# Physics of Electronics

## Problem Set 1:
## Chapters 01 to 04

July – December 2008

# The problems were taken from:

¥ D. Griffiths, "Introduction to Quantum Mechanics," 1994.

¶ J. Allison, *"Electronic Engineering: Semiconductors and Devices,"* 1990.

Ħ C. Kittel, *"Introduction to Solid State Physics,"*, 1996.

# Introduction to Q.M.

¥ **Problem 1.7** At time $t = 0$ a particle is represented by the wave function

$$\Psi(x,0) = \begin{cases} Ax/a, & \text{if } 0 \le x \le a, \\ A(b-x)/(b-a), & \text{if } a \le x \le b, \\ 0, & \text{otherwise,} \end{cases}$$

where $A$, $a$, and $b$ are constants.

**(a)** Normalize $\Psi$ (that is, find $A$ in terms of $a$ and $b$).

**(b)** Sketch $\Psi(x, 0)$ as a function of $x$.

**(c)** Where is the particle most likely to be found, at $t = 0$?

**(d)** What is the probability of finding the particle to the left of $a$? Check your result in the limiting cases $b = a$ and $b = 2a$.

**(e)** What is the expectation value of $x$?

¥ ∗**Problem 1.8** Consider the wave function

$$\Psi(x,t) = Ae^{-\lambda|x|}e^{-i\omega t},$$

where $A$, $\lambda$, and $\omega$ are positive real constants. [We'll see in Chapter 2 what potential ($V$) actually produces such a wave function.]

**(a)** Normalize $\Psi$.

**(b)** Determine the expectation values of $x$ and $x^2$.

**(c)** Find the standard deviation of $x$. Sketch the graph of $|\Psi|^2$, as a function of $x$, and mark the points $(\langle x\rangle + \sigma)$ and $(\langle x\rangle - \sigma)$ to illustrate the sense in which $\sigma$ represents the "spread" in $x$. What is the probability that the particle would be found outside this range?

# Introduction to Q.M.

¥ *Problem 1.14** A particle of mass $m$ is in the state

$$\Psi(x,t) = Ae^{-a[(mx^2/\hbar)+it]},$$

where $A$ and $a$ are positive real constants.

**(a)** Find $A$.

**(b)** For what potential energy function $V(x)$ does $\Psi$ satisfy the Schrödinger equation?

**(c)** Calculate the expectation values of $x$, $x^2$, $p$, and $p^2$.

Not obligatory but extra points!!!

**(d)** Find $\sigma_x$ and $\sigma_p$. Is their product consistent with the uncertainty principle?

**Problem 2.44** (*Attention*: This is a strictly qualitative problem—no calculations allowed!) Consider the "double square well" potential (Figure 2.17). Suppose the depth $V_0$ and the width $a$ are fixed, and great enough so that several bound states occur.

**(a)** Sketch the ground-state wave function $\psi_1$ and the first excited state $\psi_2$, (i) for the case $b = 0$, (ii) for $b \approx a$, and (iii) for $b \gg a$.

**(b)** Qualitatively, how do the corresponding energies ($E_1$ and $E_2$) vary, as $b$ goes from 0 to $\infty$? Sketch $E_1(b)$ and $E_2(b)$ on the same graph.

**(c)** The double well is a very primitive one-dimensional model for the potential experienced by an electron in a diatomic molecule (the two wells represent the attractive force of the nuclei). If the nuclei are free to move, they will adopt the configuration of minimum energy. In view of your conclusions in (b), does the electron tend to draw the nuclei together, or push them apart? (Of course, there is also the internuclear repulsion to consider, but that's a separate problem.)

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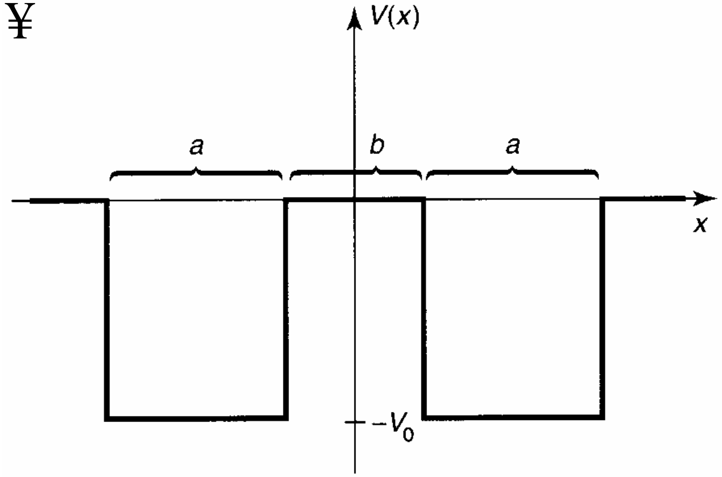


**Figure 2.17:** The double square well (Problem 2.44).

# Electronic Structure of Atoms

¥ **Problem 2.4** Solve the time-independent Schrödinger equation with appropriate boundary conditions for an infinite square well centered at the origin [$V(x) = 0$, for $-a/2 < x < +a/2$; $V(x) = \infty$ otherwise]. Check that your allowed energies are consistent with mine (Equation 2.23), and confirm that your $\psi$'s can be obtained from mine (Equation 2.24) by the substitution $x \to x - a/2$.

$$E_n = \frac{\hbar^2 k_n^2}{2m} = \frac{n^2\pi^2\hbar^2}{2ma^2}. \quad \textbf{(Eq. 2.23)}$$

$$\psi_n(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi}{a}x\right). \quad \textbf{(Eq. 2.24)}$$

¥ **Problem 4.3** Use Equations 4.27, 4.28, and 4.32 to construct $Y_0^0$ and $Y_2^1$. Check that they are normalized and orthogonal.

$$P_l^m(x) \equiv (1 - x^2)^{|m|/2} \left(\frac{d}{dx}\right)^{|m|} P_l(x) \quad \textbf{(Eq. 4.27)}$$

$$P_l(x) \equiv \frac{1}{2^l l!} \left(\frac{d}{dx}\right)^l (x^2 - 1)^l. \quad \textbf{(Eq. 4.28)}$$

$$Y_l^m(\theta, \phi) = \epsilon \sqrt{\frac{(2l + 1)}{4\pi} \frac{(l - |m|)!}{(l + |m|)!}} e^{im\phi} P_l^m(\cos\theta), \quad \textbf{(Eq. 4.32)}$$

¥ **Problem 4.13**

**(a)** Find $\langle r \rangle$ and $\langle r^2 \rangle$ for an electron in the ground state of hydrogen. Express your answers in terms of the Bohr radius $a$.

**(b)** Find $\langle x \rangle$ and $\langle x^2 \rangle$ for an electron in the ground state of hydrogen. *Hint*: This requires no new integration—note that $r^2 = x^2 + y^2 + z^2$, and exploit the symmetry of the ground state.

**(c)** Find $\langle x^2 \rangle$ in the state $n = 2$, $l = 1$, $m = 1$. *Hint*: This state is *not* symmetrical in $x, y, z$. Use $x = r \sin\theta \cos\phi$.

**Not obligatory but extra points!!!**

# Assembly of Classical Particles

¶ A hypothetical gas with $N$ molecules per cubic metre has a speed distribution function

$$f(v) = Cv^2 \quad \text{for } v_0 > v > 0$$
$$f(v) = 0 \quad \text{for } v > v_0$$

Find the mean-square fluctuation of the speeds, which is defined as the mean-square speed minus the square of the mean speed.

*Ans.* $0.04v_0^2$

¶ At $T = 0$ K the electron energy levels in a metal are all occupied for $E < E_F$ and are empty for $E > E_F$. The energy distribution is then of the form

$$\Delta N/N = CE^{1/2}\Delta E \quad \text{for } E < E_F$$
$$\Delta N/N = 0 \quad \text{for } E > E_F$$

where $C$ is a constant. Find (a) the average electron energy under these conditions and (b) the percentage of the total number of electrons with energies between $0.1E_F$ and $0.2E_F$.

*Ans.* $0.6E_F$, 5.8 per cent.

# Conduction in Metals

¶ **2.** The Fermi level in copper at 0 K is 7.0 eV. Estimate the number of free electrons per unit volume in copper at this temperature.

*Ans.* $8.4 \times 10^{28}\ \mathrm{m}^{-3}$

¶ **3.** Calculate the Fermi energy at 0 K in copper given that there is one conduction electron per atom, that the density of copper is $8920\ \mathrm{kg\ m}^{-3}$ and its atomic weight is 63.54.

*Ans.* 7.06 eV

**1.** Find the wave functions and their energies for a 2D electron gas.

**2**. Find the density of states for a 2D electron gas.

**5.** ***Liquid He$^3$.*** The atom He$^3$ has spin $\frac{1}{2}$ and is a fermion. The density of liquid He$^3$ is $0.081\ \mathrm{g\ cm}^{-3}$ near absolute zero. Calculate the Fermi energy $\epsilon_F$ and the Fermi temperature $T_F$.

**6.** ***Frequency dependence of the electrical conductivity.*** Use the equation $m(dv/dt + v/\tau) = -eE$ for the electron drift velocity $v$ to show that the conductivity at frequency $\omega$ is

$$\sigma(\omega) = \sigma(0)\left(\frac{1 + i\omega\tau}{1 + (\omega\tau)^2}\right) , \qquad (62)$$

where $\sigma(0) = ne^2\tau/m$ .