Physics of Electronics:

6. Semiconductors

July – December 2008

Contents overview

- Density of carriers in extrinsic semiconductors.
- Compensation doping.
- Electrical conduction in semiconductors.
- Continuity equation.
- Semiconductor measurements.

Density of carriers in extrinsic sC

- General considerations:
  - Concentration of ionized impurities:

ionized impurities = impurity concentration × prob. finding e/h at the imp. level

$$N_a^- = \frac{N_a}{1+\exp[(E_A - E_F)/kT]} \quad \& \quad N_d^+ = N_d\left(1 - \frac{1}{1+\exp[(E_D - E_F)/kT]}\right)$$

  - Neutrality condition

holes + ionized donors = electrons + ionized acceptors

$$p + N_d^+ = n + N_a^-$$

$$\Rightarrow N_v \exp(-E_F/kT) + \frac{N_d}{1+\exp[-(E_D - E_F)/kT]} = N_c \exp[-(E_g - E_F)/kT] + \frac{N_a}{1+\exp[(E_A - E_F)/kT]}$$

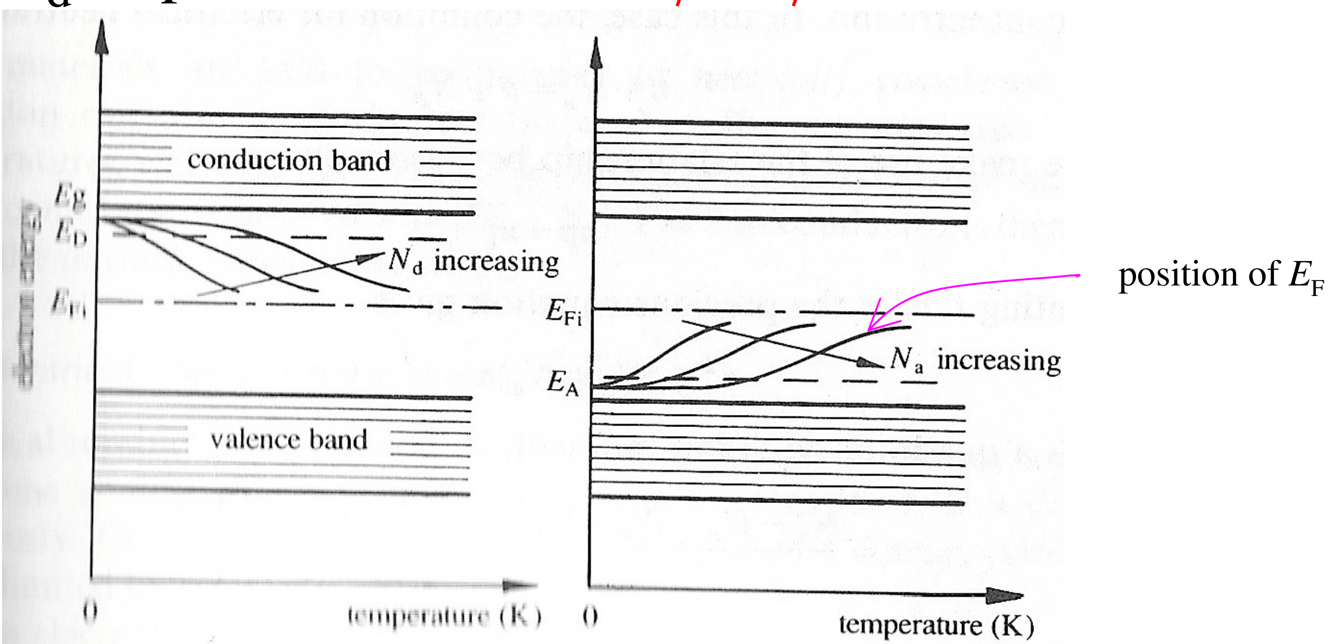
which can be solved for E_F

Density of carriers in extrinsic sC

- n-type material:
  - $N_d >> N_a$ & $n >> p$ $\Rightarrow$ $\overset{0}{\cancel{p}} + N_d^+ = n + \overset{0}{\cancel{N_a^-}}$

$$\Rightarrow \exp(E_F/kT) = \frac{-1+\{1+(4N_d/N_c)\exp[(E_g-E_D)/kT]\}^{1/2}}{2\exp(-E_D/kT)}$$

- p-type material:
  - $N_a >> N_d$ & $p >> n$ $\Rightarrow$ $p + \overset{0}{\cancel{N_d^+}} = \overset{0}{\cancel{n}} + N_a^-$



Compensation doping

- Compensation: both donors and acceptors
  - Suppose all impurities are ionized ($N_a = N_a^+$ & $N_d = N_d^+$)
  - Neutrality: $n + N_a = p + N_d \xrightarrow{np = n_i^2} n^2 - (N_d - N_a)n - n_i^2 = 0$

$$\Rightarrow \quad n = \frac{N_d - N_a}{2} + \frac{N_d - N_a}{2}\left[1 + \left(\frac{2n_i}{N_d - N_a}\right)^2\right]^{1/2}$$

(Idem for p)

- Extrinsic compensated $n_i \ll |N_d - N_a|$

for $N_d > N_a$: $n \simeq N_d - N_a$, $p \simeq n_i^2/(N_d - N_a)$

for $N_a > N_d$: $p \simeq N_a - N_d$, $n \simeq n_i^2/(N_a - N_d)$

- Intrinsic compensated $n_i \gg |N_d - N_a|$

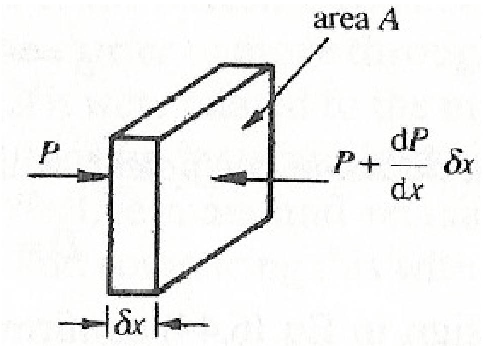
$$n = n_i + (N_d - N_a)/2 \qquad p = n_i + (N_d - N_a)/2$$

Electrical conduction in sC.

- Conductivity
  - In a sC we have e^{-}'s and h^{+}'s moving:

$$J_e = ne\mu_e \mathscr{E} \quad \& \quad J_h = pe\mu_h \mathscr{E} \quad \Rightarrow \quad J = J_e + J_h = \overbrace{e(n\mu_e + p\mu_h)}^{\sigma}\mathscr{E}$$

  - In a n-type sC ($n>>p$): $\sigma_n \simeq en\mu_e \simeq eN_d\mu_e$
  - In a p-type sC ($p>>n$): $\sigma_p \simeq ep\mu_h \simeq eN_a\mu_h$
- Diffusion (when density gradients are present):
  - In a neutral gas with gradient in the x direction:



Force on the elemental volume

$$\Rightarrow \left[P(x) - \left(P + \frac{dP}{dx}\delta x\right)\right]A = -\frac{dP}{dx}\delta x A$$

Force on each particle

$$\Rightarrow F_D = -\frac{1}{N}\frac{dP}{dx}$$

$$\Rightarrow v_D = -\frac{\tau_r}{M}\frac{1}{N}\frac{dP}{dx} \xrightarrow{P = NkT} v_D = -\overbrace{\frac{\tau_r kT}{M}}^{D}\frac{1}{N}\frac{dN}{dx}$$

Electrical conduction in sC.

- Diffusion of charge carriers
  - In a sC with a concentration gradient of the e^- and h^+:

$$v_{\mathrm{De}} = -D_{\mathrm{e}} \frac{1}{n} \frac{\mathrm{d}n}{\mathrm{d}x} \Rightarrow J_{\mathrm{De}} = -nev_{\mathrm{D}} = eD_{\mathrm{e}}\,\mathrm{d}n/\mathrm{d}x$$

$$v_{\mathrm{D_h}} = -D_{\mathrm{h}} \frac{1}{p} \frac{\mathrm{d}p}{\mathrm{d}x} \Rightarrow J_{\mathrm{Dh}} = -eD_{\mathrm{h}}\,\mathrm{d}p/\mathrm{d}x$$

  - Diffusion coefficient is related with mobility:

$$D = \frac{\tau_{\mathrm{r}} kT}{M} \xrightarrow{\mu = e\tau_{\mathrm{r}}/m} \begin{matrix} D_{\mathrm{e}} = (kT/e)\mu_{\mathrm{e}} \\ D_{\mathrm{h}} = (kT/e)\mu_{\mathrm{h}} \end{matrix} \Rightarrow \boxed{D_{\mathrm{e}}/\mu_{\mathrm{e}} = D_{\mathrm{h}}/\mu_{\mathrm{h}} = kT/e}$$

  - Total current flow

$$\boxed{J_{\mathrm{e}} = ne\mu_{\mathrm{e}}\mathscr{E} + eD_{\mathrm{e}}\nabla n}$$

$$\boxed{J_{\mathrm{h}} = pe\mu_{\mathrm{h}}\mathscr{E} - eD_{\mathrm{h}}\nabla p} \xrightarrow{J_{\mathrm{h}} = 0} \mathscr{E} = D_{\mathrm{h}}\nabla p/p\mu_{\mathrm{h}}$$

current can be compensated at a particular electrical field

Continuity Equation for Minority Carriers

- General considerations:
  - Describes how charge density varies in time.
  - Two main causes for the variation:
    - Rate of generation and loss
    - Drift of carriers
- No current flow (no gradient & no $\mathcal{E}$)
  - Let's consider a **p-type** material:

$$dn/dt = G - R$$

Variation = Generation – Recombination

In this case, only T dependent

$$R = r\, n(t)\, p(t)$$

Continuity Equation for Minority Carriers

- No current flow (no gradient & no $\mathscr{E}$)
  - Consider first equilibrium:

$$dn/dt = 0 \Rightarrow G(T) = r n_0 p_0 = r n_i^2$$

  - Now consider variations from the equilibrium:

$$n = n_0 + \delta n \Leftrightarrow p = p_0 + \delta p \quad \text{with } \delta n = \delta p$$

To mantain charge neutrality

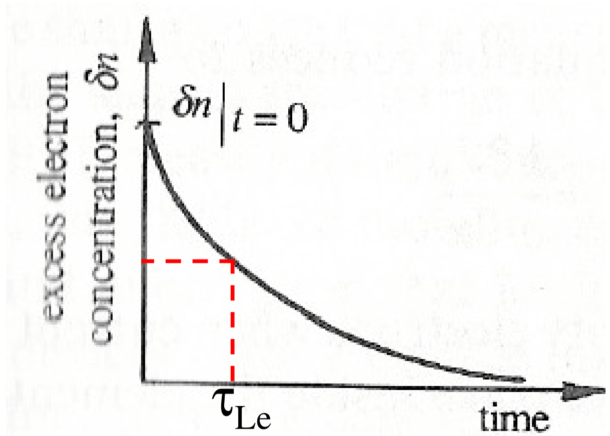
$$\Rightarrow (dn/dt)|_{J=0} = [d(\delta n)/dt]|_{J=0} = G(T) - r(n_0 + \delta n)(p_0 + \delta p)$$

$p_o >> n_o$ 0

$$\Rightarrow [d(\delta n)/dt]|_{J=0} = -\cancel{r n_0 \delta p} - r p_0 \delta n - \cancel{r \delta n \delta p}$$

Considering T=const.

$$\Rightarrow [d(\delta n)/dt]|_{J=0} = -r p_0 \delta n = -\delta n/\tau_{Le}$$

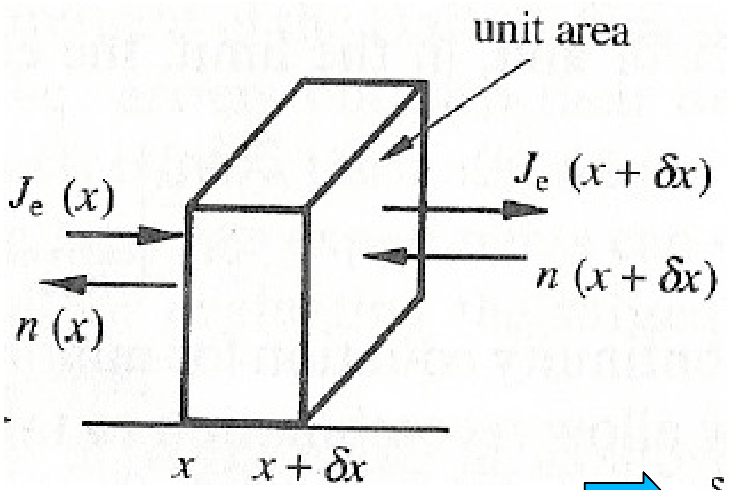


  - For a **n-type** material:

$$d(\delta p)/dt = -\delta p/\tau_{Lh}$$

Continuity Equation for Minority Carriers

- With current flow



$$J_e = -nev_D \Rightarrow \begin{cases} n_{x+\delta x} = J_e(x+\delta x)/ev_D \\ n_x = J_e(x)/ev_D \end{cases}$$

$$n_{x+\delta x} - n_x = \delta n = [J_e(x+\delta x) - J_e(x)]/ev_D$$

$$= \frac{1}{ev_D}\left(J_e(x) + \frac{\partial J_e}{\partial x}\delta x - J_e(x)\right)$$

$$\Rightarrow \delta n = \frac{1}{ev_D}\frac{\partial J_e}{\partial x}\delta x \xrightarrow{v_D = \partial x/\partial t} \left.\frac{\partial(\delta n)}{\partial t}\right|_{\text{current flow}} = \frac{1}{e}\frac{\partial J_e}{\partial x}$$

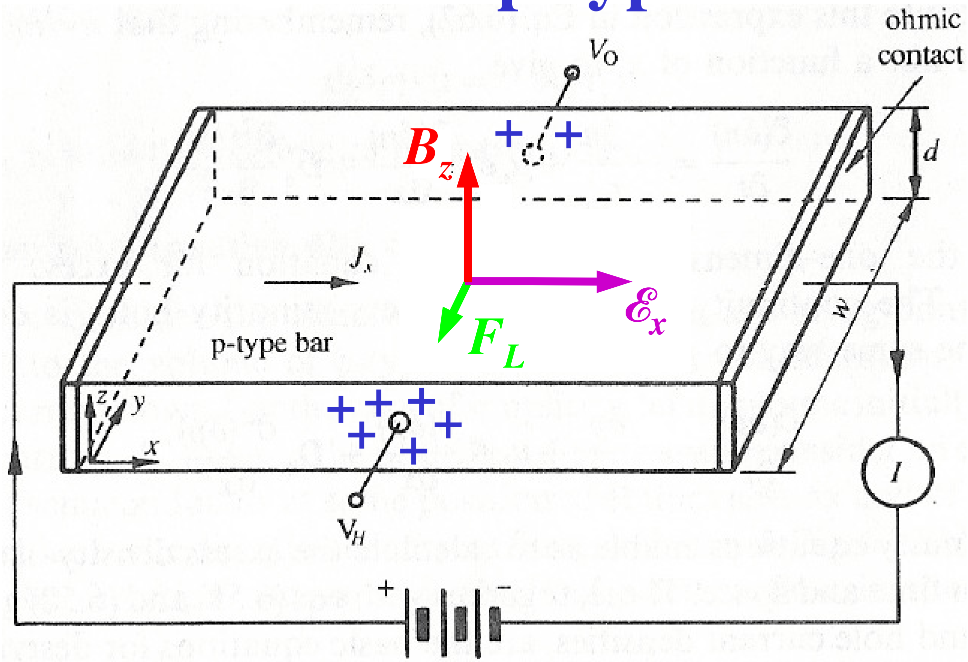
- Total continuity equation

$$\left.\frac{\partial(\delta n)}{\partial t}\right|_{\text{total}} = -\frac{\delta n}{\tau_{Le}} + \frac{1}{e}\frac{\partial J_e}{\partial x} \quad \text{(idem for holes)}$$

But $J_e = ne\mu_e\mathscr{E} + eD_e\nabla n \xrightarrow{n = n_0 + \delta n}$
$$\boxed{\frac{\partial(\delta n)}{\partial t} = -\frac{\delta n}{\tau_{Le}} + \mu_e\mathscr{E}_x\frac{\partial(\delta n)}{\partial x} + D_e\frac{\partial^2(\delta n)}{\partial x^2}}$$

Semiconductor Measurements

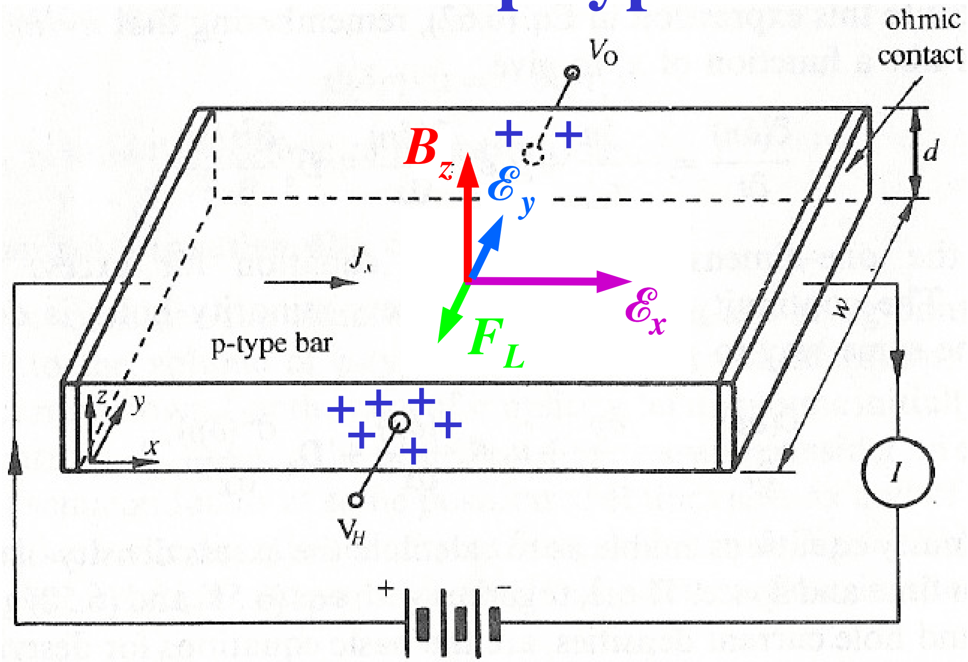
- Hall effect
  - Consider **p-type** material



$$F_L = ev \times B \Rightarrow |F_L| = e\, v_{Dx} B_z$$

Semiconductor Measurements

- Hall effect
  - Consider **p-type** material

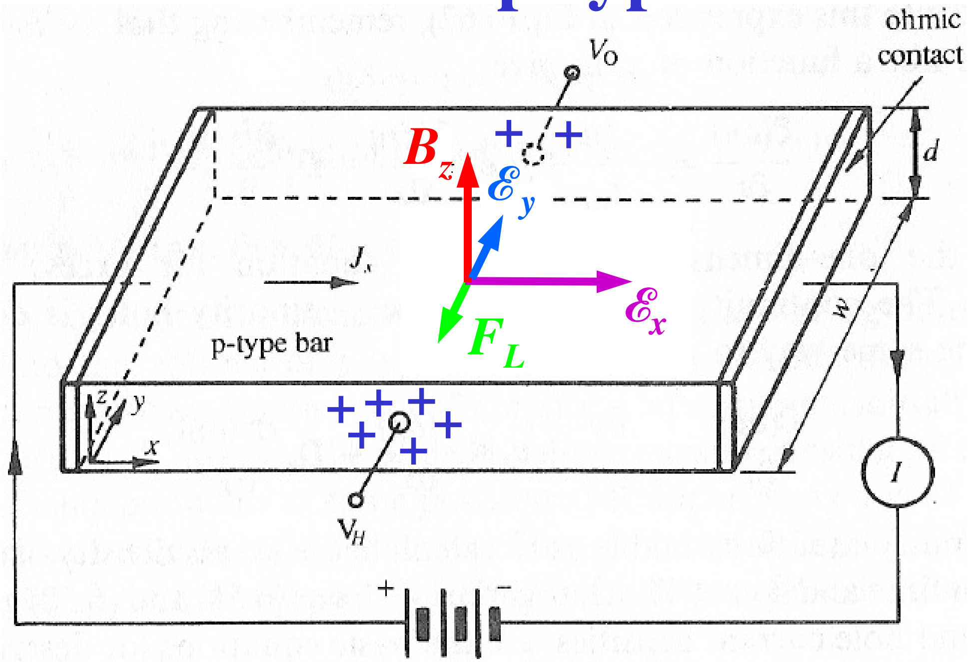


$$F_L = ev \times B \Rightarrow |F_L| = e\, v_{Dx}\, B_z$$

$$\Rightarrow e\mathscr{E}_y = F_L = ev_{Dx} B_z$$

Semiconductor Measurements

- Hall effect
  - Consider **p-type** material



$$F_L = ev \times B \Rightarrow |F_L| = e\, v_{Dx}\, B_z$$

$$\Rightarrow \left.\begin{aligned} e\mathscr{E}_y &= F_L = ev_{Dx}B_z \\ \& \quad J_x &\simeq pev_{Dx} \end{aligned}\right\} \mathscr{E}_y = J_x B_z / pe$$

The Hall coefficient is defined:

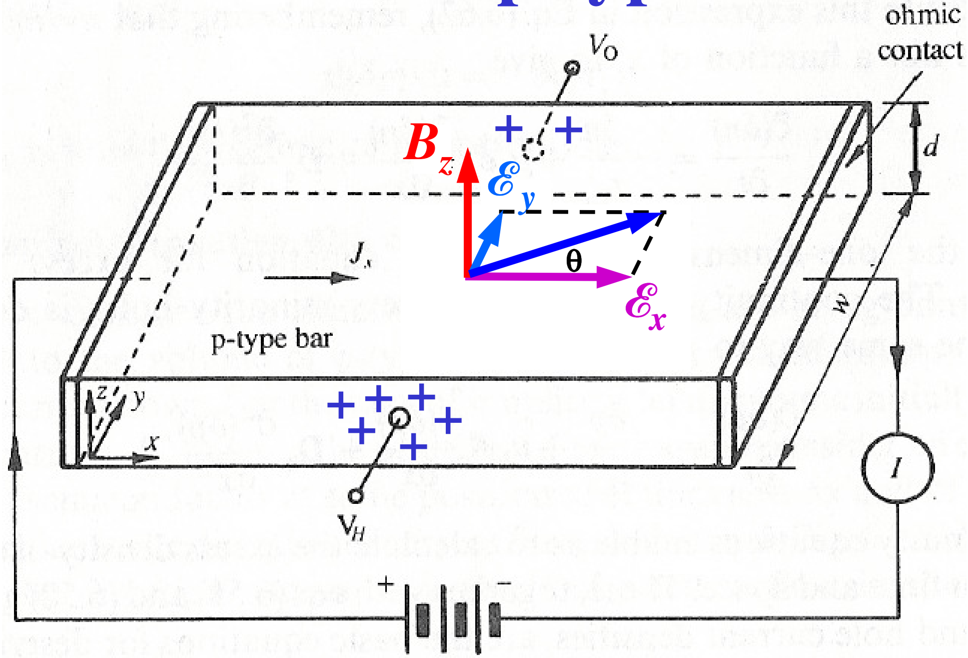
$$R_H = \mathscr{E}_y / J_x B_z = 1/pe$$

- If V_H and I are measured:

$$\left.\begin{aligned} V_H &= \mathscr{E}_y w \\ I &= J_x\, w\, d \end{aligned}\right\} R_H = \frac{V_H}{wIB_z} wd = \frac{V_H d}{IB_z} = \frac{1}{pe}$$

Semiconductor Measurements

- Hall effect
  - Consider **p-type** material



The Hall angle is defined:

$$\tan\theta = \mathscr{E}_y / \mathscr{E}_x$$

$$\Rightarrow \tan\theta = \frac{J_x B_z}{pe}\frac{\sigma}{J_x} = \mu_h B_z$$

$$\Rightarrow \mu_h = R_H \sigma$$

  - For a **n-type** material:

$$R_{He} = -1/ne$$

Semiconductor Measurements

- Hall effect
  - Consider both type of carriers to be present:

$$v_{Dh} = \mu_H \mathscr{E}_x \qquad F_h = -e(v_{Dh} \times B) = -e v_{Dh} B_z$$

$$v_{De} = -\mu_H \mathscr{E}_x \qquad F_e = e(v_{De} \times B) = -e v_{De} B_z$$

    - A net current is created in the *y* direction

$$\sigma \mathscr{E}_y = e(p v_{yh} - n v_{ye})$$

    - Now, using the expressions for the Hall angle

$$v_{yh} = \mu_h \mathscr{E}_y = \mu_h(\mathscr{E}_x \tan\theta) = \mu_h(\mathscr{E}_x \mu_h B_z) \qquad \& \qquad v_{ye} = \mu_e^2 \mathscr{E}_x B_z$$

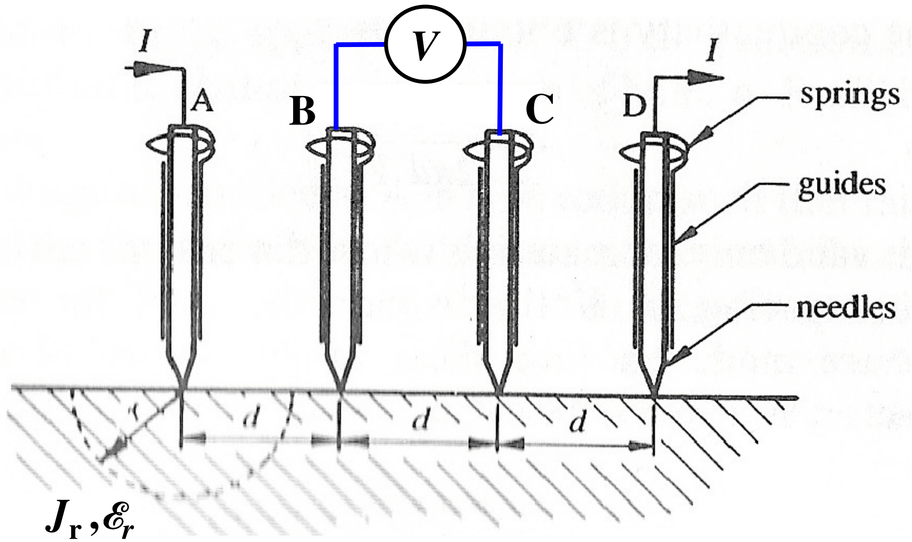
$$\Rightarrow \quad \sigma \mathscr{E}_y = e(p\mu_h^2 - n\mu_e^2)\mathscr{E}_x B_z$$

    - Hall coefficient

$$R_H = \frac{\mathscr{E}_y}{J_x B_z} = \frac{e(p\mu_h^2 - n\mu_e^2)}{\sigma^2} \quad \xrightarrow{\sigma = e(n\mu_e + p\mu_h)} \quad R_H = \frac{p\mu_h^2 - n\mu_e^2}{e(p\mu_h + n\mu_e)^2}$$

Semiconductor Measurements

- Four-point probe method for conductivity meas.
  - Consider $d << L$



For the current leaving $\boldsymbol{A}$:

$$J_r = I/(2\pi r^2) \Rightarrow \mathscr{E}_r = J/\sigma = I/(2\pi\sigma r^2)$$

Potential at a distance a from $\boldsymbol{A}$:

$$V_a = \int_{-\infty}^{a} \mathscr{E}_r \, \mathrm{d}r = -\frac{I}{2\pi\sigma}\int_{-\infty}^{a} \frac{1}{r^2}\,\mathrm{d}r = \frac{I}{2\pi\sigma a}$$

$$\Rightarrow V_{BC} = \frac{I}{2\pi\sigma d} - \frac{I}{2\pi\sigma(2d)} = \frac{I}{4\pi\sigma d}$$

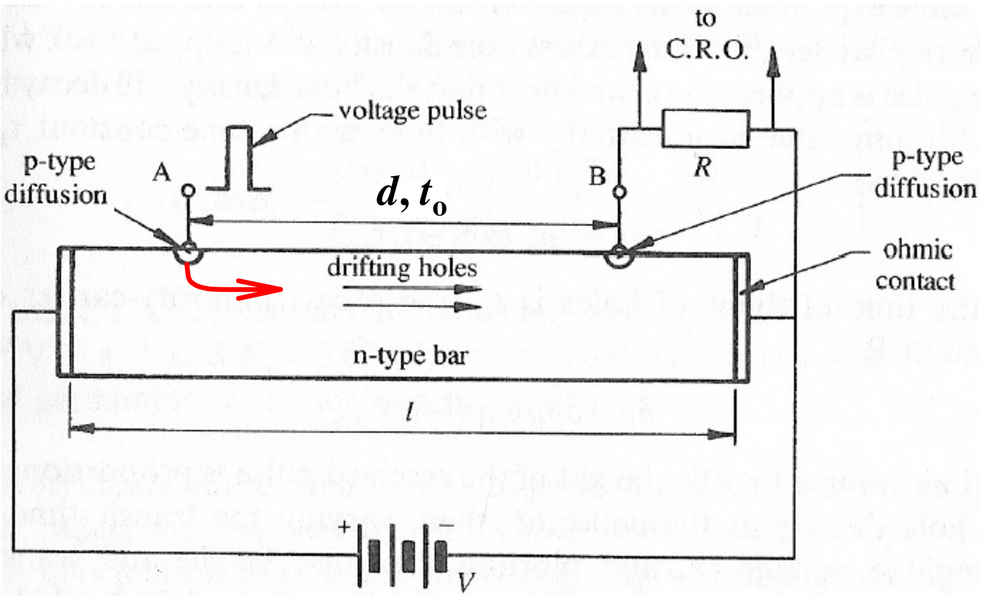
Identically for the current entering $\boldsymbol{D}$, then:

$$V = 2V_{BC} = \frac{I}{(2\pi\sigma d)}$$

$$\Rightarrow \sigma = \frac{1}{2\pi d}\frac{I}{V}$$

Semiconductor Measurements

- Minority carrier life-time and mobility.



Mobility:

$$v_D = d/t_0 \Rightarrow \mu E = \mu V/l = d/t_0$$

$$\Rightarrow \boxed{\mu = ld/t_0 V}$$

Life-time:

$$d(\delta p)/dt = -\delta p/\tau_{Lh} \Rightarrow \delta p = \delta p_0 \exp(-t/\tau_{Lh})$$