CONTROL Nº 2

EL 31-A ANALISIS DE REDES I

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1. Para una red lineal e invariante, se tiene la respuesta al impulso $h(t)$ de la Figura 1. Encuentre la respuesta de estado cero para la entrada $f(t)$ de la Figura 2, utilizando convolución gráfica.

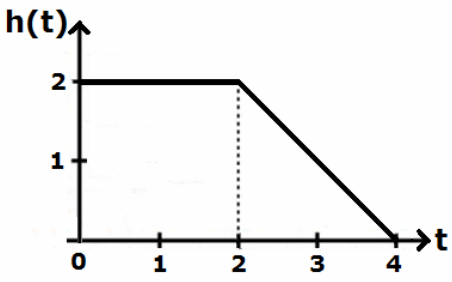


Figura 1

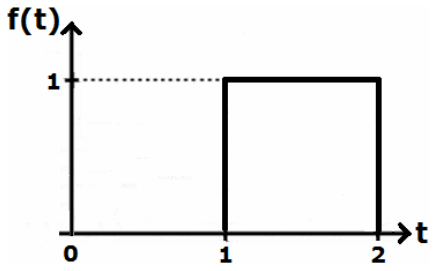
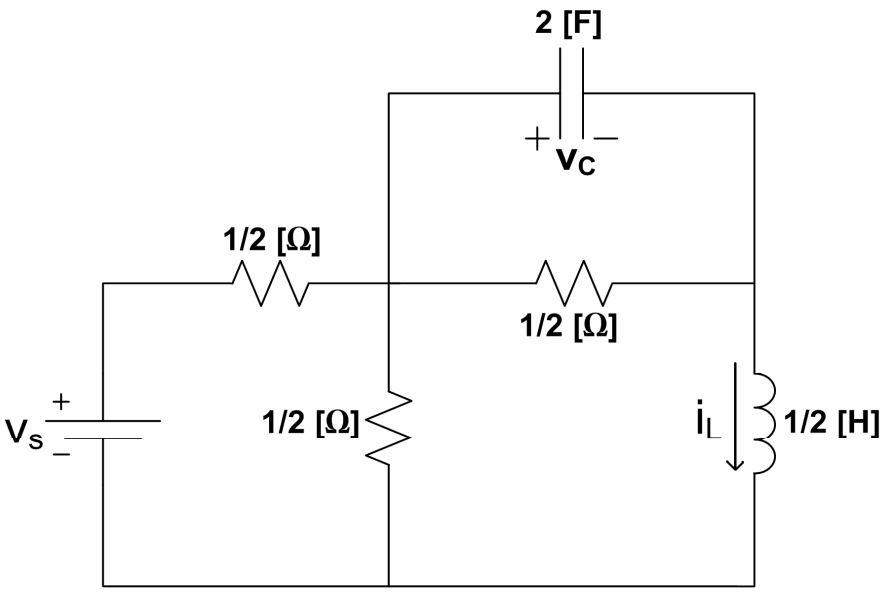


Figura 2

2. Para la red lineal e invariante de la figura, determine:

   i) La ecuación diferencial para la variable $v_c(t)$, ocupando el método de los nodos y el operador D (no considere las condiciones iniciales como fuentes). Indicando las condiciones iniciales para $v_c(t)$ y su derivada a partir del circuito en *t=0*.
   ii) Considerando que $V_s(t)=2e^{-t}u(t)$, calcule la respuesta completa $v_c(t)$ para $t\geq 0$



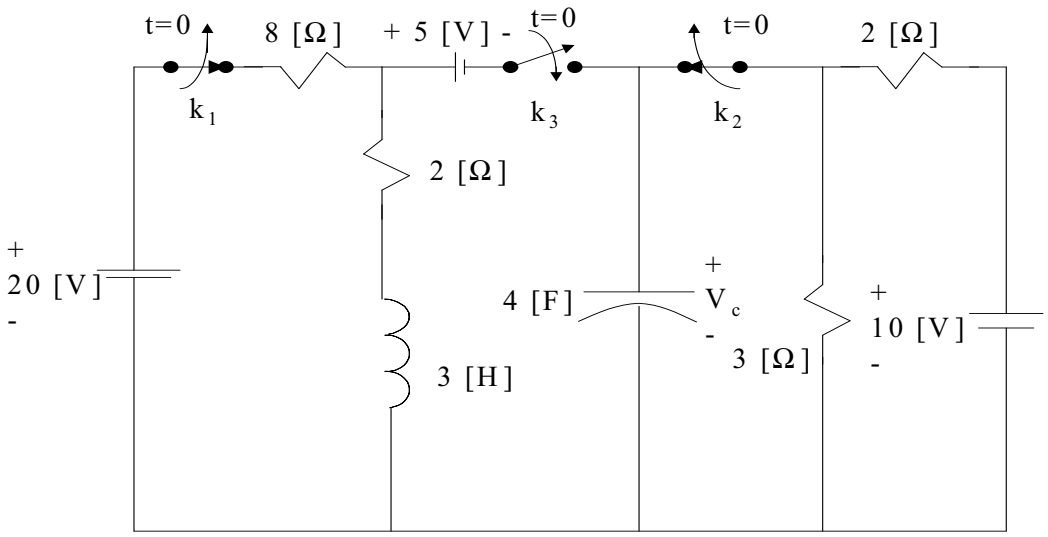
$v_C(0) = 1[V]$ $i_L(0) = 4[A]$

3.- En la red de la figura, los interruptores k_1 y k_2 han estado cerrados y k_3 abierto, por un tiempo muy largo (hasta alcanzar el régimen permanente). En $t=^-0$ simultáneamente se abren los interruptores k_1 y k_2 y se cierra k_3. Para $v_c(t)$, para $t\geq 0$, determine:

a) La respuesta de entrada cero.

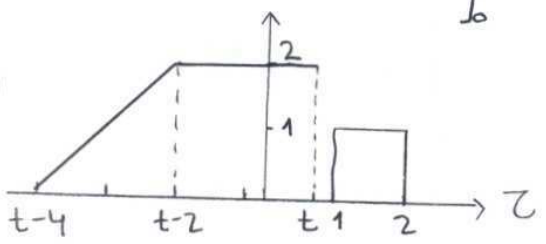
b) La respuesta de estado cero.

c) La respuesta completa, identificando su parte permanente y su parte transitoria.



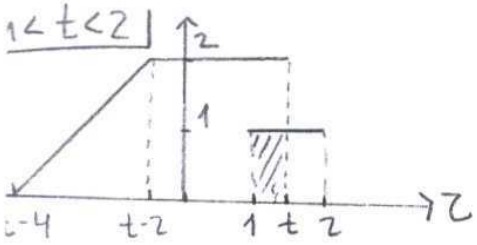
[M1] $y(t) = h(t) * f(t) \Rightarrow \int_0^{\cdot} h(t-\tau) f(\tau)\, d\tau$

$t<1$



$\Rightarrow$ salida = $\boxed{y=0}$ Pues no hay intersección. $(t<1)$

$1<t<2$

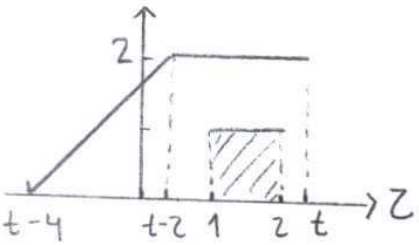


$f \longrightarrow 1\,(1,\dots,t)$

$h \longrightarrow 2\,(1,\dots,t)$

$$\Rightarrow y = \int_1^t 2 \cdot 1\, d\tau = 2\tau \Big|_1^t = \boxed{2t-2 = y \quad (1<t<2)}$$

$2<t<3$

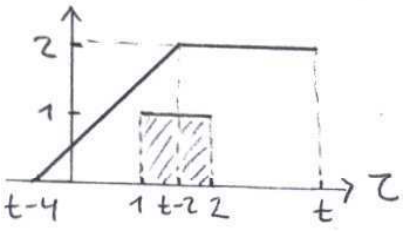


$f \longrightarrow 1\,(1,\dots,2)$

$h \longrightarrow 2\,(1,\dots,2)$

$$\Rightarrow y = \int_1^2 2 \cdot 1\, d\tau = 2\tau \Big|_1^2 = 4-2 \Rightarrow \boxed{y = 2 \quad (2<t<3)}$$

$3<t<4$



$f \longrightarrow 1\,(1,\dots,2)$

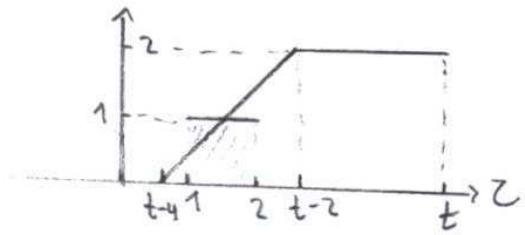
$h \longrightarrow \tau+4-t\,(1,\dots,t-2)$

$\quad\quad 2\,(t-2,\dots,2)$

$$\Rightarrow y = \int_1^{t-2} (\tau+4-t)\, d\tau + \int_{t-2}^2 2\, d\tau$$

$$= \frac{\tau^2}{2}\Big|_1^{t-2} + (4-t)\tau \Big|_1^{t-2} + 2\tau \Big|_{t-2}^2 = \boxed{\frac{-t^2}{2} + 3t - \frac{5}{2} = y \quad (3<t<4)}$$

$4<t<5$

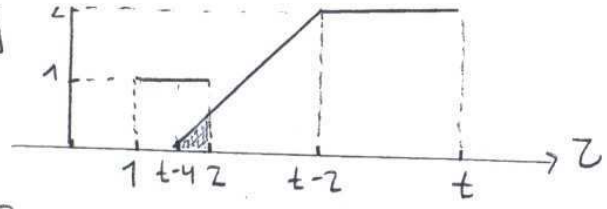


$f \longrightarrow 1\,(1,\dots,2)$

$h \longrightarrow \tau+4-t\,(1,\dots,2)$

$$\Rightarrow y = \int_1^2 (\tau+4-t)\, d\tau = \boxed{\frac{11}{2} - t = y \quad (4<t<5)}$$

$5<t<6$



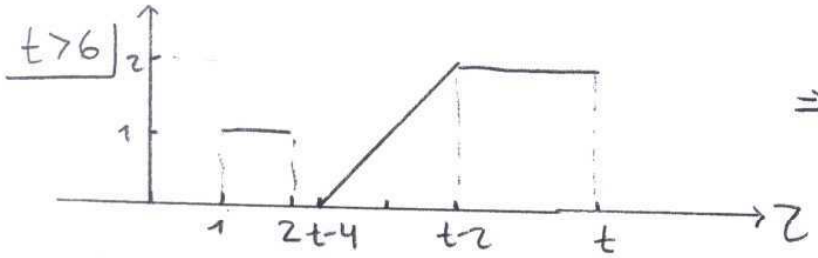
$f \rightarrow 1\,(t-4,\ldots,2)$

$h \rightarrow z+4-t\,(t-4,\ldots,2)$

$$\Rightarrow y = \int_{t-4}^{2} (z+4-t)\,dz = \frac{z^2}{2}\Big|_{t-4}^{2} + (4-t)z\Big|_{t-4}^{2} = \boxed{\frac{t^2}{2} - 6t + 18 = y}$$

$(5<t<6)$

$t>6$

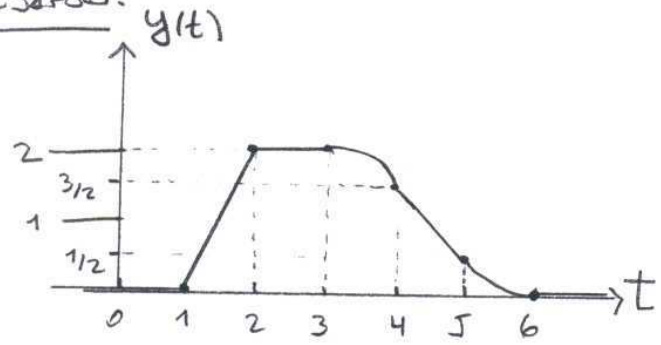


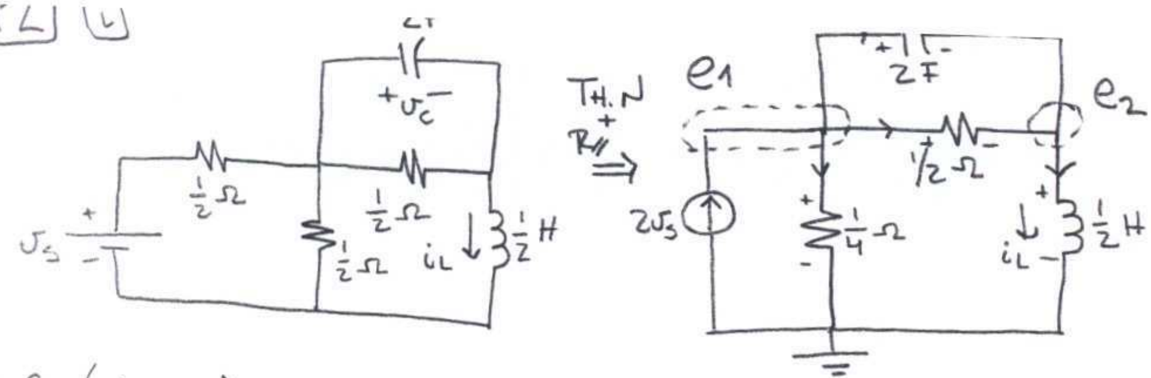
$\Rightarrow \boxed{y = 0 \;.\; (t>6)}$

$$\Rightarrow y(t) = \begin{cases} 0 & t<1 \\ 2t-2 & 1<t<2 \\ 2 & 2<t<3 \\ -\frac{t^2}{2} + 3t - \frac{5}{2} & 3<t<4 \\ \frac{11}{2} - t & 4<t<5 \\ \frac{t^2}{2} - 6t + 18 & 5<t<6 \\ 0 & t>6 \end{cases}$$

Respuesta de estado cero.

Bosquejando:





$i_L(0) = 4[A]$

$v_c(0) = 1[V]$

① $e_1(6+2D) + e_2(-2-2D) = 2v_s \quad /:2$

$$\Rightarrow \boxed{e_1(3+D) + e_2(-1-D) = v_s}$$

② $e_1(-2D-2) + e_2\left(2D+2+\frac{2}{D}\right) = 0 \quad /:2$

$$\boxed{e_1(-1-D) + e_2\left(D+1+\frac{1}{D}\right) = 0}$$

$$\Rightarrow \begin{bmatrix} 3+D & -1-D \\ -1-D & D+1+\frac{1}{D} \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} v_s \\ 0 \end{bmatrix}$$

buscamos $v_c = e_1 - e_2$.

$\Rightarrow e_1 = \frac{\Delta_1}{\Delta}$ y $e_2 = \frac{\Delta_2}{\Delta}$ (Por Regla de Cramer)

$$\Delta = (3+D)\left(D+1+\frac{1}{D}\right) - (-1-D)^2 \Rightarrow \boxed{\Delta = 2D+3+\frac{3}{D}}$$

$$\boxed{\Delta_1 = v_s\left(D+1+\frac{1}{D}\right)} \quad \boxed{\Delta_2 = v_s(1+D)}$$

$$\Rightarrow e_1 = \frac{v_s\left(D+1+\frac{1}{D}\right)}{2D+3+\frac{3}{D}} \Rightarrow \boxed{\frac{v_s(D^2+D+1)}{2D^2+3D+3} = e_1}$$

$$\Rightarrow e_2 = \frac{v_s(1+D)}{2D+3+\frac{3}{D}} \Rightarrow \boxed{\frac{v_s(D^2+D)}{2D^2+3D+3} = e_2}$$

$$\Rightarrow e_1 - e_2 = \frac{v_s(\cancel{D^2}+\cancel{D}+1-\cancel{D^2}-\cancel{D})}{2D^2+3D+3} \Rightarrow \boxed{v_c = \frac{v_s}{2D^2+3D+3}}$$

$$\Rightarrow \boxed{2\frac{\partial^2 v_c}{\partial t^2} + 3\frac{\partial v_c}{\partial t} + 3v_c = v_s} \quad \boxed{v_c(0) = 1\,[V].}$$

¿$\frac{\partial v_c}{\partial t}(0)$? de LCK en ②: $i_R + i_C = i_L$ Pero $v_R = v_c$

$\Rightarrow \frac{1}{2} i_R = v_c \Rightarrow i_R = 2v_c$. Y como $i_C = 2\frac{\partial v_c}{\partial t}$

$$\Rightarrow 2\frac{\partial v_c}{\partial t} = i_L - 2v_c \Rightarrow \frac{\partial v_c}{\partial t}(0) = \frac{\cancel{i_L(0)}^{\,4}}{2} - \cancel{v_c(0)}^{\,1} \Rightarrow \boxed{\frac{\partial v_c}{\partial t}(0) = 1}$$

ii) $\Rightarrow$ como $v_s = 2e^{-t}u(t)$

$$\Rightarrow \boxed{2\frac{\partial^2 v_c}{\partial t^2} + 3\frac{\partial v_c}{\partial t} + 3v_c = 2e^{-t}} \quad v_c(0) = 1 \quad \frac{\partial v_c}{\partial t}(0) = 1$$

Respuesta Completa = RESC + RENC.

RESC $\Rightarrow$ $v_c(0) = 0$, $i_L(0) = 0$ $\Rightarrow \frac{\partial v_c}{\partial t} = 0$

Homogénea: $2\lambda^2 + 3\lambda + 3 = 0 \Rightarrow \lambda = \frac{-3 \pm \sqrt{9-24}}{4}$ $\Rightarrow$ Raíces complejas conjugadas.

$\Rightarrow v_c(t) = K_1 e^{-\frac{3}{4}t}\cos(\omega_d t + \phi)$ con $\omega_d = \sqrt{\frac{3}{2} - \frac{9}{16}} = \boxed{\sqrt{\frac{15}{16}} \approx 1. = \omega_d}$ (aproximación)

$\Rightarrow \boxed{v_c(t)_H = K_1 e^{-\frac{3}{4}t}\cos(t+\phi)}$

Particular: $v_c = Ce^{-t} \Rightarrow$ en la ec. d.f. $2Ce^{-t} - 3Ce^{-t} + 3Ce^{-t} = 2e^{-t} \Rightarrow \boxed{C = 1}$

$\Rightarrow \boxed{v_c(t)_P = e^{-t}}$

$\Rightarrow \boxed{v_{cresc}(t) = K_1 e^{-\frac{3}{4}t}\cos(t+\phi) + e^{-t}}$ $v_c(0) = 0$, $\frac{\partial v_c}{\partial t}(0) = 0$ ¿K_1, ϕ?

$$\frac{\partial v_c}{\partial t}(t) = -\frac{3}{4}K_1 e^{-\frac{3}{4}t}\cos(t+\phi) - K_1 e^{-\frac{3}{4}t}\operatorname{sen}(t+\phi) - e^{-t}$$

$\Rightarrow v_c(0) = 0 = K_1\cos(\phi) + 1 \Rightarrow \boxed{K_1\cos(\phi) = -1}$ (1)

$$\frac{\partial v_c}{\partial t}(0) = 0 = -\frac{3}{4}\underbrace{K_1\cos(\phi)}_{-1} - K_1\operatorname{sen}(\phi) - 1 \Rightarrow K_1 = \frac{-1}{4\operatorname{sen}(\phi)}$$ en (1):

(1): $\frac{\cos(\phi)}{4\,\mathrm{sen}(\phi)} = 1 \Rightarrow tg(\phi) = \frac{1}{4} \Rightarrow \boxed{\phi = tg^{-1}\left(\frac{1}{4}\right)}$

$\Rightarrow (\phi = 14{,}04°)$

$\Rightarrow$ en (1): $K_1 = \frac{-1}{4\cos(14{,}04°)} \Rightarrow \boxed{K_1 = -0{,}257}$

REAL: $2\frac{\partial^2 v_c}{\partial t^2} + 3\frac{\partial v_c}{\partial t} + 3v_c = 0$ $\quad \left(v_c(0) = 1,\ \frac{\partial v_c}{\partial t}(0) = 1\right)$

$\Rightarrow v_c(t) = K_2 e^{-\frac{3}{4}t}\cos(t+\phi_2) \Rightarrow \frac{\partial v_c}{\partial t}(t) = -\frac{3}{4}K_2 e^{-\frac{3}{4}t}\cos(t+\phi_2) - K_2 e^{-\frac{3}{4}t}\mathrm{sen}(t+\phi_2)$

$\Rightarrow 1 = K_2\cos(\phi_2)$

$1 = -\frac{3}{4}\underbrace{K_2\cos(\phi_2)}_{=1} - K_2\,\mathrm{sen}(\phi_2) \Rightarrow K_2 = \frac{-7}{4\,\mathrm{sen}(\phi_2)}$

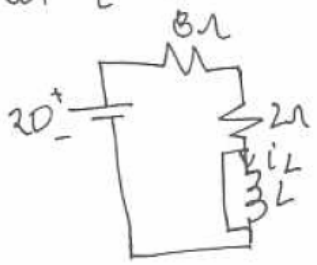
$\Rightarrow 1 = \frac{-7\cos(\phi_2)}{4\,\mathrm{sen}(\phi_2)} \Rightarrow tg(\phi_2) = \frac{-7}{4} \Rightarrow \phi_2 = tg^{-1}\left(\frac{-7}{4}\right) \Rightarrow \boxed{-60{,}26° = \phi_2}$

$\Rightarrow K_2 = \frac{1}{\cos(\phi_2)} \Rightarrow \boxed{K_2 = 2{,}016}$

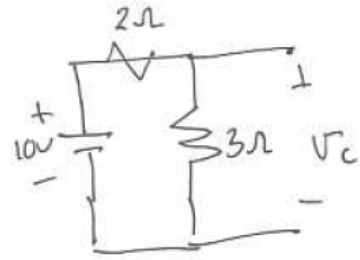
Resp. Completa = RESC + REAL

$\Rightarrow \boxed{v_c(t) = \left[e^{-t} - 0{,}257\cdot e^{-\frac{3}{4}t}\cos(t+14{,}04°) + 2{,}016\cdot e^{-\frac{3}{4}t}\cos(t-60{,}26°)\right]\cdot u(t)}$

2) En $t = 0^-$

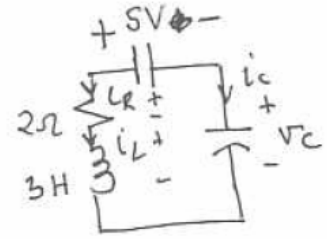


$i_L(0^-) = \frac{20}{10} = 2A$



$V_c(0^-) = \frac{10}{5} \times 3 = 6[V]$

En $t \geq 0$ tenemos un circuito serie



$i_R + i_c = 0 \qquad i_R = i_L$

$V_c - V_L - V_R + 5 = 0$

$V_c - L\frac{di_L}{dt} - Ri_L = -5$

$i_L = -i_c \qquad i_R = -i_c$

$\Rightarrow L\frac{di_c}{dt} + Ri_c + V_c = -5$

$$i_c = C\frac{dV_c}{dt}$$

$$\Rightarrow LC\frac{d^2V_c}{dt^2} + RC\frac{dV_c}{dt} + V_c = -5 \quad \Rightarrow \quad \frac{d^2V_c}{dt^2} + \frac{R}{L}\frac{dV_c}{dt} + \frac{1}{LC}V_c = \frac{-5}{LC}$$

reemplazando números

$$\frac{d^2V_c}{dt^2} + \frac{2}{3}\frac{dV_c}{dt} + \frac{1}{12}V_c = -\frac{5}{12}$$

$V_c(0^+) = 6\,V$

$\frac{dV_c}{dt}(0^+) = -1/2$

$V_c(0) = 6V$

$i_L(0^+) = 2A \Rightarrow i_c(0^+) = -2A$

$C\frac{dV_c}{dt}(0^+) = -2$

$\Rightarrow \frac{dV_c}{dt}(0^+) = -\frac{1}{2}$

REN C

$$V_c(t) = K_1 e^{-1/2\,t} + K_2 e^{-1/6\,t} \quad \Rightarrow \quad K_1 + K_2 = 6$$

$$-\frac{1}{2}K_1 - \frac{1}{6}K_2 = -1/2$$

$$\Rightarrow K_2 = \frac{15}{2} \quad K_1 = -\frac{3}{2}$$

$$V_c(t) = \left(-\frac{3}{2}e^{-1/2\,t} + \frac{15}{2}e^{-1/6\,t}\right)u(t)$$ //

RESC

$$V_{c_4} = K_3 e^{-1/2\,t} + K_4 e^{-1/6\,t}$$

$V_{cp} = -5$

$$V_c(t) = K_3 e^{-1/2\,t} + K_4 e^{-1/6\,t} - 5 \quad \Rightarrow \quad K_3 + K_4 - 5 = 0$$

$$-1/2\,K_3 - 1/6\,K_4 = 0$$

$$\Rightarrow K_3 = -\frac{5}{2} \quad K_4 = \frac{15}{2}$$

$$V_c(t) = \left(-\frac{5}{2}e^{-1/2\,t} + \frac{15}{2}e^{-1/6\,t} - 5\right)u(t)$$ //

Respuesta completa

$$V_c(t) = \left(-4e^{-1/2\,t} + 15e^{-1/6\,t} - 5\right)u(t)$$

Respuesta permanente

$\Rightarrow V_c(t)$ cuando $t \to \infty$

$\Rightarrow V_{c\,perm} = -5$ //

$$V_{c_{Transitorio}} = \left(-4e^{-1/2\,t} + 15e^{-1/6\,t}\right)$$ //