Physics of Electronics:

7. Junction Diodes

July – December 2008

Contents overview

- Continuity equation.
- Semiconductor measurements.
- Pn junction in equilibrium.
- Current flow in a pn junction with forward bias.

Continuity Equation for Minority Carriers

- General considerations:
  - Two main causes for the variation: Generation and loss & Drift
- No current flow (no gradient & no $\mathscr{E}$)
  - Let's consider a **p-type** material:

$$dn/dt = G - R \quad \textbf{with: } G = G(T) \;\; \& \; R = r\, n(t)\, p(t)$$

  - Equilibrium:

$$dn/dt = 0 \Rightarrow G(T) = r n_0 p_0 = r n_i^2$$

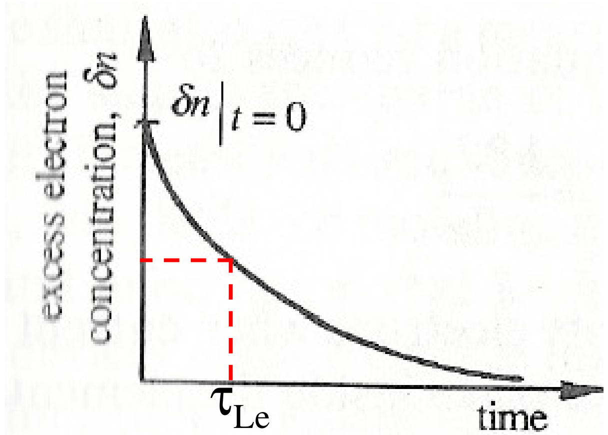
  - Variations from the equilibrium:

$$n = n_0 + \delta n \Leftrightarrow p = p_0 + \delta p \quad \text{with } \delta n = \delta p$$

$$\Rightarrow \boxed{[d(\delta n)/dt]|_{J=0} = -r p_0 \delta n = -\delta n/\tau_{Le}}$$

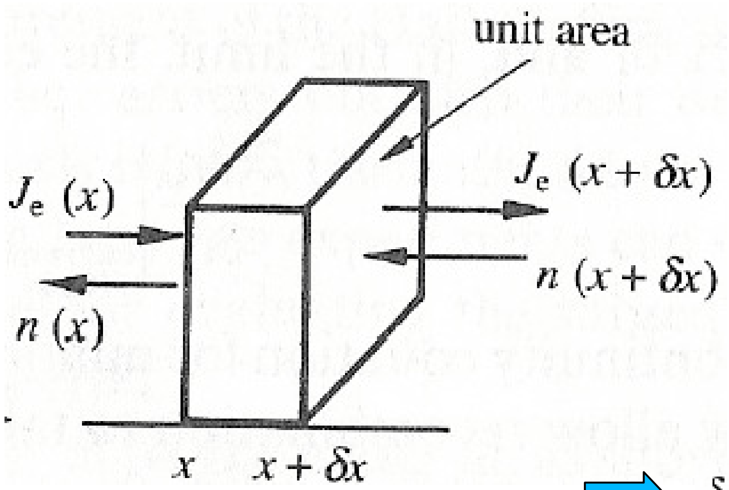
  - For a **n-type** material:

$$d(\delta p)/dt = -\delta p/\tau_{Lh}$$



Continuity Equation for Minority Carriers

- With current flow



$$J_e = -nev_D \Rightarrow \begin{cases} n_{x+\delta x} = J_e(x+\delta x)/ev_D \\ n_x = J_e(x)/ev_D \end{cases}$$

$$n_{x+\delta x} - n_x = \delta n = [J_e(x+\delta x) - J_e(x)]/ev_D$$

$$= \frac{1}{ev_D}\left(J_e(x) + \frac{\partial J_e}{\partial x}\delta x - J_e(x)\right)$$

$$\Rightarrow \delta n = \frac{1}{ev_D}\frac{\partial J_e}{\partial x}\delta x \xrightarrow{v_D = \partial x/\partial t} \boxed{\left.\frac{\partial(\delta n)}{\partial t}\right|_{\substack{\text{current}\\\text{flow}}} = \frac{1}{e}\frac{\partial J_e}{\partial x}}$$

- Total continuity equation

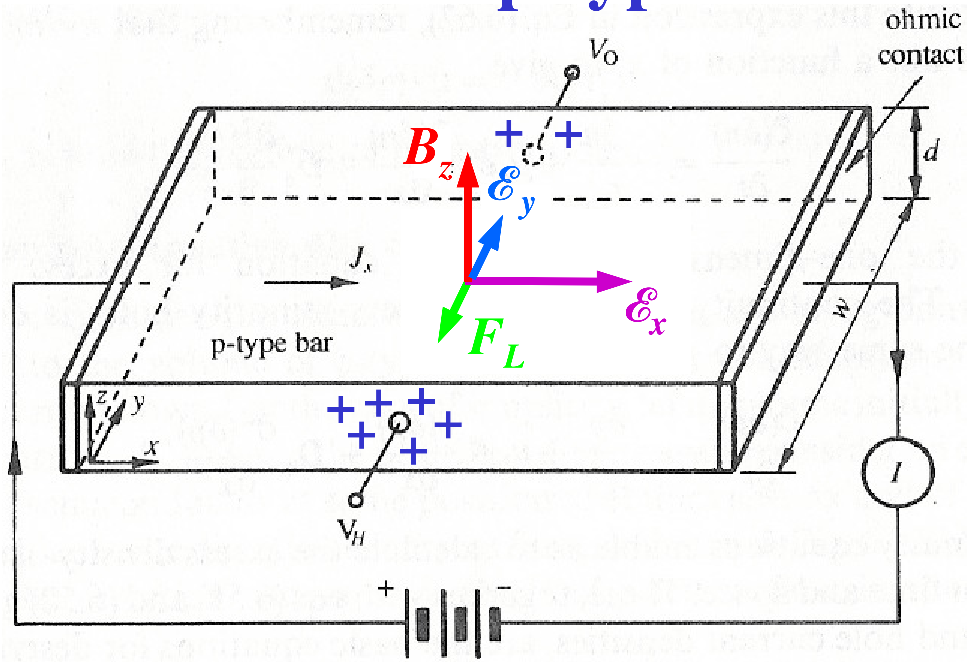
$$\left.\frac{\partial(\delta n)}{\partial t}\right|_{\text{total}} = -\frac{\delta n}{\tau_{Le}} + \frac{1}{e}\frac{\partial J_e}{\partial x} \quad \text{(idem for holes)}$$

But $J_e = ne\mu_e\mathscr{E} + eD_e\nabla n \xrightarrow{n = n_0 + \delta n}$

$$\boxed{\frac{\partial(\delta n)}{\partial t} = -\frac{\delta n}{\tau_{Le}} + \mu_e\mathscr{E}_x\frac{\partial(\delta n)}{\partial x} + D_e\frac{\partial^2(\delta n)}{\partial x^2}}$$

Semiconductor Measurements

- Hall effect
  - Consider **p-type** material



$$F_L = ev \times B \Rightarrow |F_L| = e\, v_{Dx}\, B_z$$

$$\Rightarrow \left.\begin{array}{l} e\mathscr{E}_y = F_L = ev_{Dx}B_z \\ \& \quad J_x \simeq pev_{Dx} \end{array}\right\} \mathscr{E}_y = J_x B_z / pe$$

The Hall coefficient is defined:

$$R_H = \mathscr{E}_y / J_x B_z = 1/pe$$

$$\Rightarrow \boxed{R_H = \frac{V_H}{wIB_z} wd = \frac{V_H d}{IB_z} = \frac{1}{pe}}$$

- Using he Hall angle:

$$\tan\theta = \mathscr{E}_y / \mathscr{E}_x \Rightarrow \tan\theta = \frac{J_x B_z}{pe}\frac{\sigma}{J_x} = \mu_h B_z \Rightarrow \boxed{\mu_h = R_H \sigma}$$

Semiconductor Measurements

- Hall effect
  - Consider both type of carriers to be present:

$$v_{\mathrm{Dh}} = \mu_{\mathrm{H}} \mathscr{E}_{\mathrm{x}} \qquad F_{\mathrm{h}} = -e(v_{\mathrm{Dh}} \times B) = -e v_{\mathrm{Dh}} B_z$$

$$v_{\mathrm{De}} = -\mu_{\mathrm{H}} \mathscr{E}_{\mathrm{x}} \quad \Rightarrow \quad F_{\mathrm{e}} = e(v_{\mathrm{De}} \times B) = -e v_{\mathrm{De}} B_z$$

- A net current is created in the *y* direction

$$\sigma \mathscr{E}_y = e(p v_{y\mathrm{h}} - n v_{y\mathrm{e}})$$

- Now, using the expressions for the Hall angle

$$v_{y\mathrm{h}} = \mu_{\mathrm{h}} \mathscr{E}_y = \mu_{\mathrm{h}}(\mathscr{E}_x \tan\theta) = \mu_{\mathrm{h}}(\mathscr{E}_x \mu_{\mathrm{h}} B_z) \qquad \& \qquad v_{y\mathrm{e}} = \mu_{\mathrm{e}}^2 \mathscr{E}_x B_z$$

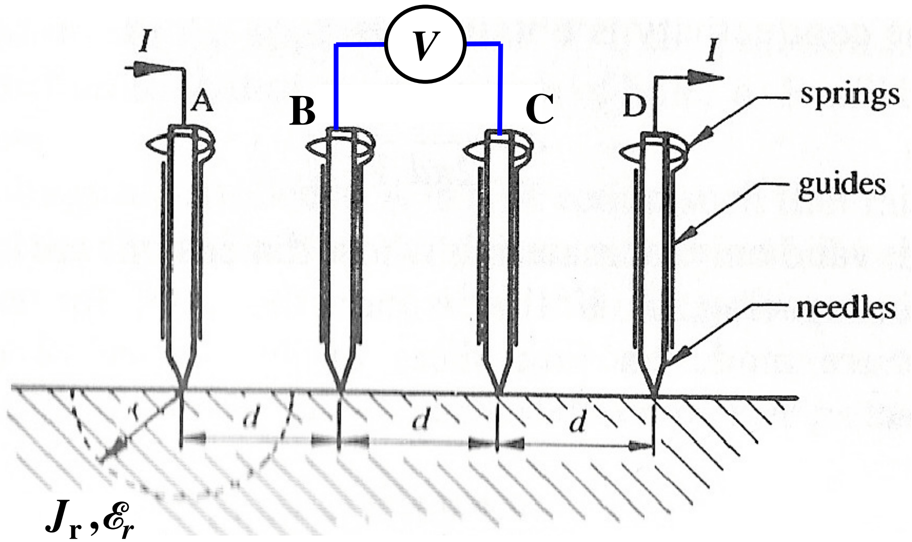
$$\Rightarrow \quad \sigma \mathscr{E}_y = e(p\mu_{\mathrm{h}}^2 - n\mu_{\mathrm{e}}^2) \mathscr{E}_x B_z$$

- Hall coefficient

$$R_{\mathrm{H}} = \frac{\mathscr{E}_y}{J_x B_z} = \frac{e(p\mu_{\mathrm{h}}^2 - n\mu_{\mathrm{e}}^2)}{\sigma^2} \quad \xrightarrow{\sigma = e(n\mu_{\mathrm{e}} + p\mu_{\mathrm{h}})} \quad \boxed{R_{\mathrm{H}} = \frac{p\mu_{\mathrm{h}}^2 - n\mu_{\mathrm{e}}^2}{e(p\mu_{\mathrm{h}} + n\mu_{\mathrm{e}})^2}}$$

Semiconductor Measurements

- Four-point probe method for conductivity meas.
  - Consider $d << L$



For the current leaving $\boldsymbol{A}$:

$$J_r = I/(2\pi r^2) \Rightarrow \mathscr{E}_r = J/\sigma = I/(2\pi\sigma r^2)$$

Potential at a distance a from $\boldsymbol{A}$:

$$V_a = \int_{-\infty}^{a} \mathscr{E}_r \,\mathrm{d}r = -\frac{I}{2\pi\sigma}\int_{-\infty}^{a}\frac{1}{r^2}\,\mathrm{d}r = \frac{I}{2\pi\sigma a}$$

$$\Rightarrow V_{\mathrm{BC}} = \frac{I}{2\pi\sigma d} - \frac{I}{2\pi\sigma(2d)} = \frac{I}{4\pi\sigma d}$$

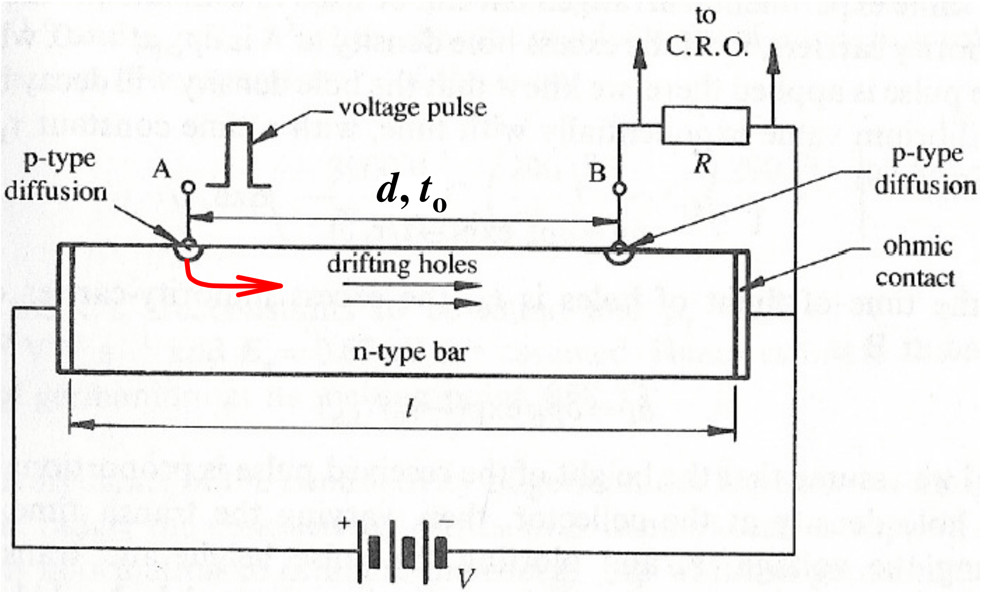
Identically for the current entering $\boldsymbol{D}$, then:

$$V = 2V_{\mathrm{BC}} = \frac{I}{(2\pi\sigma d)}$$

$$\Rightarrow \boxed{\sigma = \frac{1}{2\pi d}\frac{I}{V}}$$

Semiconductor Measurements

- Minority carrier life-time and mobility.



Mobility:

$$v_D = d/t_0 \Rightarrow \mu E = \mu V/l = d/t_0$$

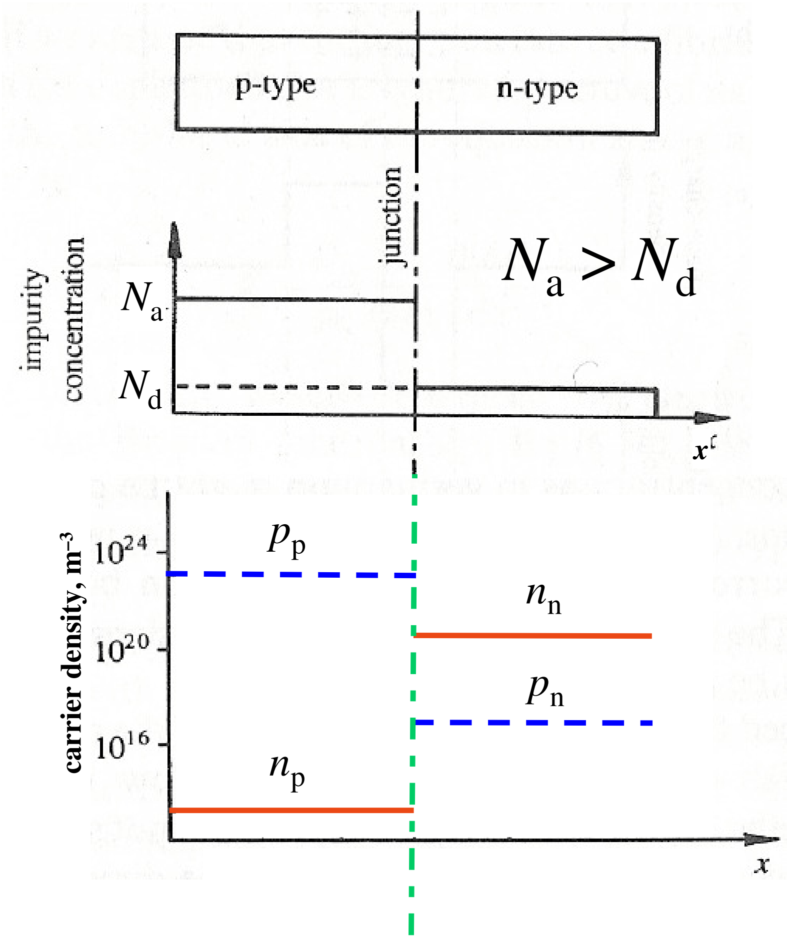
$$\Rightarrow \boxed{\mu = ld/t_0 V}$$

Life-time:

$$d(\delta p)/dt = -\delta p/\tau_{Lh} \Rightarrow \delta p = \delta p_0 \exp(-t/\tau_{Lh})$$

Pn junction in equilibrium

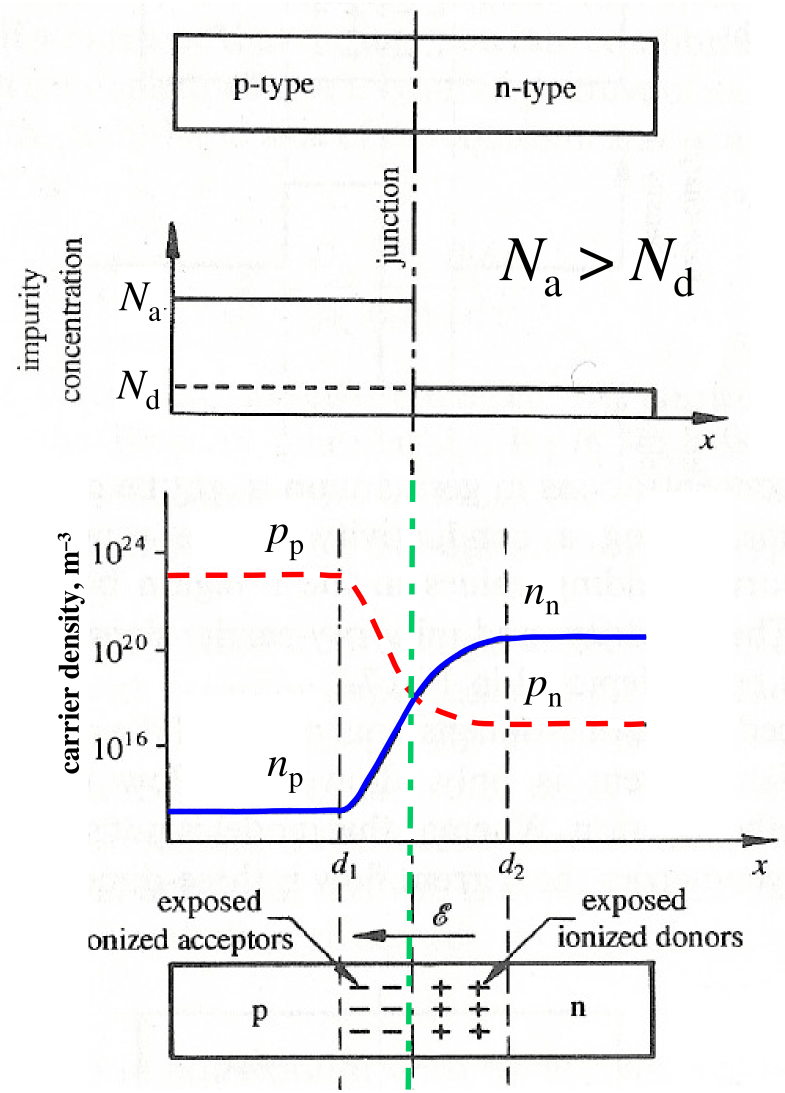
- Consider a pn junction with abrupt transition



Initially, the density of carriers change abruptly at the junction.

Pn junction in equilibrium

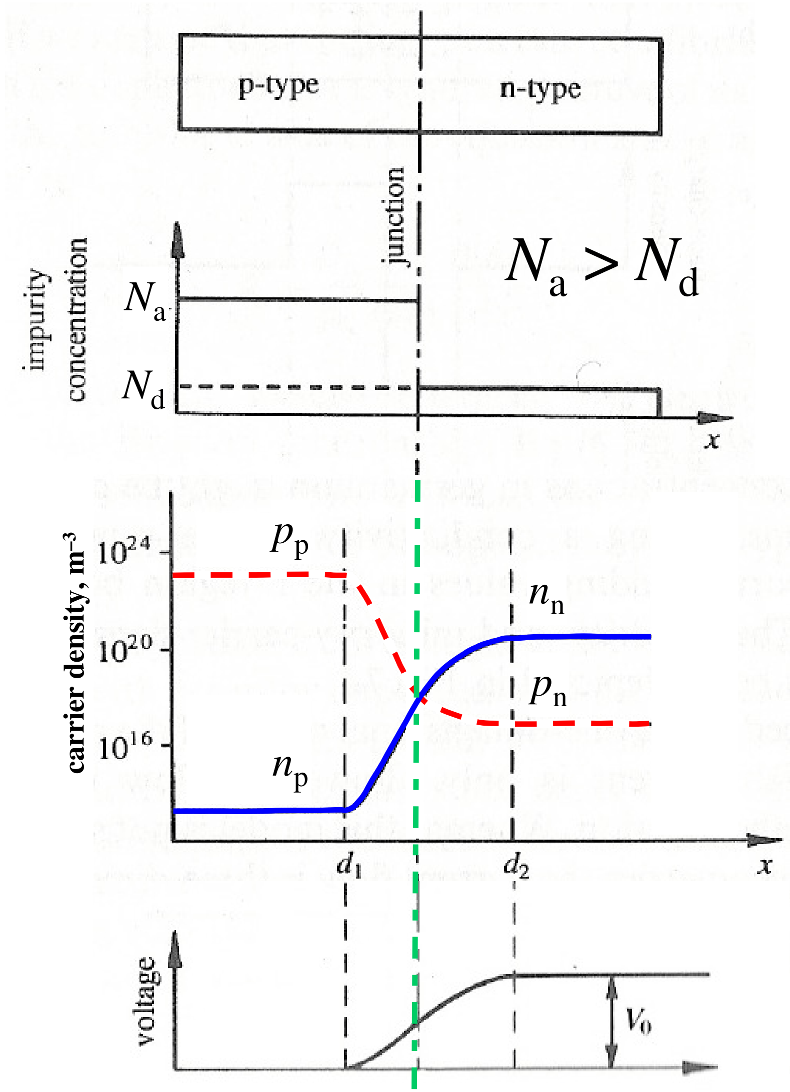
- Consider a pn junction with abrupt transition



This change of concentration causes diffusion which is stopped by exposition of ionized impurities at the transition (called depletion layer ~1μm).

Pn junction in equilibrium

- Consider a pn junction with abrupt transition



This change of concentration causes diffusion which is stopped by exposition of ionized impurities at the transition (called depletion layer ~1μm).

In equilibrium, a voltage difference is created across the depletion layer.

Pn junction in equilibrium

- Voltage across the depletion layer
  - Start with continuity eq. for holes

$$\underbrace{\cancel{\frac{\partial(\delta p)}{\partial t}}}_{0} = -\underbrace{\cancel{\frac{\delta p}{\tau_{\mathrm{Lh}}}}}_{0} + \mu_{\mathrm{h}}\mathscr{E}_x \frac{\partial(\delta p)}{\partial x} + D_{\mathrm{h}}\frac{\partial^2(\delta p)}{\partial x^2} \Rightarrow \mathscr{E}_x = \frac{D_{\mathrm{h}}}{\mu_{\mathrm{h}}}\frac{1}{(\delta p)}\frac{\mathrm{d}(\delta p)}{\mathrm{d}x}$$

$$D_{\mathrm{h}}/\mu_{\mathrm{h}} = kT/e$$

$$\Rightarrow \mathscr{E}_x = \frac{kT}{e}\frac{1}{(\delta p)}\frac{\mathrm{d}(\delta p)}{\mathrm{d}x} \Rightarrow e\int_{d_1}^{d_2}\mathscr{E}_x\,\mathrm{d}x = -kT\int_{p_{\mathrm{p}}}^{p_{\mathrm{n}}}\frac{\mathrm{d}(\delta p)}{\delta p}$$

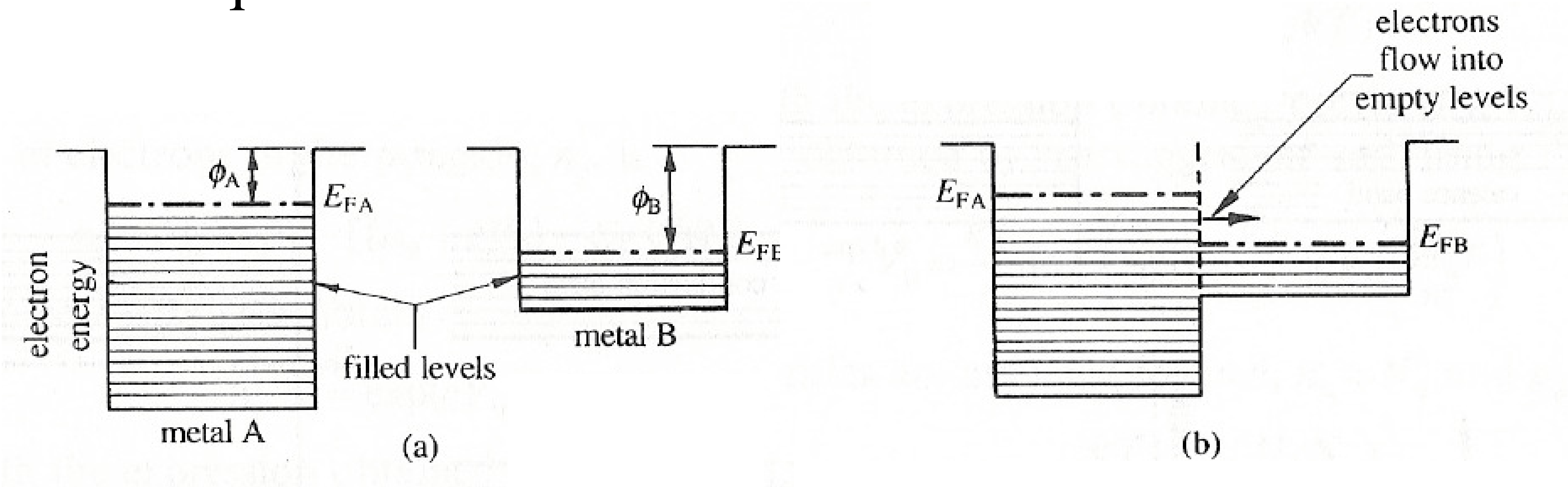
$$\Rightarrow \boxed{p_{\mathrm{p}} = p_{\mathrm{n}}\exp(eV_0/kT)}$$

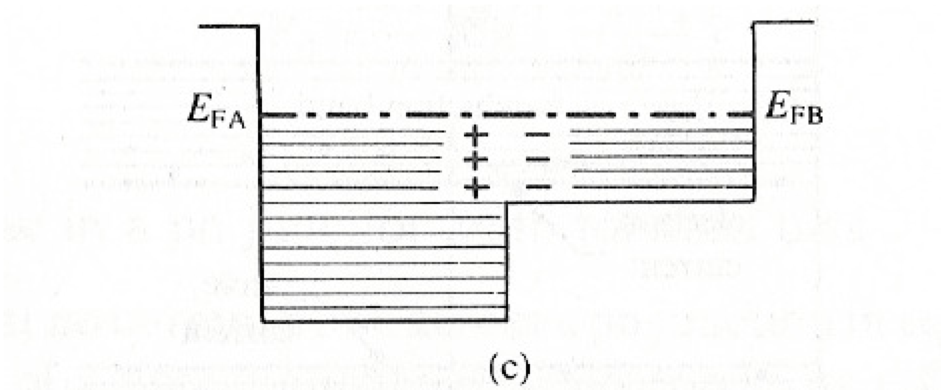
  - Idem for electrons

$$\boxed{n_{\mathrm{n}} = n_{\mathrm{p}}\exp(eV_0/kT)}$$

Pn junction in equilibrium

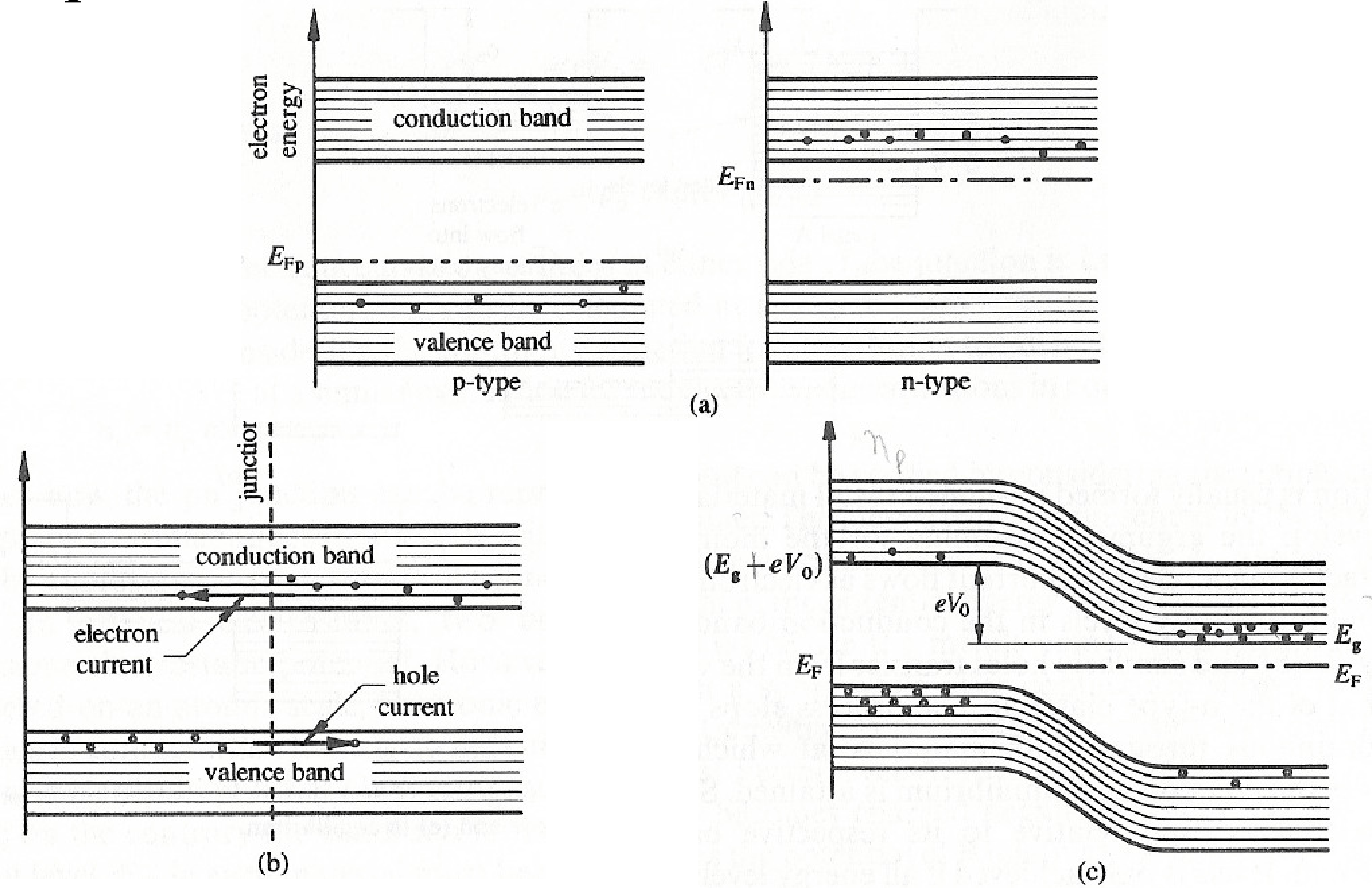
- Junctions and band structure
  - Equilibrium at atomic scale (in metals):





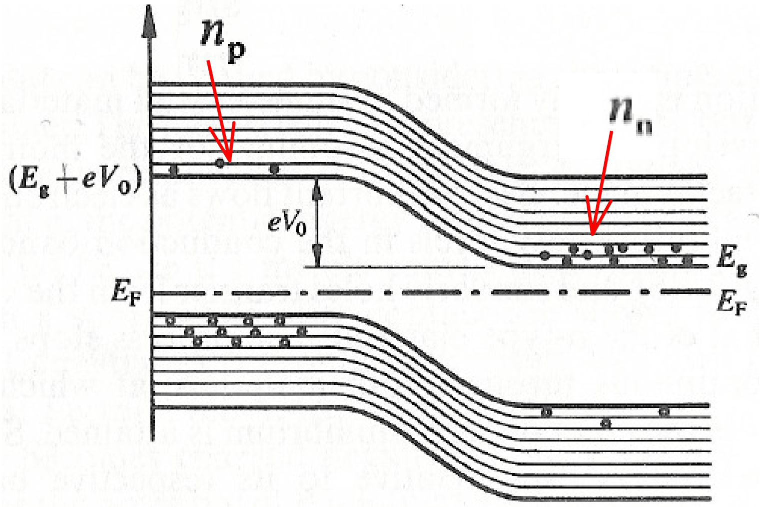
Pn junction in equilibrium

- Junctions and band structure
  - Equilibrium at atomic scale (in semiconductors)



Pn junction in equilibrium

- Number of electrons in the conduction band



$$n_n = N_c \exp[-(E_g - E_F)/kT]$$

$$n_p = N_c \exp\{-[(E_g + eV_0) - E_F]/kT\}$$

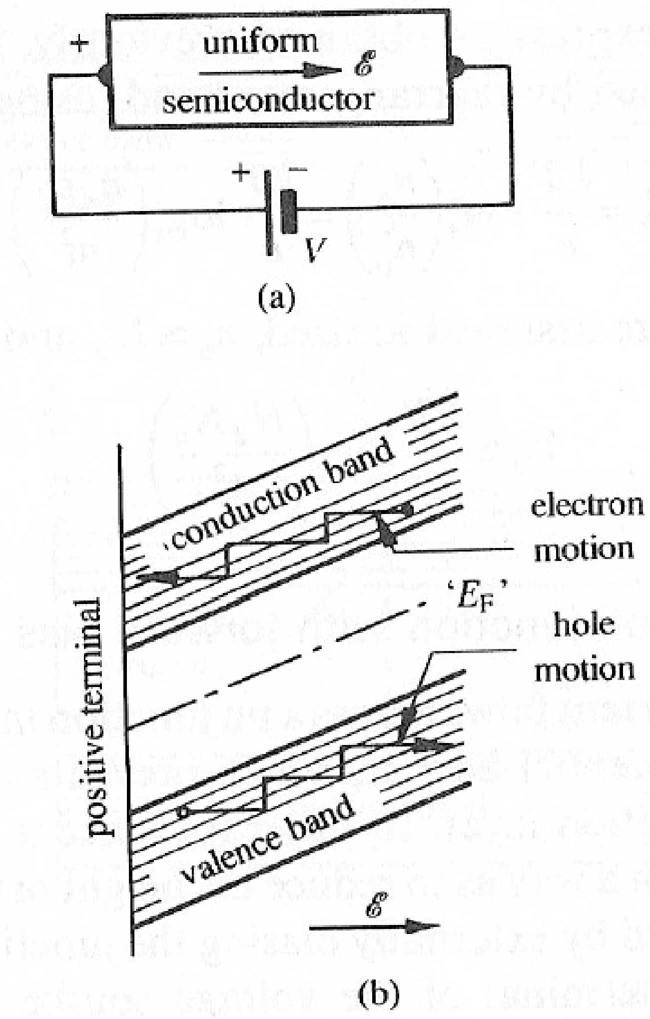
$$\Rightarrow \quad n_n/n_p = \exp(eV_0/kT)$$

- If all the impurities are ionized ($n_n \approx N_d$ and $p_p \approx N_a$):

$$\boxed{V_0 \simeq \frac{kT}{e} \log_e\left(\frac{N_d N_a}{n_i^2}\right)}$$

Pn junction with forward bias

- Consider a uniform sC biased with a voltage V:

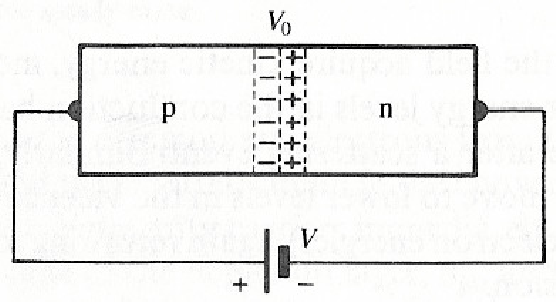


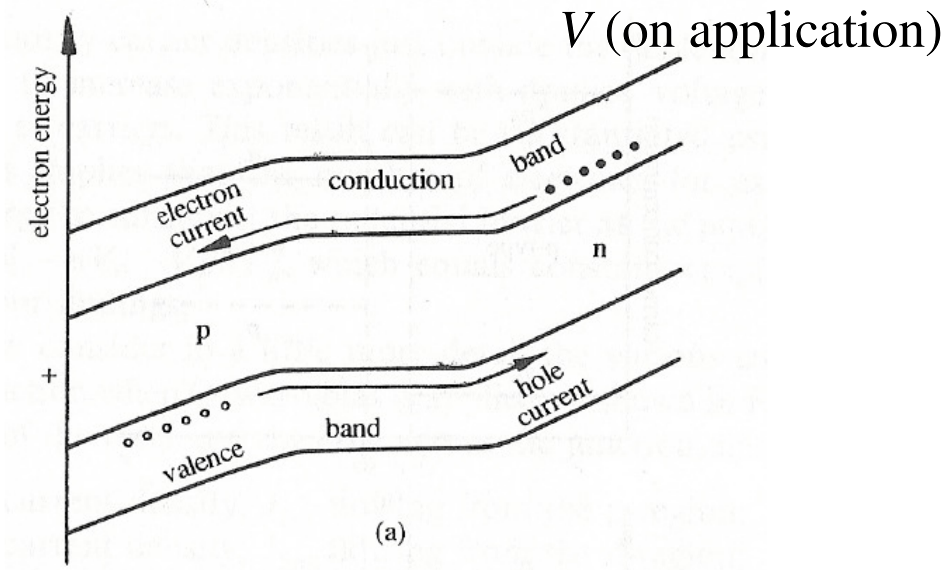
The whole band structure tilts (one end of the sC has more energy than the other) .

Electrons and holes move in the field (in opposite directions.

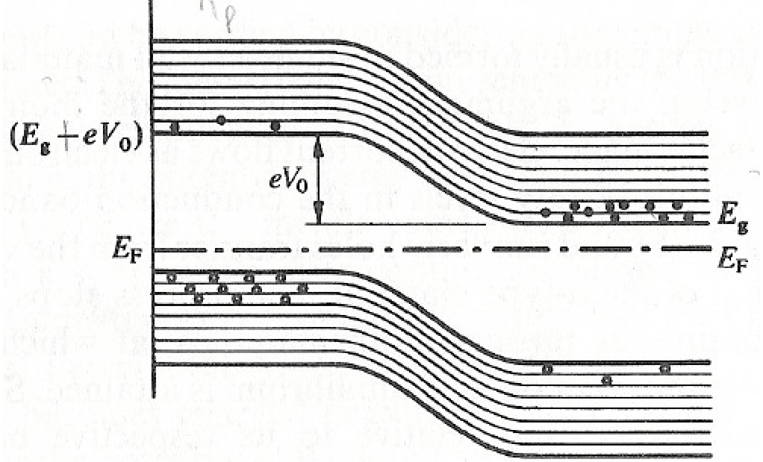
Pn junction with forward bias

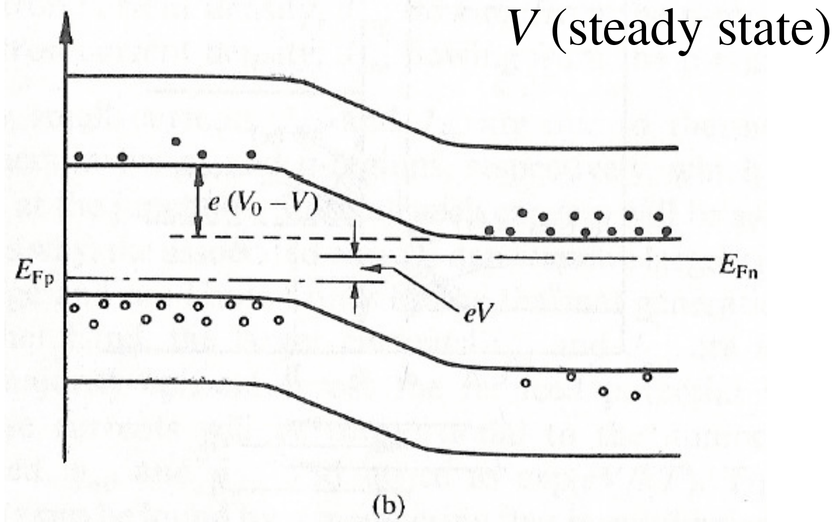
- Consider a junction biased with a voltage V:





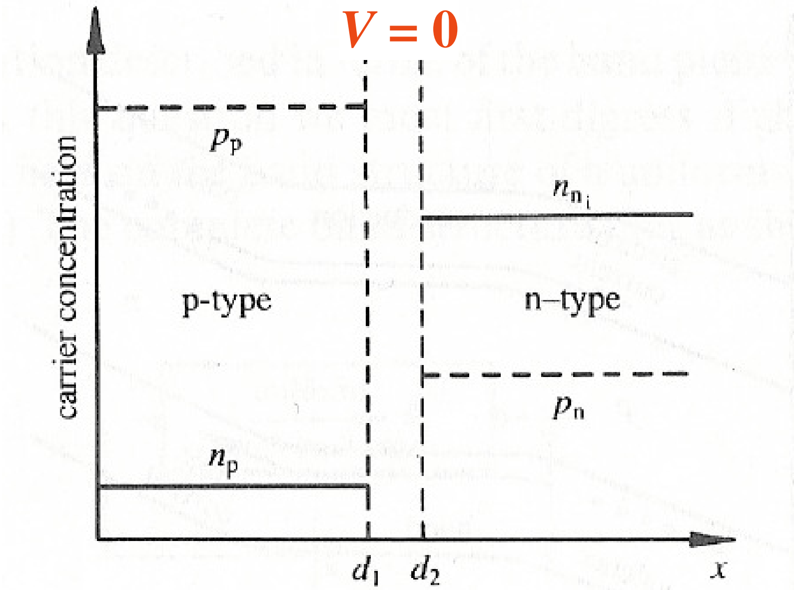
$V = 0$

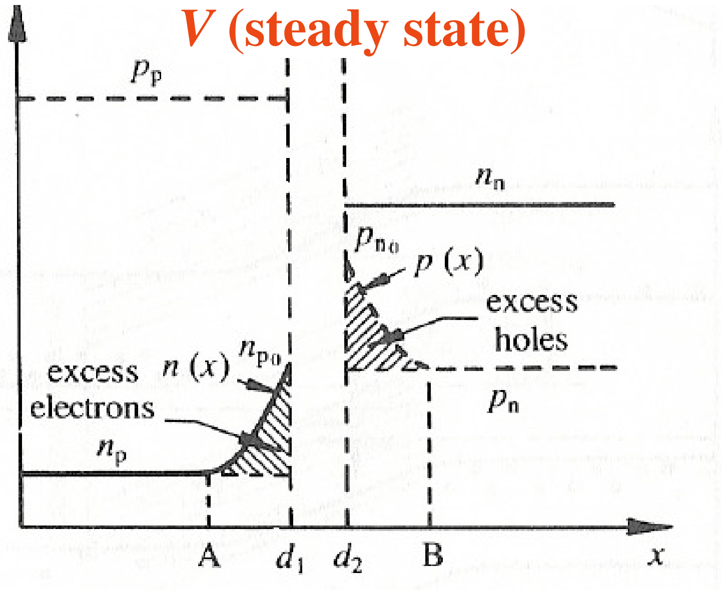




Pn junction with forward bias

- Excess carriers in a biased junctions:





$$p_p = p_n \exp(eV_0/kT)$$

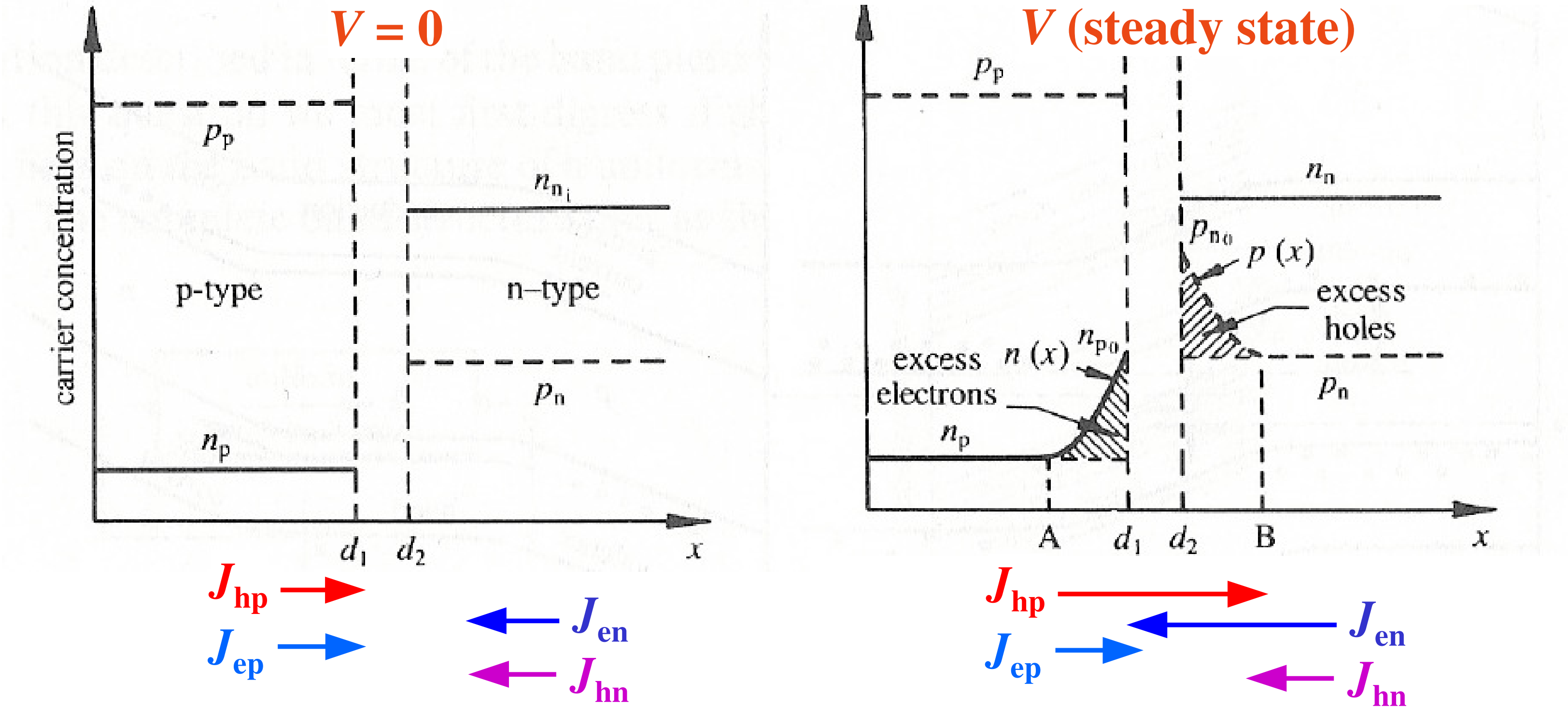
$$p_{n0} = p_p \exp[e(V - V_0)/kT]$$

$$p_{n0} = p_n \exp(eV/kT)$$

$$n_{p0} = n_p \exp(eV/kT)$$

Pn junction with forward bias

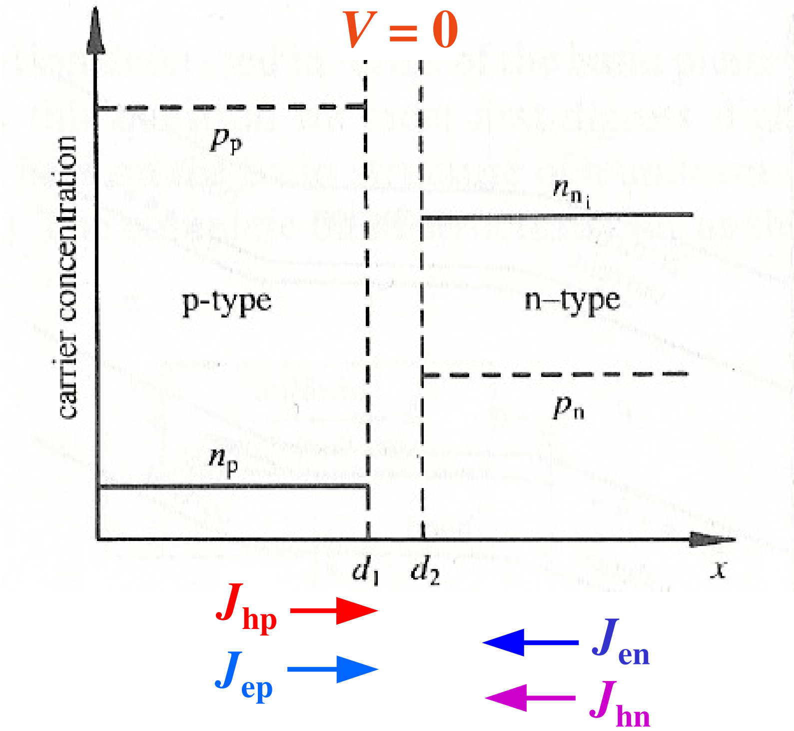
- Excess carriers in a biased junctions:

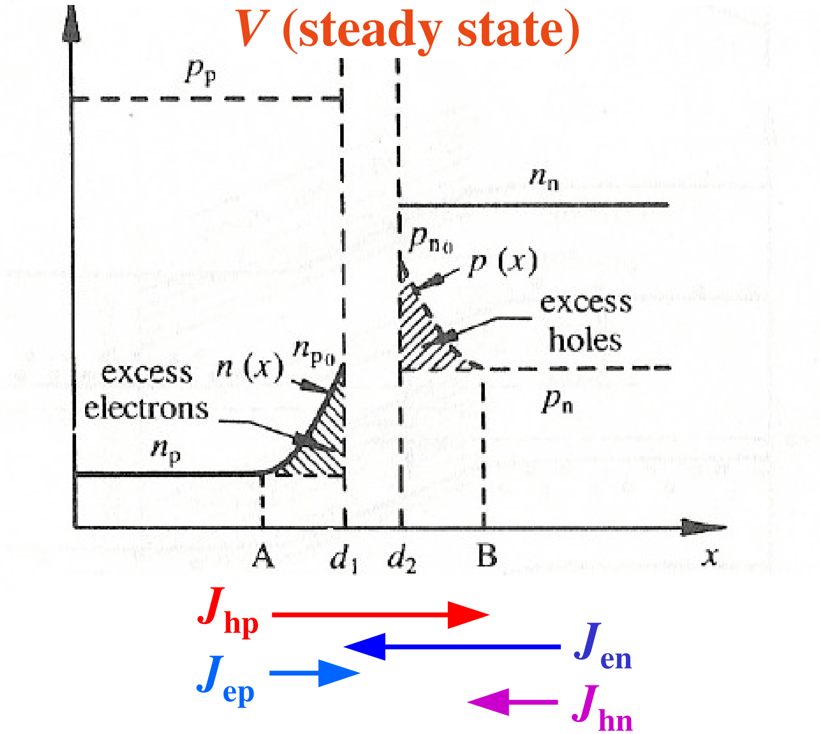


(a) a hole current density, J_{hp}, flowing from the p-region;
(b) a hole current density, J_{hn}, flowing from the n-region;
(c) an electron current density, J_{en}, flowing from the n-region;
(d) an electron current density, J_{ep}, flowing from the p-region.

Pn junction with forward bias

- Excess carriers in a biased junctions:





$$J_{hp} \propto p_{n0} = p_n \exp(eV/kT)$$

$$J_{hp} = J_{hn} \quad \Rightarrow \quad J_{hp} = J_{hn} \exp(eV/kT)$$

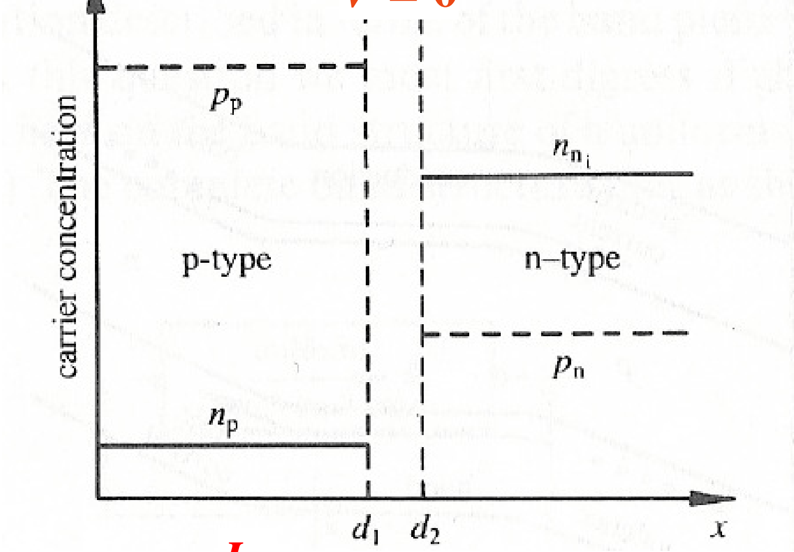
$$J_{en} \propto n_{p0} = n_p \exp(eV/kT)$$

$$J_{en} = J_{ep} \quad \Rightarrow \quad J_{en} = J_{ep} \exp(eV/kT)$$

Pn junction with forward bias

- Excess carriers in a biased junctions:

***V* = 0**



$J_{hp} \longrightarrow$

$J_{ep} \longrightarrow$

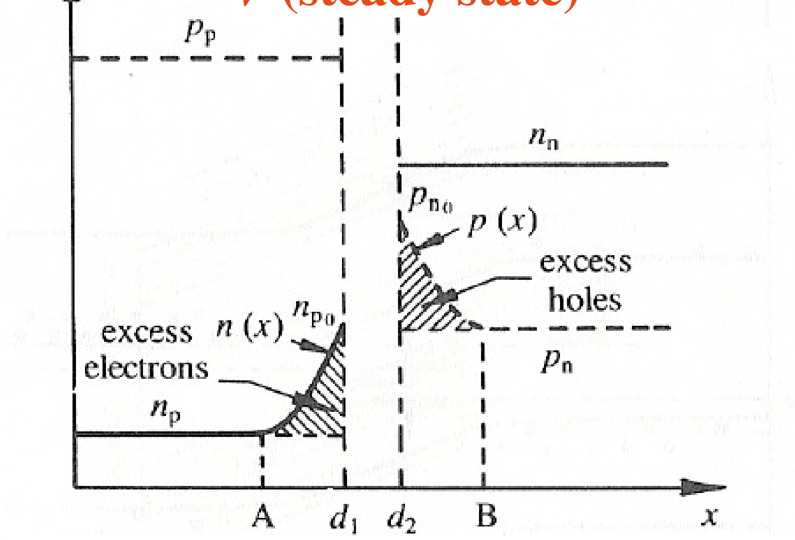
$\longleftarrow J_{en}$

$\longleftarrow J_{hn}$

$$J_{hp} = J_{hn}$$

$$J_{en} = J_{ep}$$

***V* (steady state)**



$J_{hp} \longrightarrow$

$\longleftarrow J_{en}$

$J_{ep} \longrightarrow$

$\longleftarrow J_{hn}$

Total currents:

$$J_h = J_{hp} - J_{hn} = J_{hn}[\exp(eV/kT) - 1]$$

$$J_e = J_{en} - J_{ep} = J_{ep}[\exp(eV/kT) - 1]$$

$$\Rightarrow \boxed{J = J_0[\exp(eV/kT) - 1]}$$ Diode equation

$$J_0 = J_{hn} + J_{ep}$$