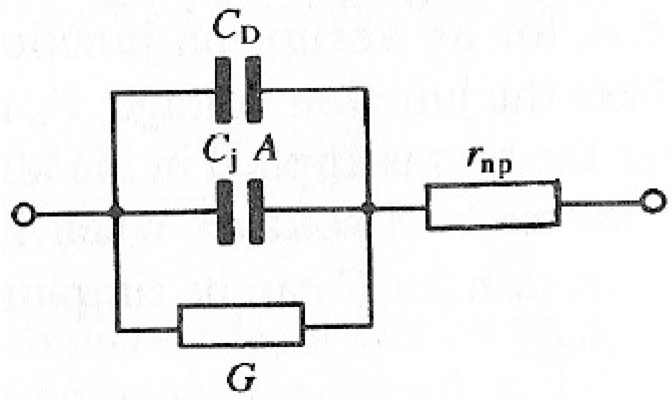


$$\left.\begin{array}{l} V_{\text{tot}} = V + V_1 \exp(i\omega t) \\ p_{n0} = p_n \exp(eV/kT) \end{array}\right\} \Rightarrow p_{n0} = p_n \exp\left(\frac{e}{kT}[V + V_1 \exp(j\omega t)]\right)$$

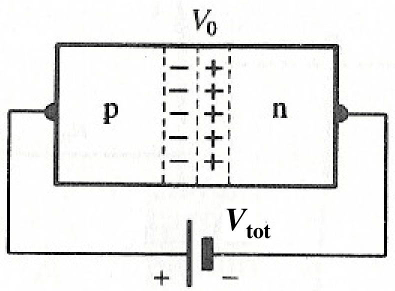
$$\xRightarrow{V_1 \ll V_0} \quad p_{n0} = \underbrace{p_n \exp\left(\frac{eV}{kT}\right)}_{\text{DC}} \underbrace{\left(1 + \frac{eV_1}{kT}\exp(j\omega t)\right)}_{\text{AC}}$$

- Therefore we can assume:

$$\delta p = p(x,t) - p_n = \underbrace{p_0(x)}_{\text{DC}} + \underbrace{p_1(x)\exp(j\omega t)}_{\text{AC}} - p_n$$

Small-AC-signal equivalent circuit

- Forward biased junction



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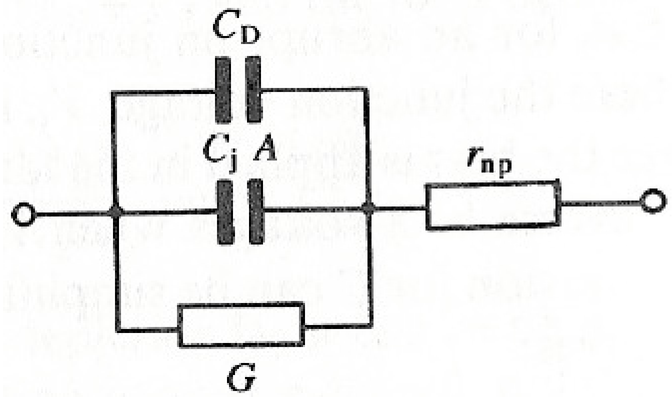


  – Applying continuity equation:

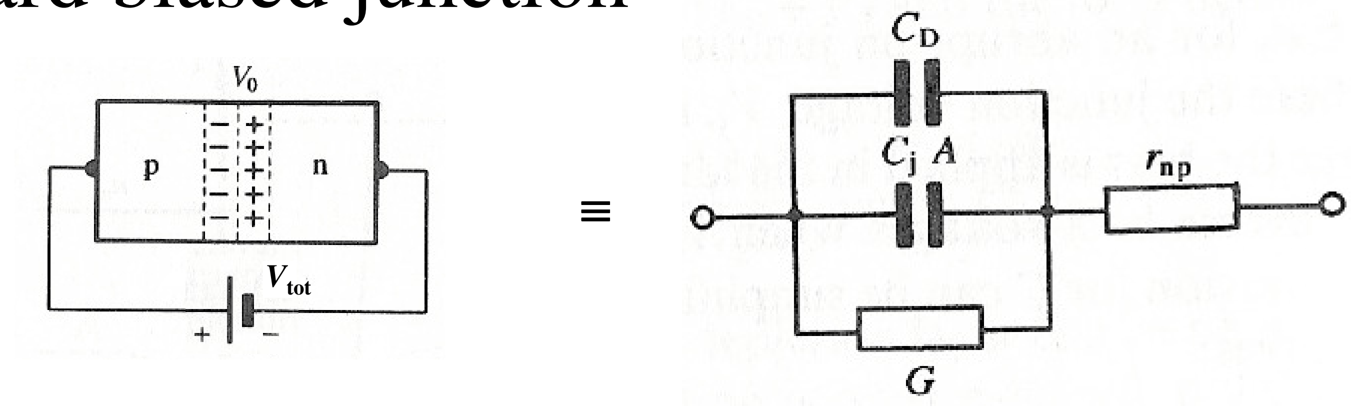
$$\frac{\partial(\delta p)}{\partial t} = -\frac{\delta p}{\tau_{Lh}} + \cancelto{0}{\mu_h \mathscr{E}_x \frac{\partial(\delta p)}{\partial x}} + D_h \frac{\partial^2(\delta p)}{\partial x^2} \Rightarrow \mathrm{j}\omega p_1 = \frac{-p_1}{\tau_{Lh}} + D_h \frac{\partial^2 p_1}{\partial x^2}$$

$$\Rightarrow \frac{\partial^2 p_1}{\partial x^2} = \frac{1+\mathrm{j}\omega\tau_{Lh}}{L_h^2} p_1 \Rightarrow p_1 = C\exp\left(-\frac{(1+\mathrm{j}\omega\tau_{Lh})^{1/2}x}{L_h}\right)$$

- But $p_1(x=0) = p_{n0}$ $\Rightarrow$ $\boxed{p_1 = p_{n}\left(\frac{eV_1}{kT}\right)\exp\left(\frac{eV}{kT}\right)\exp\left(-\frac{(1+\mathrm{j}\omega\tau_{Lh})^{1/2}x}{L_h}\right)\exp(\mathrm{j}\omega t)}$

Small-AC-signal equivalent circuit

- Forward biased junction



  - Alternating currents of holes injected into n-region:

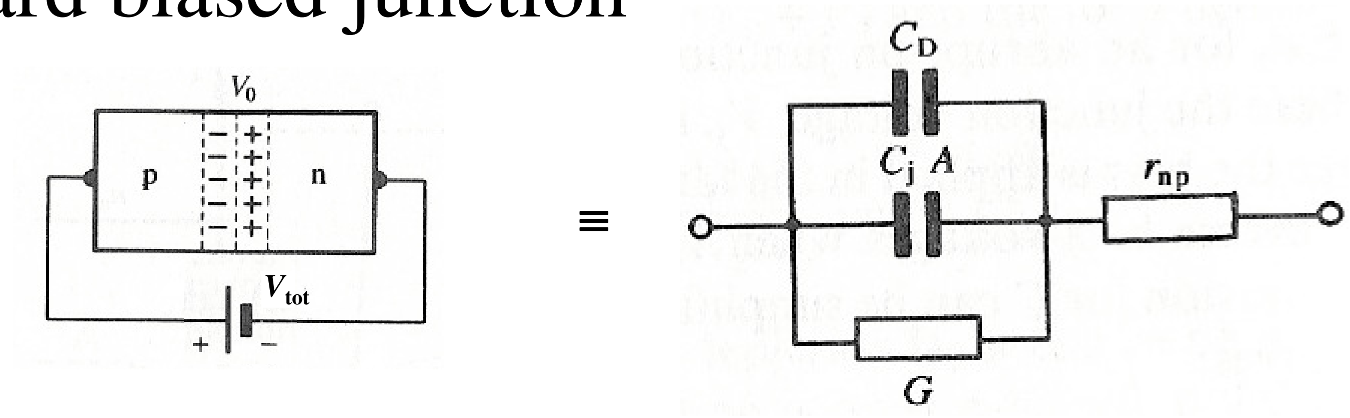
$$J_{h1} = -eD_h\left(\frac{dp_1(x)}{dx}\right)\bigg|_{x=0} = \frac{(1+i\,\omega\tau_{Lh})^{1/2}}{L_h}\,\frac{p_n D_h e^2 V_1}{kT}\exp\left(\frac{eV}{kT}\right)\exp(j\omega t)$$

  - Alternating currents of electrons injected into p-region:

$$J_{e1} = -eD_e\left(\frac{dn(x)}{dx}\right)\bigg|_{x=0} = \frac{(1+i\,\omega\tau_{Le})^{1/2}}{L_e}\,\frac{n_p D_e e^2 V_1}{kT}\exp\left(\frac{eV}{kT}\right)\exp(j\omega t)$$

Small-AC-signal equivalent circuit

- Forward biased junction



  – Total alternating currents of minority injected carriers:

$$J_1 = J_{h1} + J_{e1}$$

  – Admittance ($Y_1 = J_1/V_1$) for $\omega\tau \ll 1$:

$$Y_1 \cong \underbrace{\frac{e^2}{kT}\exp\left(\frac{eV}{kT}\right)\left(\frac{D_h p_n}{L_h} + \frac{D_e n_p}{L_e}\right)}_{G} + \underbrace{\frac{j\omega e^2}{2kT}\left(\frac{D_h p_n \tau_{Lh}}{L_h} + \frac{D_e n_p \tau_{Le}}{L_e}\right)}_{C_D}$$