Physics of Electronics:

7. Junction Diodes

July – December 2008

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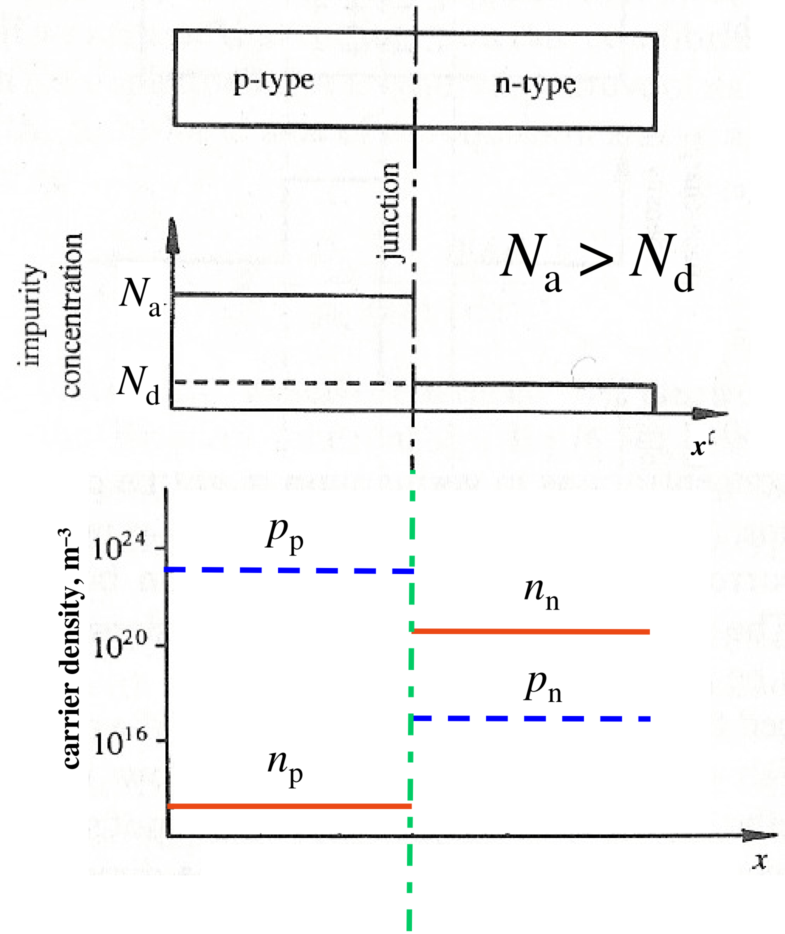
- Pn junction in equilibrium.
- Current flow in a pn junction with forward bias.
- Current flow in a pn junction with reversed bias.
- IV charachteristics of a junction diode
- Electron and hole efficiencies.
- Pn junction with finite dimensions.
- Depletion-layer capacitance

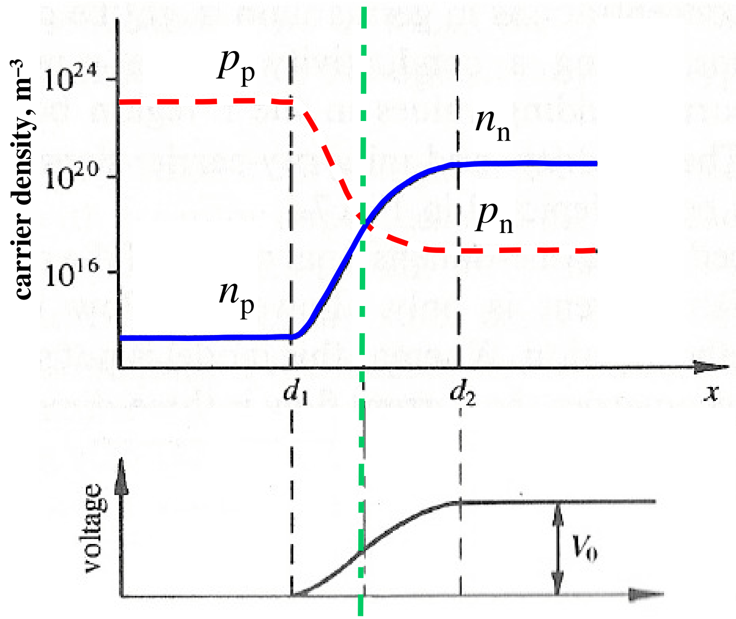
Pn junction in equilibrium

- Consider a pn junction with abrupt transition



Initially, the density of carriers change abruptly at the junction.

This change of concentration causes diffusion which is stopped by exposition of ionized impurities at the depletion layer.
In equilibrium, a voltage difference is created across the depletion layer.

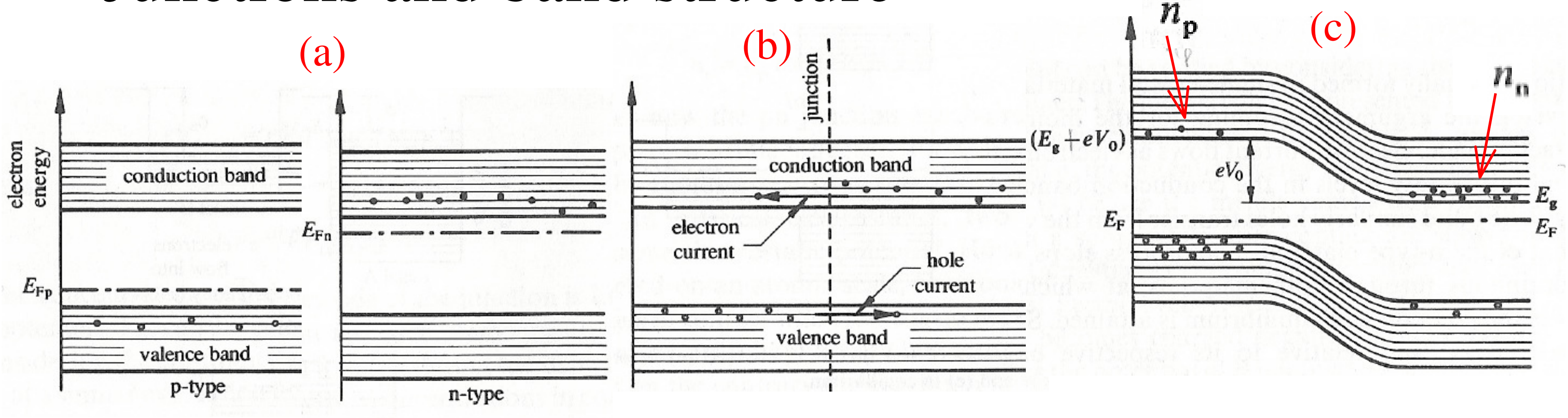


Pn junction in equilibrium

- Voltage across the depletion layer
  - Start with continuity eq. for holes

$$\cancelto{0}{\frac{\partial(\delta p)}{\partial t}} = -\cancelto{0}{\frac{\delta p}{\tau_{Lh}}} + \mu_h \mathscr{E}_x \frac{\partial(\delta p)}{\partial x} + D_h \frac{\partial^2(\delta p)}{\partial x^2} \xrightarrow[\text{\& integrating}]{D_h/\mu_h = kT/e} \boxed{p_p = p_n \exp(eV_0/kT)}$$

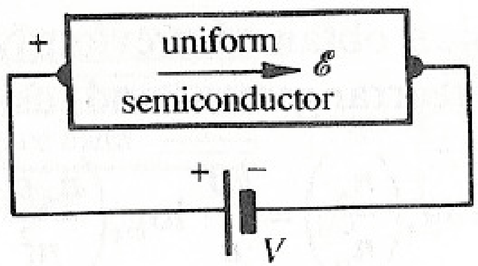
- Junctions and band structure



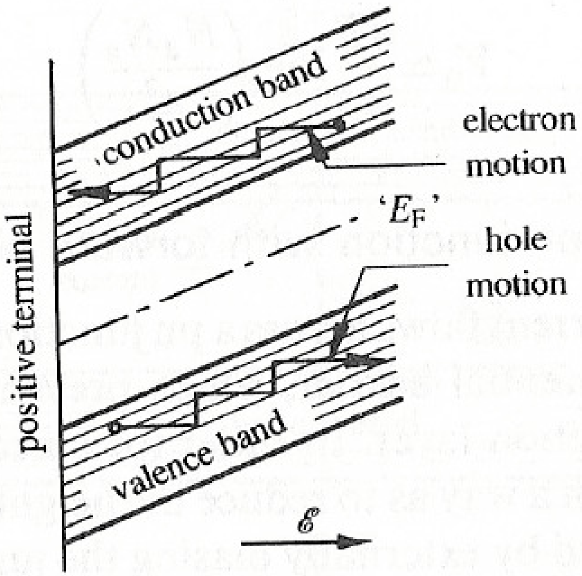
$$n_n/n_p = \exp(eV_0/kT)$$

Pn junction with forward bias

- Consider a uniform sC biased with a voltage V:



(a)



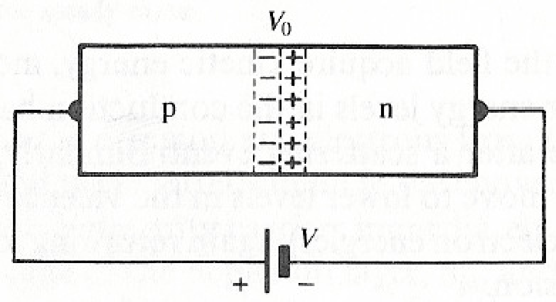
(b)

The whole band structure tilts (one end of the sC has more energy than the other) .

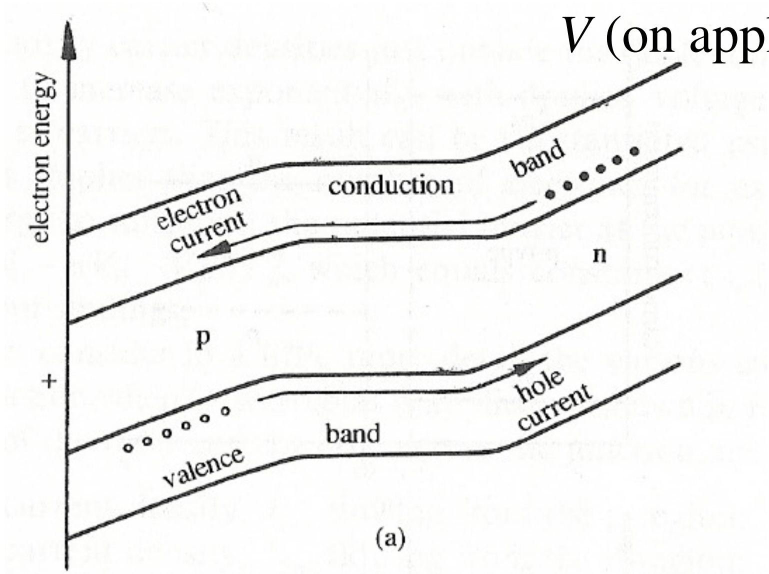
Electrons and holes move in the field (in opposite directions.

Pn junction with forward bias

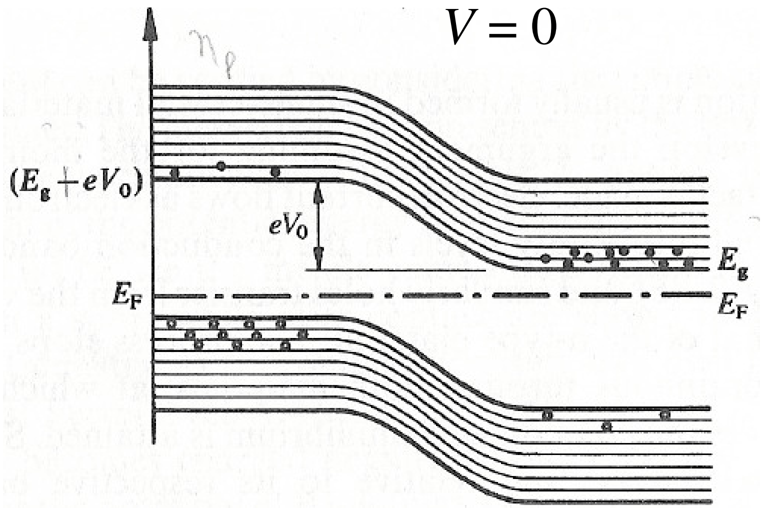
- Consider a junction biased with a voltage V:



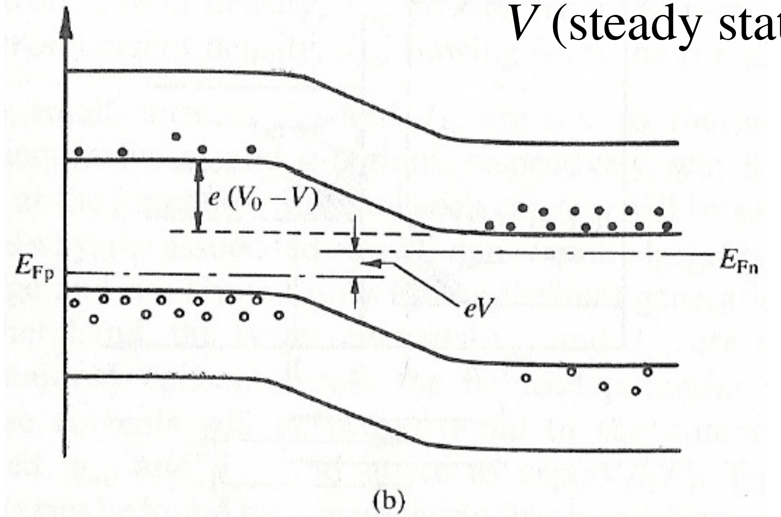
V (on application)



$V = 0$

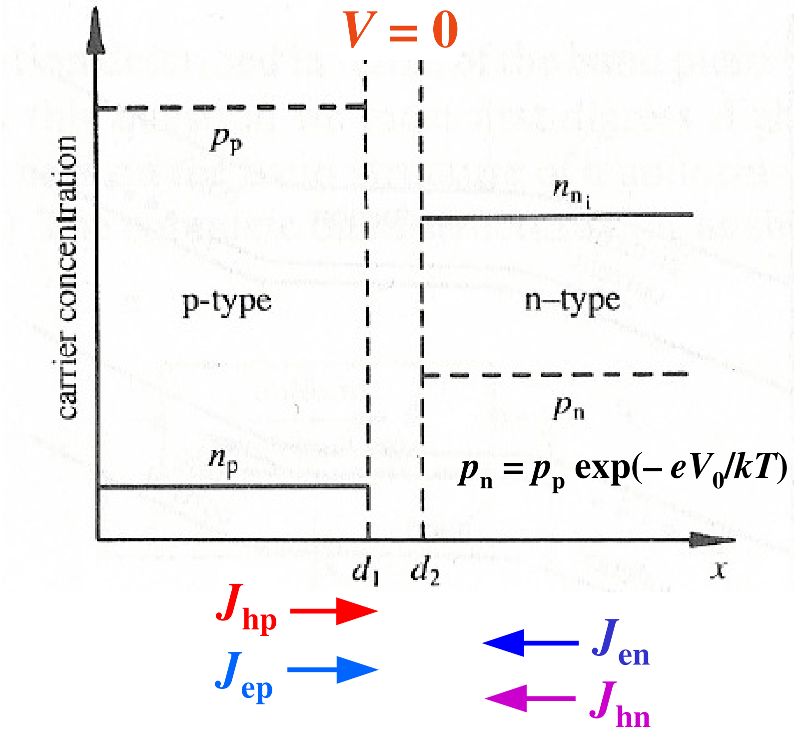


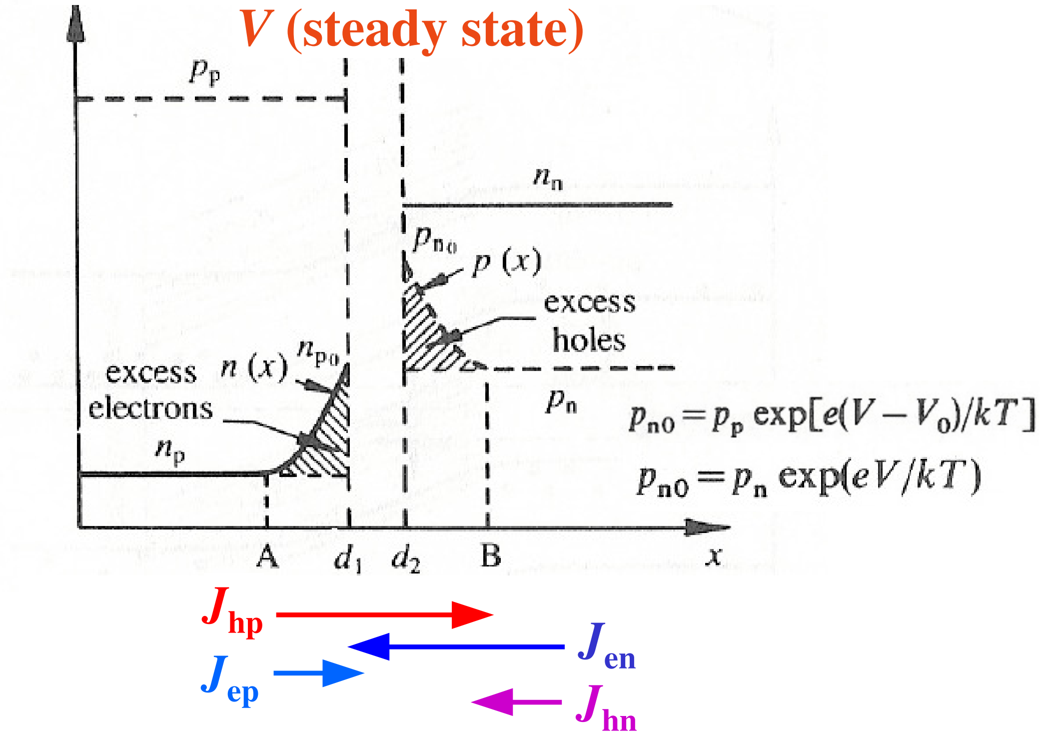
V (steady state)



Pn junction with forward bias

- Excess carriers in a biased junctions:





$$J_{hp} \propto p_{n0} = p_n \exp(eV/kT)$$

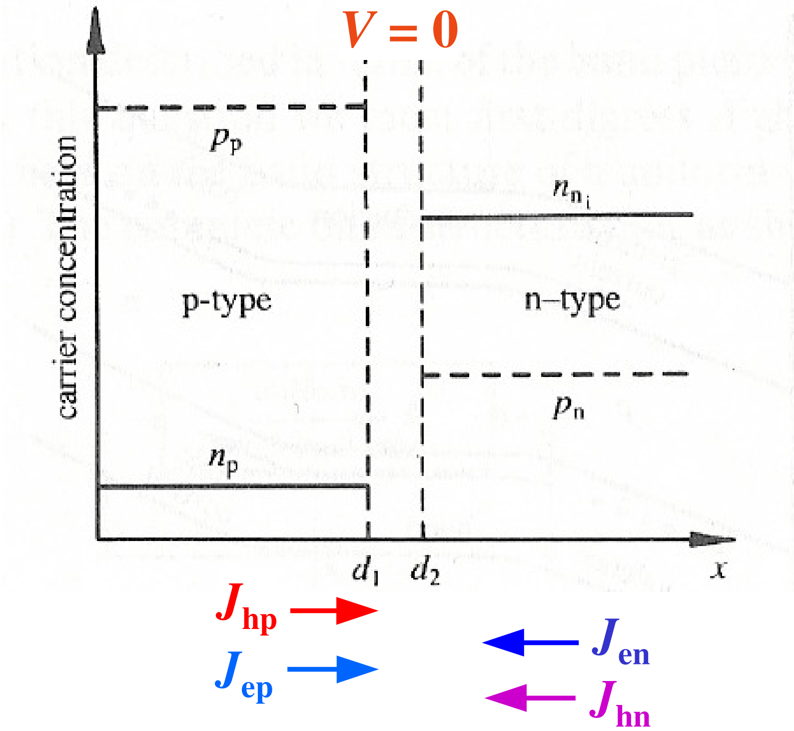
$$J_{hp} = J_{hn} \Rightarrow J_{hp} = J_{hn} \exp(eV/kT)$$

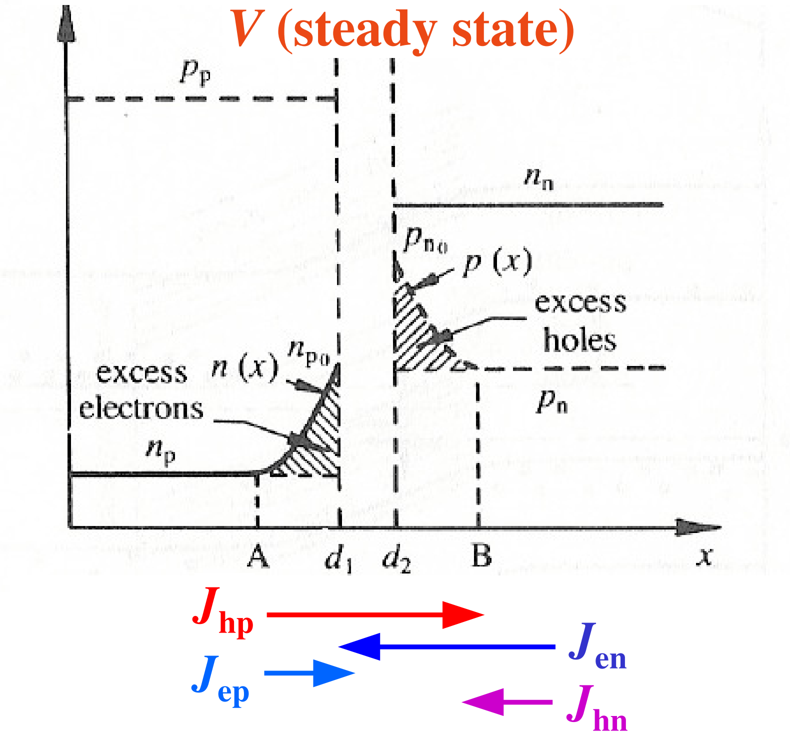
$$J_{en} \propto n_{p0} = n_p \exp(eV/kT)$$

$$J_{en} = J_{ep} \Rightarrow J_{en} = J_{ep} \exp(eV/kT)$$

Pn junction with forward bias

- Excess carriers in a biased junctions:





$$J_{hp} = J_{hn}$$

$$J_{en} = J_{ep}$$

Total currents:

$$J_h = J_{hp} - J_{hn} = J_{hn}[\exp(eV/kT) - 1]$$

$$J_e = J_{en} - J_{ep} = J_{ep}[\exp(eV/kT) - 1]$$

$$\boxed{J = J_0[\exp(eV/kT) - 1]}$$

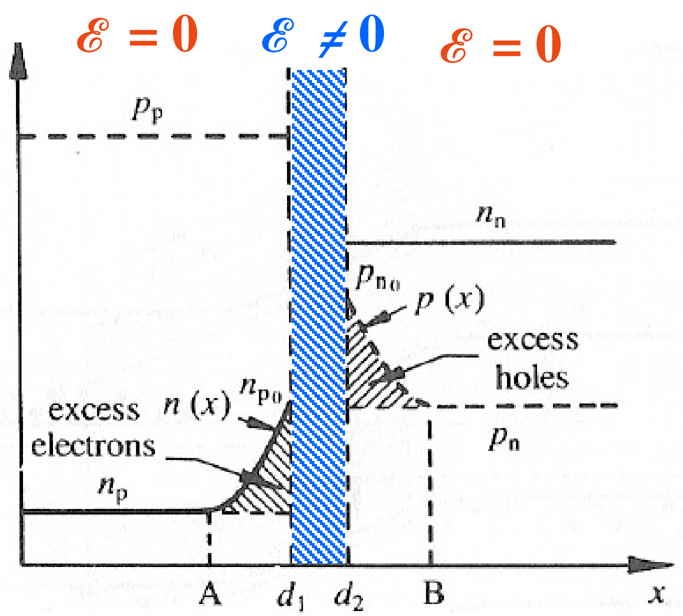
Diode equation

$$J_0 = J_{hn} + J_{ep}$$

Pn junction with forward bias

- Diode equation:
  - Continuity eq. for h^+ at $x \geq d_2$

V **(steady state)**



$$\cancelto{0}{\frac{\partial(\delta p)}{\partial t}} = -\frac{\delta p}{\tau_{Lh}} + \cancelto{0}{\mu_h \mathscr{E}_x \frac{\partial(\delta p)}{\partial x}} + D_h \frac{\partial^2(\delta p)}{\partial x^2}$$

$$\Rightarrow \quad \frac{d^2(\delta p)}{dx^2} = \frac{\delta p}{\tau_{Lh} D_h} = \frac{\delta p}{L_h^2}$$

$$\delta p = C_1 \exp(-x/L_h) + C_2 \exp(+x/L_h)$$

But $\delta p = p(x) - p_n \to 0$ when $x \to \infty$, then $C_2 = 0$ and changing origin to $x = d_2$ where $p(x) = p_{n0}$:

$$\Rightarrow \quad p(x) = (p_{n0} - p_n)\exp(-x/L_h) + p_n$$

- Since $\mathscr{E} = 0$, there is only diffusion:

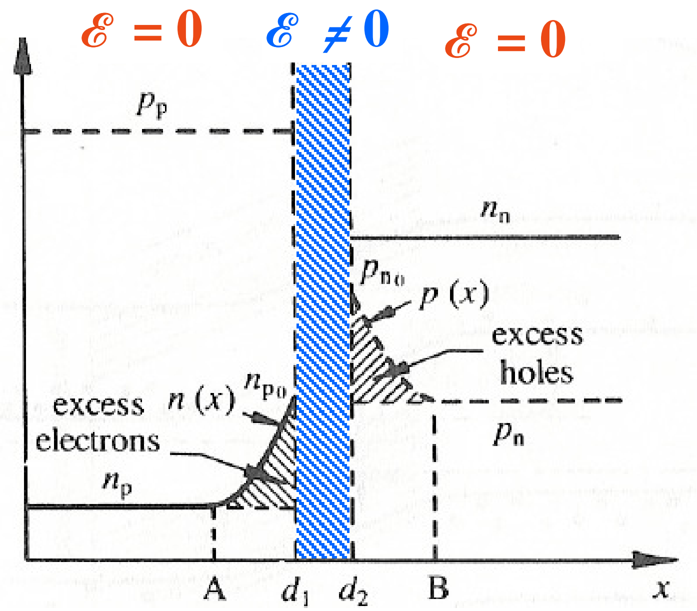
$$J_{Dh} = -eD_h dp/dx \quad \Rightarrow \quad J_h = -eD_h[-(1/L_h)(p_{n0} - p_n)\exp(-x/L_h)]$$

$$\xrightarrow{x=0} \quad J_h|_{d_2} = (eD_h/L_h)(p_{n0} - p_n) \quad \xrightarrow{p_{n0} = p_n \exp(eV/kT)} \quad J_h|_{d_2} = (eD_h/L_h)p_n\,[\exp(eV/kT) - 1]$$

Pn junction with forward bias

- Diode equation:
  - Continuity eq. for e^- at $x \leq d_1$:

V (steady state)



$$J_e|_{d_1} = (eD_e n_p/L_e)\,[\exp(eV/kT) - 1]$$

Total current is then:

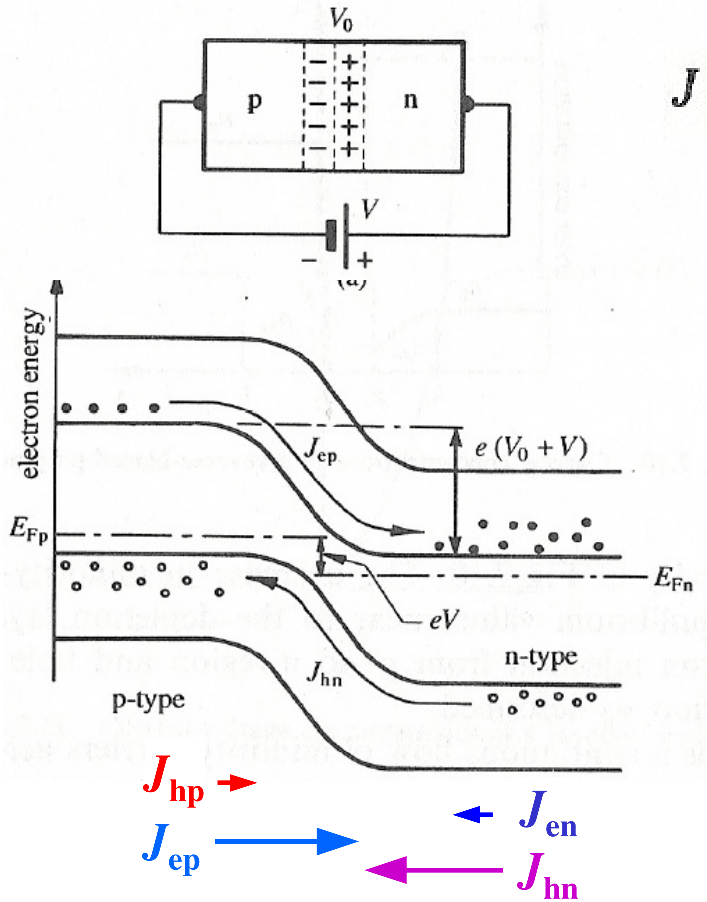
$$\boxed{J = J_h + J_e = e\underbrace{\left(\frac{D_h p_n}{L_h} + \frac{D_e n_p}{L_e}\right)}_{J_0}[\exp(eV/kT) - 1]}$$

For $V >> kT$:

$$J \simeq J_0 \exp(eV/kT)$$

Pn junction with reversed bias

- Currents through the junction :
  - Applying diode equation (with $-V$):



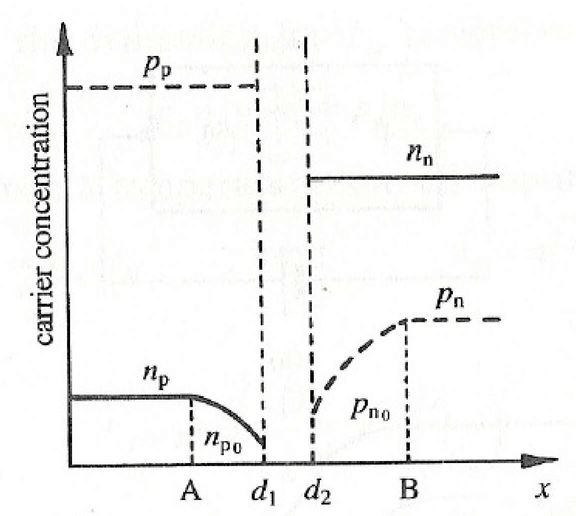
$$J = J_0[\exp(-eV/kT) - 1] = -J_0[1 - \exp(-eV/kT)]$$

For $V >> kT$: $\quad J \approx -J_0$

$$J = \cancelto{0}{J_{hp}} - J_{hn} + \cancelto{0}{J_{en}} - J_{ep} \approx -J_0$$

Pn junction with reversed bias

- Minority carriers:
  - As before (with –V):



$$n_{p0} = n_p \exp(-eV/kT)$$

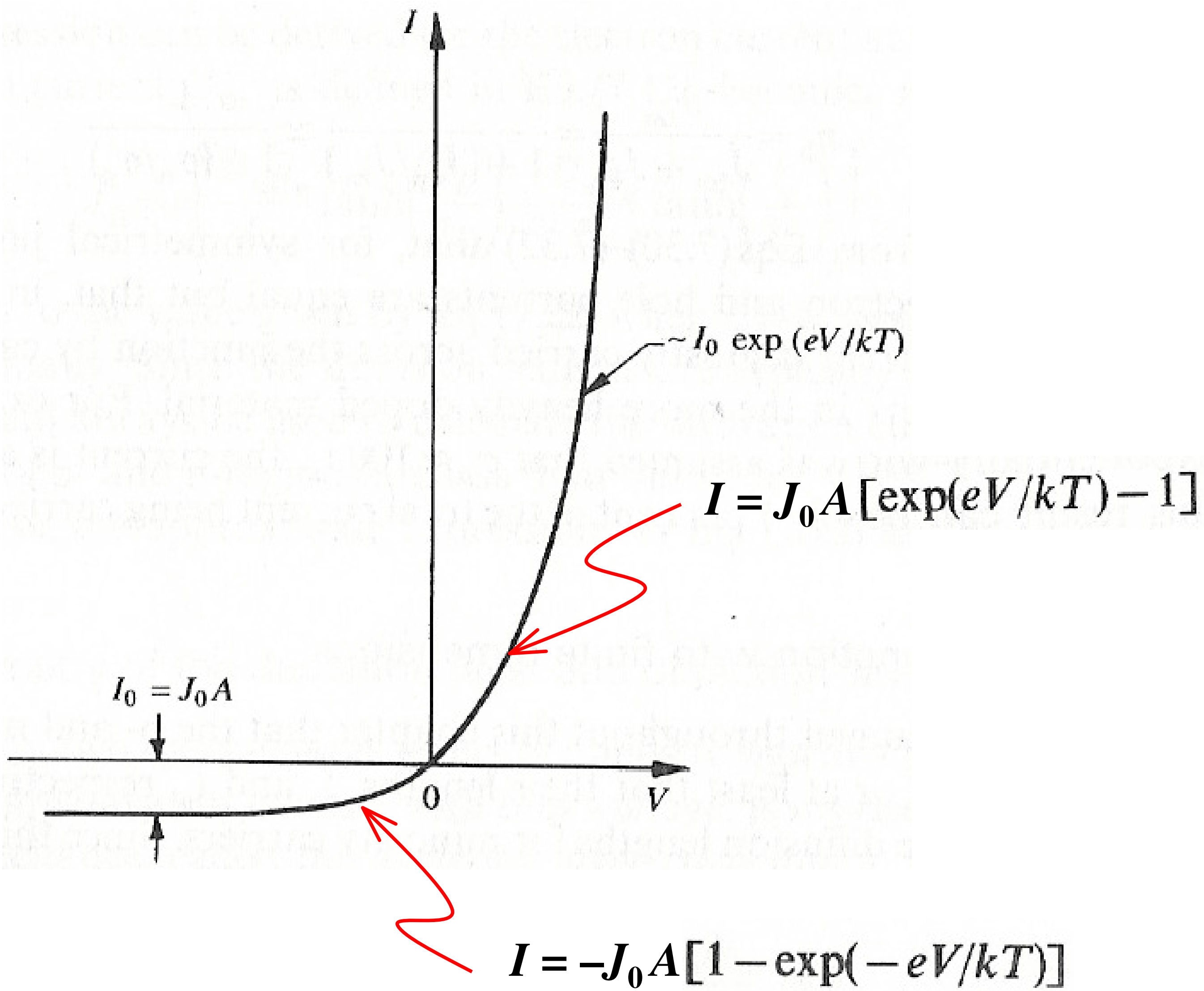
$$p_{n0} = p_n \exp(-eV/kT)$$

  - Applying continuity eq. as before

$$p(x) = p_n\{[\exp(-eV/kT) - 1]\exp(-x/L_h) + 1\}$$

I-V curve of a junction diode

- Applying the results for the current density:



Electron-holes efficiencies

- Relative hole and electron currents:
  - Excess carriers:

$$\left.\begin{aligned} J_h &= J_{hp} - J_{hn} = J_{hn}[\exp(eV/kT)-1] \\ J_e &= J_{en} - J_{ep} = J_{ep}[\exp(eV/kT)-1] \end{aligned}\right\} \qquad \frac{J_h}{J_e} = \frac{J_{hn}}{J_{ep}}$$

  - Continuity equations:

$$\left.\begin{aligned} J_h|_{d_2} &= (eD_h/L_h)p_n\,[\exp(eV/kT)-1] \\ J_e|_{d_1} &= (eD_e n_p/L_e)\,[\exp(eV/kT)-1] \end{aligned}\right\} \qquad \frac{J_h}{J_e} = \frac{J_{hn}}{J_{ep}} = \frac{D_h}{L_h N_d}\frac{L_e N_a}{D_e}$$

  - Einstein ($D_e/\mu_e = D_h/\mu_h = kT/e$) and conductivity ($\sigma = e(n\mu_e + p\mu_h)$) eqs.:

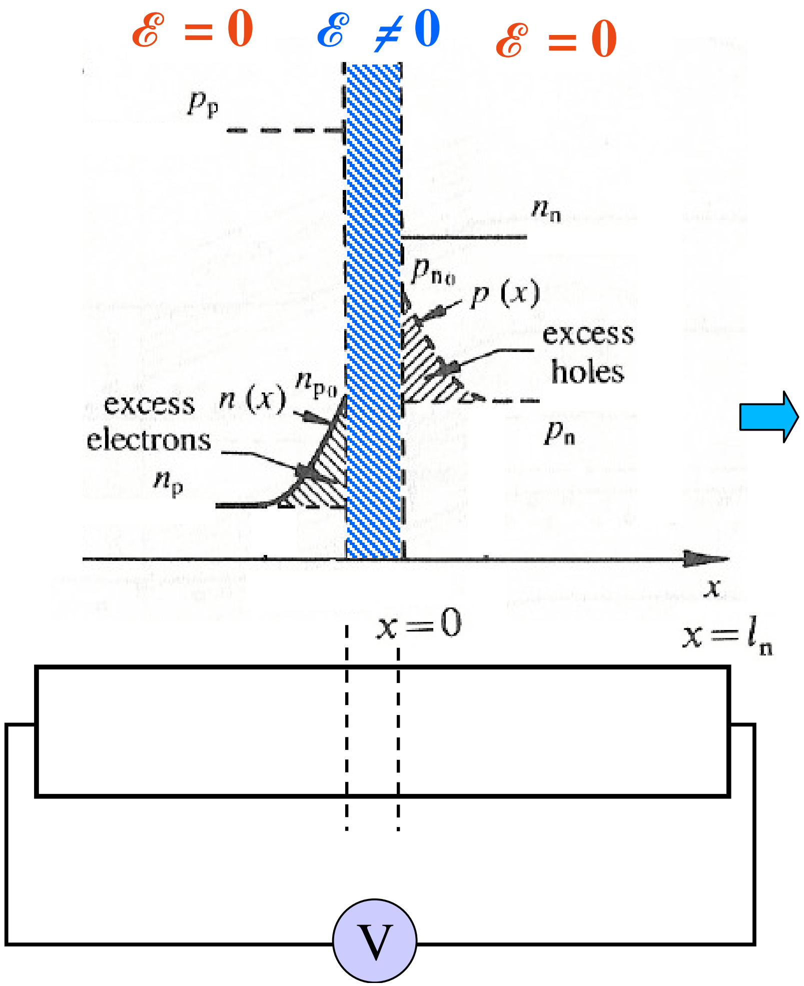
$$\frac{J_h}{J_e} = \frac{L_e}{L_h}\frac{\mu_h N_a}{\mu_e N_d} \quad \Rightarrow \quad \boxed{J_h/J_e \simeq \sigma_p/\sigma_n}$$

  - Hole efficiency:

$$\eta_h \equiv \frac{J_{hn}}{J_{hn}+J_{ep}} = \frac{1}{1+(J_{ep}/J_{hn})} \simeq \frac{1}{1+(\sigma_n/\sigma_p)} \qquad \eta_h \equiv \frac{J_{ep}}{J_{hn}+J_{ep}} = \frac{1}{1+(J_{hn}/J_{ep})} \simeq \frac{1}{1+(\sigma_p/\sigma_n)}$$

Pn junction with finite dimensions

- Current in a finite junction:
  - From the continuity eq. we obtained



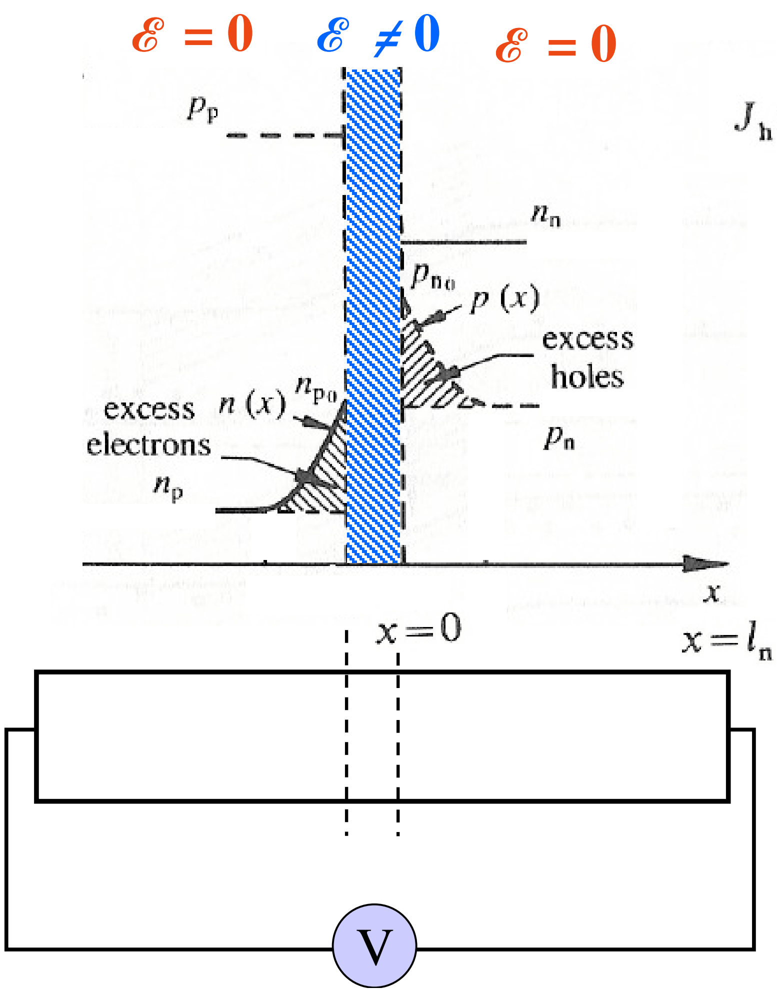
$$\delta p = C_1 \exp(-x/L_h) + C_2 \exp(+x/L_h)$$

$$p = p_{n0} = p_n \exp(eV/kT) \qquad \text{when } x=0$$
$$p = 0 \qquad \text{when } x = l_n$$

$$p(x) = \left(\frac{p_n \exp(-l_n/L_h) + p_{n0} - p_n}{1-\exp(-2l_n/L_h)}\right)\exp(-x/L_h)$$
$$-\left(\frac{p_n \exp(-l_n/L_h) + p_{n0} - p_n}{1-\exp(-2l_n/L_h)} - (p_{n0}-p_n)\right)\exp(+x/L_h) + p_n$$

Pn junction with finite dimensions

- Current in a finite junction:
  - Currents:



For holes:

$$J_h = -eD_h\left(\frac{dp(x)}{dx}\right)\bigg|_{x=0} = \frac{eD_h}{L_h}p_n \tanh\left(\frac{l_n}{L_h}\right)[\exp(eV/kT)-1]$$

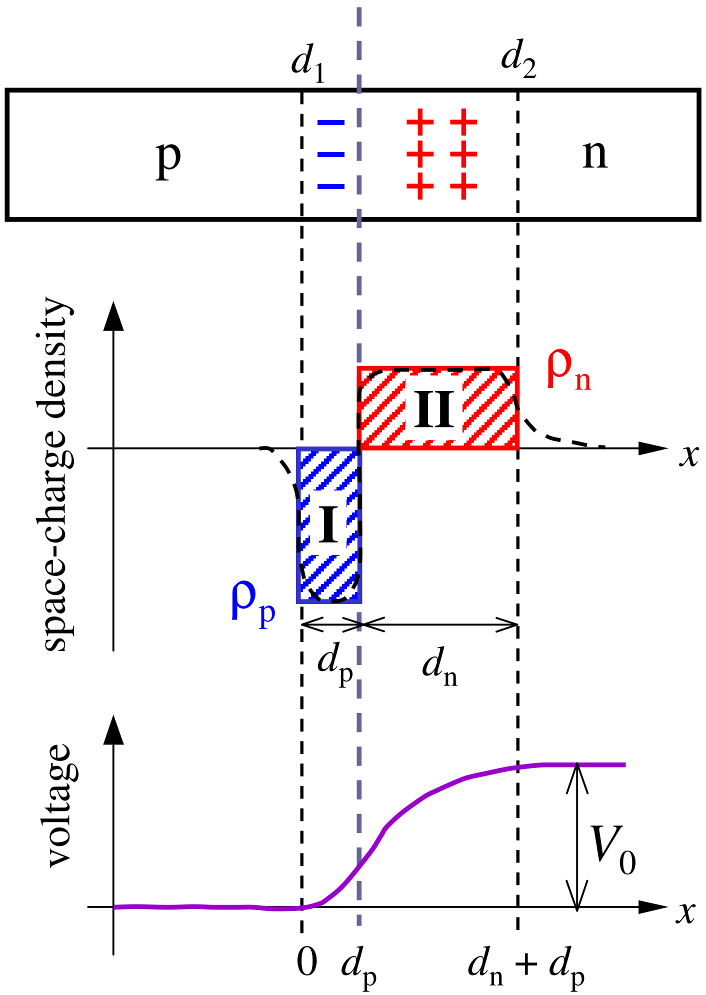
Idem for the electrons. Therefore the saturation current is:

$$J_0 = e\left[\frac{D_h p_n}{L_h}\underbrace{\tanh\left(\frac{l_n}{L_h}\right)} + \frac{D_e n_p}{L_e}\underbrace{\tanh\left(\frac{l_p}{L_e}\right)}\right]$$

→0 when *l* » L
(L ~ 1 mm)

Depletion-layer capacitance

- The depletion layer forms a capacitance (important for high frequency applications)



Using Poisson relation ($\partial^2 V/\partial x^2 = -\rho/\epsilon$):

I:

$\rho_p \simeq -N_a e \Rightarrow \partial^2 V_1/\partial x^2 = eN_a/\epsilon$

$V = 0$ & $\partial V/\partial x = 0$ at $x = 0$ $\Rightarrow V_1(x) = eN_a x^2/2\epsilon$

II:

$\rho_n \simeq N_d e \Rightarrow \partial^2 V_2/\partial x^2 = -eN_d/\epsilon$

$\Rightarrow V_2 = -eN_d x^2/2\epsilon + C_1 x + C_2$

$(\partial V_1/\partial x)|_{d_p} = (\partial V_2/\partial x)|_{d_p}$

$V_1|_{d_p} = V_2|_{d_p}$

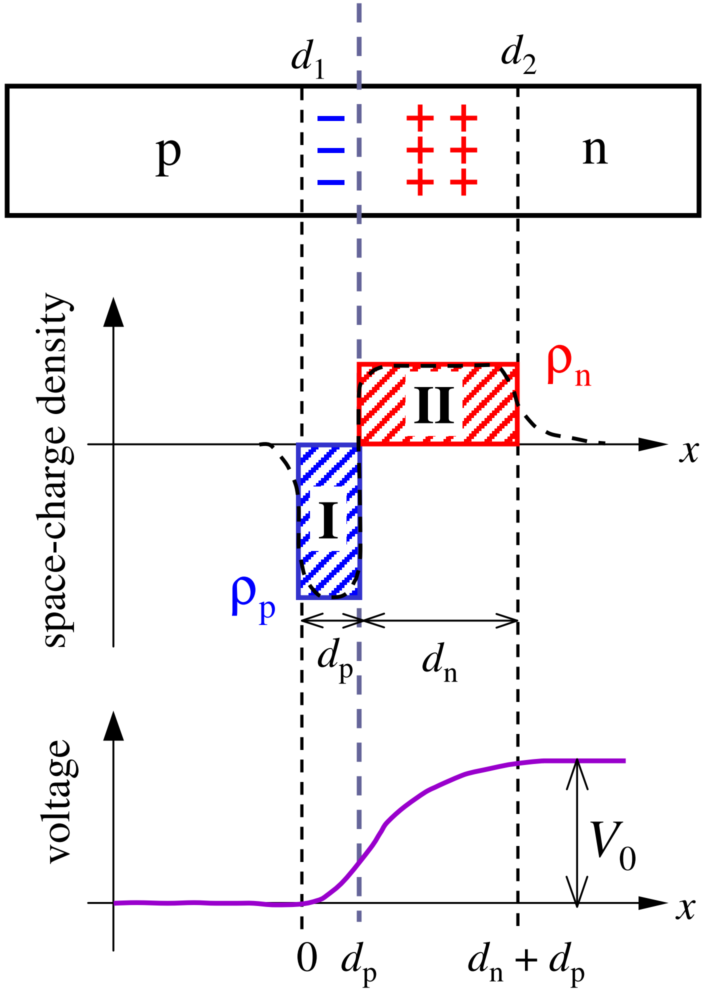
$\Rightarrow$

$$C_1 = ed_p(N_a + N_d)/\epsilon$$

$$C_2 = -\frac{ed_p^2(N_a + N_d)}{2\epsilon}$$

Depletion-layer capacitance

- The depletion layer forms a capacitance (important for high frequency applications)



Using Poisson relation ($\partial^2 V/\partial x^2 = -\rho/\epsilon$) and continuity for voltage and its derivative:

I:

$$\rho_p \simeq -N_a e \Rightarrow \partial^2 V_1/\partial x^2 = eN_a/\epsilon$$

$V = 0$ & $\partial V/\partial x = 0$ at $x = 0$

$$\Rightarrow V_1(x) = eN_a x^2/2\epsilon$$

II:

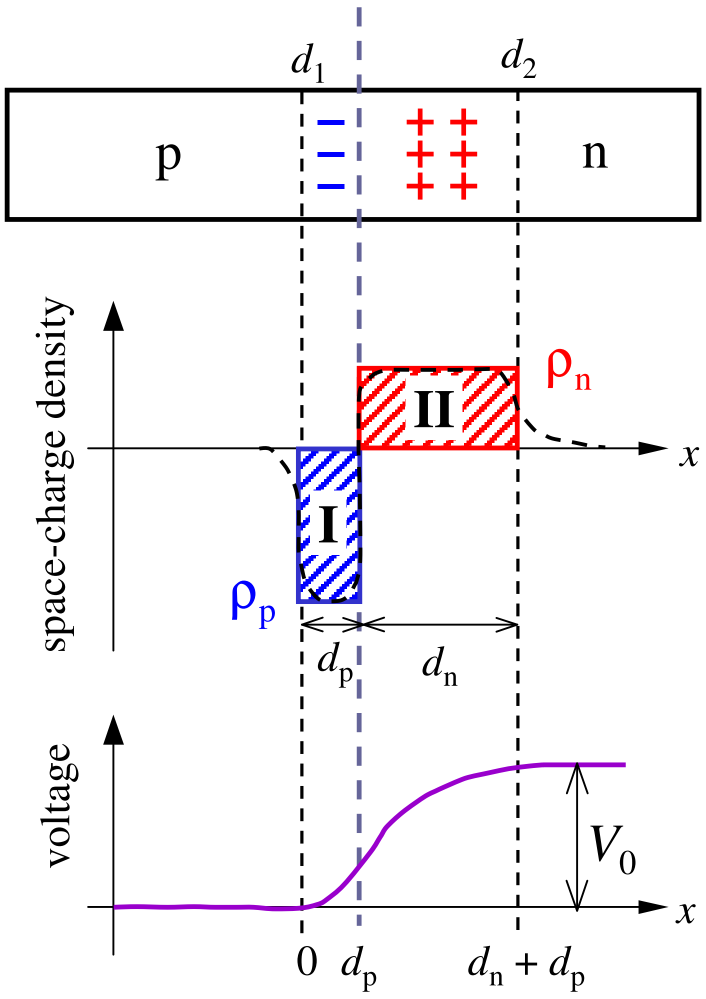
$$\rho_n \simeq N_d e \Rightarrow \partial^2 V_2/\partial x^2 = -eN_d/\epsilon$$

$$\Rightarrow V_2 = -eN_d x^2/2\epsilon + C_1 x + C_2$$

$$(\partial V_1/\partial x)|_{d_p} = (\partial V_2/\partial x)|_{d_p}, \quad V_1|_{d_p} = V_2|_{d_p} \Rightarrow \begin{cases} C_1 = ed_p(N_a + N_d)/\epsilon \\ C_2 = -\dfrac{ed_p^2(N_a + N_d)}{2\epsilon} \end{cases}$$

Depletion-layer capacitance

- Expressions for the depletion layer:



Applying $\mathscr{E} = 0$ at $x \geq d_n + d_p$:

$$(\partial V_2/\partial x)|_{d_n+d_p} = 0 \Rightarrow \boxed{d_p/d_n = N_d/N_a}$$

Applying $V = V_0$ at $x \geq d_n + d_p$ & using previous relation:

$$V_2|_{d_n+d_p} = V_0 \Rightarrow \boxed{d_p = \left(\frac{2\epsilon V_0 N_d}{eN_a(N_a+N_d)}\right)^{1/2}}$$

$$\Rightarrow \boxed{d_n = \left(\frac{2\epsilon V_0 N_a}{eN_d(N_a+N_d)}\right)^{1/2}}$$

This expresions are still applicable for a biased junction $V_0 \leftrightarrow V_0 - V$

Depletion-layer capacitance

- Capacitance of the depletion layer:
  - Charge per unit area accumulated at the depletion layer

$$Q_j = eN_d d_n = eN_a d_p \xrightarrow{V_j = V_0 \pm V} Q_j = \left(\frac{2\epsilon e V_j N_a N_d}{N_a + N_d}\right)^{1/2}$$

  - The capacitance per unit area ($C_j = dQ_j/dV_j$) is then:

$$C_j = \left(\frac{\epsilon e N_a N_d}{2(N_a + N_d)}\right)^{1/2} \frac{1}{V_j^{1/2}} \Longrightarrow \boxed{C_j = \epsilon/(d_p + d_n)}$$

$$\Big\Downarrow \; N_a \gg N_d$$

$$V_j = V_0 + V \simeq \frac{1}{C_j^2}\left(\frac{\epsilon e N_d}{2}\right)^{1/2}$$