TORSION DE SAINT VENANT: ELEMENTO TRIANGULAR DE 6 NODOS
EJE CIRCULAR CON RANURA

Coordenadas de los nodos

$ORIGIN := 1$

$k := 1 .. 61$

$x_k := 0$

$y_k := 0$

$i := 1 .. 24$

$\alpha_i := (i-1)\cdot\frac{\pi}{12}$

$x_i := \sin(\alpha_i)$ $\quad y_i := -\cos(\alpha_i)$

$i := 1 .. 12$ $\quad \alpha_i := (i-1)\cdot\frac{\pi}{6}$ $\quad x_{i+24} := \frac{\sin(\alpha_i)}{2}$ $\quad y_{i+24} := \frac{-\cos(\alpha_i)}{2}$

$b := 2\cdot\cos\left(\pi\cdot\frac{75}{180}\right)$ $\quad b = 0.518$

$x_{18} := b\cdot\cos\left(37.5\cdot\frac{\pi}{180}\right) - 1$ $\quad x_{18} = -0.589$

$y_{18} := b\cdot\sin\left(37.5\cdot\frac{\pi}{180}\right)$ $\quad y_{18} = 0.259$

$x_{19} := -0.482$ $\quad y_{19} := 0$ $\quad x_{20} := x_{18}$ $\quad y_{20} := -y_{18}$ $\quad x_{61} := 0$ $\quad y_{61} := 0$

$f(a,b) := \frac{a+b}{2}$

$x_{33} := f(x_{32}, x_{19})$ $\quad x_{34} := f(x_{61}, x_{19})$

$y_{33} := f(y_{32}, y_{19})$ $\quad y_{34} := f(y_{61}, y_{19})$

$x_{35} := f(x_{36}, x_{19})$ $\quad x_{37} := f(x_{25}, x_1)$ $\quad x_{38} := f(x_3, x_{25})$ $\quad x_{39} := f(x_3, x_{27})$

$y_{35} := f(y_{36}, y_{19})$ $\quad y_{37} := f(y_{25}, y_1)$ $\quad y_{38} := f(y_3, y_{25})$ $\quad y_{39} := f(y_3, y_{27})$

$x_{40} := f(x_5, x_{27})$ $\quad x_{41} := f(x_7, x_{27})$ $\quad x_{42} := f(x_7, x_{29})$ $\quad x_{43} := f(x_9, x_{29})$

$y_{40} := f(y_5, y_{27})$ $\quad y_{41} := f(y_7, y_{27})$ $\quad y_{42} := f(y_7, y_{29})$ $\quad y_{43} := f(y_9, y_{29})$

$x_{44} := f(x_{11}, x_{29})$ $x_{45} := f(x_{11}, x_{31})$ $x_{46} := f(x_{13}, x_{31})$ $x_{47} := f(x_{15}, x_{31})$

$y_{44} := f(y_{11}, y_{29})$ $y_{45} := f(y_{11}, y_{31})$ $y_{46} := f(y_{13}, y_{31})$ $y_{47} := f(y_{15}, y_{31})$

$x_{48} := f(x_{15}, x_{32})$ $x_{49} := f(x_{17}, x_{32})$ $x_{50} := f(x_{21}, x_{36})$ $x_{51} := f(x_{23}, x_{36})$

$y_{48} := f(y_{15}, y_{32})$ $y_{49} := f(y_{17}, y_{32})$ $y_{50} := f(y_{21}, y_{36})$ $y_{51} := f(y_{23}, y_{36})$

$x_{52} := f(x_{23}, x_{25})$ $x_{53} := f(x_{25}, x_{61})$ $x_{54} := f(x_{27}, x_{61})$ $x_{55} := f(x_{29}, x_{61})$

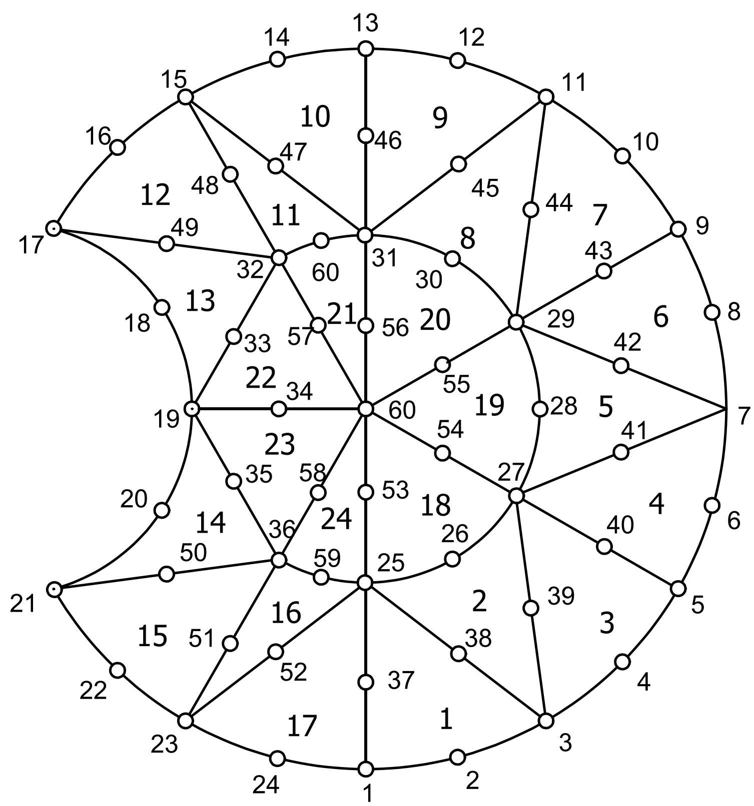
$y_{52} := f(y_{23}, y_{25})$ $y_{53} := f(y_{25}, y_{61})$ $y_{54} := f(y_{27}, y_{61})$ $y_{55} := f(y_{29}, y_{61})$

$x_{56} := f(x_{31}, x_{61})$ $x_{57} := f(x_{32}, x_{61})$ $x_{58} := f(x_{36}, x_{61})$ $x_{59} := f(x_{36}, x_{25})$

$y_{56} := f(y_{31}, y_{61})$ $y_{57} := f(y_{32}, y_{61})$ $y_{58} := f(y_{36}, y_{61})$ $y_{59} := f(y_{36}, y_{25})$

$x_{60} := f(x_{32}, x_{31})$

$y_{60} := f(y_{32}, y_{31})$



CONECTIVIDAD DE LOS ELEMENTOS

$e := 1 .. 24 \qquad i := 1 .. 6$

$cc_{i,e} := 0$

$p := 1 .. 8$

$c_{i,p} := A1_{(p-1)\cdot 6+i}$

$p := 9 .. 16$

$c_{i,p} := A2_{(p-9)\cdot 6+i}$

$p := 17 .. 24$

$c_{i,p} := Au_{(p-17)\cdot 6+i}$

$ii := 1 .. 48 \qquad kk := 1 .. 48$

$A1_{ii} :=$, $A2_{ii} :=$, $Au_{kk} :=$

$A1_{ii}$	$A2_{ii}$	Au_{kk}
1	11	23
3	13	1
25	31	25
2	12	24
38	46	37
37	45	52
3	13	25
27	15	27
25	31	61
39	14	26
26	47	54
38	46	53
3	5	27
5	32	29
27	31	61
4	48	28
40	60	55
39	47	54
5	15	29
7	17	31
27	32	61
6	16	30
41	49	56
40	48	55
7	17	31
29	19	32
27	32	61
42	18	60
28	33	57
41	49	56
7	19	32
9	21	19
29	36	61
8	20	33
43	50	34
42	35	57
9	21	19
11	23	36
29	36	61
10	22	35
44	51	58
43	50	34
11	23	36
31	25	25
29	36	61
45	52	59
30	59	53
44	51	58

$$c = \begin{pmatrix} 1 & 3 & 3 & 5 & 7 & 7 & 9 & 11 & 11 & 13 & 5 & 15 & 17 & 19 & 21 & 23 & 23 & 25 & 27 & 29 & 31 & 32 & 19 & 36 \\ 3 & 27 & 5 & 7 & 29 & 9 & 11 & 31 & 13 & 15 & 32 & 17 & 19 & 21 & 23 & 25 & 1 & 27 & 29 & 31 & 32 & 19 & 36 & 25 \\ 25 & 25 & 27 & 27 & 27 & 29 & 29 & 29 & 31 & 31 & 31 & 32 & 32 & 36 & 36 & 36 & 25 & 61 & 61 & 61 & 61 & 61 & 61 & 61 \\ 2 & 39 & 4 & 6 & 42 & 8 & 10 & 45 & 12 & 14 & 48 & 16 & 18 & 20 & 22 & 52 & 24 & 26 & 28 & 30 & 60 & 33 & 35 & 59 \\ 38 & 26 & 40 & 41 & 28 & 43 & 44 & 30 & 46 & 47 & 60 & 49 & 33 & 50 & 51 & 59 & 37 & 54 & 55 & 56 & 57 & 34 & 58 & 53 \\ 37 & 38 & 39 & 40 & 41 & 42 & 43 & 44 & 45 & 46 & 47 & 48 & 49 & 35 & 50 & 51 & 52 & 53 & 54 & 55 & 56 & 57 & 34 & 58 \end{pmatrix}$$

FUNCIONES DE INTERPOLACION

Coordenadas naturales de los nodos -->

$$\xi := \begin{pmatrix} 1 \\ 0 \\ 0 \\ \frac{1}{2} \\ 0 \\ \frac{1}{2} \end{pmatrix} \qquad \eta := \begin{pmatrix} 0 \\ 1 \\ 0 \\ \frac{1}{2} \\ \frac{1}{2} \\ 0 \end{pmatrix}$$

$$N_{1,i} := \xi_i \cdot (2 \cdot \xi_i - 1)$$

$$N_{2,i} := \eta_i \cdot (2 \cdot \eta_i - 1)$$

$$N_{3,i} := (1 - \xi_i - \eta_i) \cdot [2 \cdot (1 - \xi_i - \eta_i) - 1]$$

$$N_{4,i} := 4 \cdot \xi_i \cdot \eta_i$$

$$N_{5,i} := 4 \cdot \eta_i \cdot (1 - \xi_i - \eta_i)$$

$$N_{6,i} := 4 \cdot \xi_i \cdot (1 - \xi_i - \eta_i)$$

Comprobación:

$$N = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

COORDENADAS DE LOS PUNTOS DE LA INTEGRACION DE GAUSS

$$\xi := \begin{pmatrix} 0.8168475730 \\ 0.0915762135 \\ 0.0915762135 \\ 0.1081030182 \\ 0.4459484909 \\ 0.4459484909 \end{pmatrix} \qquad \eta := \begin{pmatrix} 0.0915762135 \\ 0.8168475730 \\ 0.0915762135 \\ 0.4459484909 \\ 0.1081030182 \\ 0.4459484909 \end{pmatrix}$$

EVALUACION DE LAS FUNCIONES DE INTERPOLACION

$$N_{1,i} := \xi_i \cdot (2 \cdot \xi_i - 1)$$

$$N_{2,i} := \eta_i \cdot (2 \cdot \eta_i - 1)$$

$$N_{3,i} := (1 - \xi_i - \eta_i) \cdot [2 \cdot (1 - \xi_i - \eta_i) - 1]$$

$$N_{4,i} := 4 \cdot \xi_i \cdot \eta_i$$

$$N_{5,i} := 4 \cdot \eta_i \cdot (1 - \xi_i - \eta_i)$$

$$N_{6,i} := 4 \cdot \xi_i \cdot (1 - \xi_i - \eta_i)$$

DERIVADAS DE LAS FUNCIONES DE INTERPOLACION

$N\xi_{1,i} := 4 \cdot \xi_i - 1$ $\quad N\eta_{1,i} := 0$

$N\xi_{2,i} := 0$ $\quad N\eta_{2,i} := 4 \cdot \eta_i - 1$

$N\xi_{3,i} := -3 + 4 \cdot \eta_i + 4 \cdot \xi_i$ $\quad N\eta_{3,i} := -3 + 4 \cdot \xi_i + 4 \cdot \eta_i$

$N\xi_{4,i} := 4 \cdot \eta_i$ $\quad N\eta_{4,i} := 4 \cdot \xi_i$

$N\xi_{5,i} := -4 \cdot \eta_i$ $\quad N\eta_{5,i} := 4 - 8 \cdot \eta_i - 4 \cdot \xi_i$

$N\xi_{6,i} := 4 - 8 \cdot \xi_i - 4 \cdot \eta_i$ $\quad N\eta_{6,i} := -4 \cdot \xi_i$

COEFICIENTES DE GAUSS $\quad W_i :=$

0.05497587185
0.05497587185
0.05497587185
0.11169079485
0.11169079485
0.11169079485

JACOBIANOS PARA LOS PUNTOS DE INTEGRACION

$$j := 1 .. 6$$

$$J11_{i,e} := \sum_j N\xi_{j,i} \cdot x_{(c_{j,e})} \quad J12_{i,e} := \sum_j N\xi_{j,i} \cdot y_{(c_{j,e})}$$

$$J21_{i,e} := \sum_j N\eta_{j,i} \cdot x_{(c_{j,e})} \quad J22_{i,e} := \sum_j N\eta_{j,i} \cdot y_{(c_{j,e})}$$

DETERMINANTE DEL JACOBIANO

$$detJ_{i,e} := J11_{i,e} \cdot J22_{i,e} - J12_{i,e} \cdot J21_{i,e}$$

MATRIZ T

$$T11_{i,e} := \left(J22_{i,e}\right)^2 + \left(J21_{i,e}\right)^2$$

$$T12_{i,e} := -\left(J12_{i,e} \cdot J22_{i,e} + J11_{i,e} \cdot J21_{i,e}\right)$$

$$T22_{i,e} := \left(J11_{i,e}\right)^2 + \left(J12_{i,e}\right)^2$$

$$T21_{i,e} := T12_{i,e}$$

MATRIZ TxN,

$$Q1_{(e-1)\cdot 6+i,j} := T11_{i,e} \cdot N\xi_{j,i} + T12_{i,e} \cdot N\eta_{j,i}$$

$$Q2_{(e-1)\cdot 6+i,j} := T21_{i,e} \cdot N\xi_{j,i} + T22_{i,e} \cdot N\eta_{j,i}$$

MATRIZ K

$$k := 1..6$$

$$K_{(e-1)\cdot 6+k,j} := \sum_i \frac{W_i}{detJ_{i,e}} \cdot \left[N\xi_{k,i} \cdot Q1_{(e-1)\cdot 6+i,j} + N\eta_{k,i} \cdot Q2_{(e-1)\cdot 6+i,j}\right]$$

VECTOR DE FUERZAS EXTERNAS

$$P_{j,e} := 2 \cdot \sum_i W_i \cdot detJ_{i,e} \cdot N_{j,i}$$

FORMACION DE LA MATRIZ GLOBAL Y EL VECTOR DE CARGAS

$$Kt_{c_{i,e},c_{j,e}} := 0 \qquad F_{c_{i,e}} := 0$$

$$Kt_{c_{i,e},c_{j,e}} := Kt_{c_{i,e},c_{j,e}} + K_{(e-1)\cdot 6+i,j}$$

$$F_{c_{i,e}} := F_{c_{i,e}} + P_{i,e}$$

CONDICIONES DE BORDE Y SOLUCION PARA LA FUNCION DE PRANDTL

$$k := 1..24$$

$$Kt_{k,k} := 10^{10}$$

SOLUCION PARA LA FUNCION INCOGNITA

$$Fi := Kt^{-1} \cdot F$$

CALCULO DE J (dos veces el volumen bajo la superficie fi)

$$Vol := \sum_e \sum_j \sum_i W_i \cdot detJ_{i,e} \cdot N_{j,i} \cdot Fi_{\left(c_{j,e}\right)}$$

$$2 \cdot Vol = 1.061$$

$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -0 \\ 0 \\ 0 \end{pmatrix}$$

DETERMINACION DE LAS TENSIONES EN LOS NODOS

Coordenadas de los nodos:

$$\xi := \begin{pmatrix} 1 \\ 0 \\ 0 \\ \frac{1}{2} \\ 0 \\ \frac{1}{2} \end{pmatrix} \qquad \eta := \begin{pmatrix} 0 \\ 1 \\ 0 \\ \frac{1}{2} \\ \frac{1}{2} \\ 0 \end{pmatrix}$$

Derivadas de las funciones de forma:

$$N\xi_{1,i} := 4\cdot\xi_i - 1 \qquad N\eta_{1,i} := 0$$

$$N\xi_{2,i} := 0 \qquad N\eta_{2,i} := 4\cdot\eta_i - 1$$

$$N\xi_{3,i} := -3 + 4\cdot\eta_i + 4\cdot\xi_i \qquad N\eta_{3,i} := -3 + 4\cdot\xi_i + 4\cdot\eta_i$$

$$N\xi_{4,i} := 4\cdot\eta_i \qquad N\eta_{4,i} := 4\cdot\xi_i$$

$$N\xi_{5,i} := -4\cdot\eta_i \qquad N\eta_{5,i} := 4 - 8\cdot\eta_i - 4\cdot\xi_i$$

$$N\xi_{6,i} := 4 - 8\cdot\xi_i - 4\cdot\eta_i \qquad N\eta_{6,i} := -4\cdot\xi_i$$

Elementos del Jacobiano:

$$J11_{i,e} := \sum_j N\xi_{j,i}\cdot x_{(c_{j,e})} \qquad J12_{i,e} := \sum_j N\xi_{j,i}\cdot y_{(c_{j,e})}$$

$$J21_{i,e} := \sum_j N\eta_{j,i}\cdot x_{(c_{j,e})} \qquad J22_{i,e} := \sum_j N\eta_{j,i}\cdot y_{(c_{j,e})}$$

Determinante del Jacobiano:

$$detJ_{i,e} := J11_{i,e}\cdot J22_{i,e} - J12_{i,e}\cdot J21_{i,e}$$

Derivada de la función de Prandtl respecto a x en el nodo i del elemento e:

$$Fix_{i,e} := \frac{1}{detJ_{i,e}}\cdot\left[J22_{i,e}\cdot\sum_j N\xi_{j,i}\cdot Fi_{(c_{j,e})} - J12_{i,e}\cdot\sum_j N\eta_{j,i}\cdot Fi_{(c_{j,e})}\right]$$

Derivada de la función de Prandtl respecto a y en el nodo i del elemento e:

$$Fiy_{i,e} := \frac{1}{detJ_{i,e}}\cdot\left[(-J21)_{i,e}\cdot\sum_j N\xi_{j,i}\cdot Fi_{(c_{j,e})} + J11_{i,e}\cdot\sum_j N\eta_{j,i}\cdot Fi_{(c_{j,e})}\right]$$

$$Fi = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0.297 \\ 0.321 \\ 0.33 \\ 0.335 \\ 0.334 \\ 0.334 \\ 0.337 \\ 0.279 \\ 0.175 \\ 0.258 \\ 0.165 \\ 0.245 \\ 0.184 \\ 0.205 \\ 0.211 \\ 0.201 \\ 0.216 \\ 0.216 \\ 0.203 \\ 0.215 \\ 0.212 \\ 0.183 \\ 0.124 \\ 0.126 \\ 0.083 \\ 0.082 \\ 0.148 \\ 0.177 \\ 0.357 \\ 0.395 \\ 0.4 \\ 0.373 \\ 0.334 \\ 0.319 \\ 0.282 \\ 0.324 \\ 0.375 \end{pmatrix}$$

conectivida -->

$$c = \begin{pmatrix} 1 & 3 & 3 & 5 & 7 & 7 & 9 & 11 & 11 & 13 & 5 & 15 & 17 & 19 & 21 & 23 & 23 & 25 & 27 & 29 & 31 & 32 & 19 & 36 \\ 3 & 27 & 5 & 7 & 29 & 9 & 11 & 31 & 13 & 15 & 32 & 17 & 19 & 21 & 23 & 25 & 1 & 27 & 29 & 31 & 32 & 19 & 36 & 25 \\ 25 & 25 & 27 & 27 & 27 & 29 & 29 & 29 & 31 & 31 & 31 & 32 & 32 & 36 & 36 & 36 & 25 & 61 & 61 & 61 & 61 & 61 & 61 & 61 \\ 2 & 39 & 4 & 6 & 42 & 8 & 10 & 45 & 12 & 14 & 48 & 16 & 18 & 20 & 22 & 52 & 24 & 26 & 28 & 30 & 60 & 33 & 35 & 59 \\ 38 & 26 & 40 & 41 & 28 & 43 & 44 & 30 & 46 & 47 & 60 & 49 & 33 & 50 & 51 & 59 & 37 & 54 & 55 & 56 & 57 & 34 & 58 & 53 \\ 37 & 38 & 39 & 40 & 41 & 42 & 43 & 44 & 45 & 46 & 47 & 48 & 49 & 35 & 50 & 51 & 52 & 53 & 54 & 55 & 56 & 57 & 34 & 58 \end{pmatrix}$$

$$Fix^T = \begin{pmatrix} -0.004 & -0.459 & 0.083 & -0.226 & -0.16 & 0.002 \\ -0.474 & -0.313 & 0.096 & -0.489 & -0.085 & -0.122 \\ -0.457 & -0.818 & -0.314 & -0.633 & -0.548 & -0.424 \\ -0.822 & -0.94 & -0.316 & -0.881 & -0.645 & -0.588 \\ -0.937 & -0.327 & -0.317 & -0.673 & -0.322 & -0.668 \\ -0.938 & -0.828 & -0.328 & -0.883 & -0.597 & -0.65 \\ -0.824 & -0.467 & -0.328 & -0.643 & -0.438 & -0.558 \\ -0.432 & -0.03 & -0.305 & -0.177 & -0.176 & -0.466 \\ -0.448 & -0.003 & 0.081 & -0.213 & 0.022 & -0.167 \\ 0.003 & 0.141 & 0.618 & 0.131 & 0.398 & 0.315 \\ -0.206 & 0.437 & 0.034 & 0.031 & 0.235 & -0.188 \\ 0.225 & 0.081 & 0.786 & 0.177 & 0.439 & 0.485 \\ 0.129 & 1.628 & 0.809 & 0.801 & 1.068 & 0.478 \\ 1.603 & 0.202 & 0.641 & 0.842 & 0.417 & 0.943 \\ 0.128 & 0.352 & 0.665 & 0.278 & 0.523 & 0.399 \\ 0.398 & 0.124 & 0.408 & 0.261 & 0.266 & 0.403 \\ 0.36 & 0.004 & 0.143 & 0.2 & 0.107 & 0.235 \\ 0.097 & -0.354 & 0.277 & -0.094 & -0.003 & 0.137 \\ -0.359 & -0.382 & 0.301 & -0.371 & -0.086 & -0.073 \\ -0.372 & -0.024 & 0.286 & -0.152 & 0.079 & -0.004 \\ 0.022 & 0.492 & 0.377 & 0.257 & 0.434 & 0.2 \\ 0.544 & 1.367 & 0.19 & 0.955 & 0.778 & 0.367 \\ 1.367 & 0.505 & 0.19 & 0.936 & 0.348 & 0.778 \\ 0.423 & 0.12 & 0.388 & 0.272 & 0.254 & 0.406 \end{pmatrix} \quad Fiy^T = \begin{pmatrix} 0.88 & 0.802 & 0.309 & 0.844 & 0.594 & 0.594 \\ 0.781 & 0.206 & 0.327 & 0.483 & 0.28 & 0.645 \\ 0.783 & 0.477 & 0.206 & 0.633 & 0.373 & 0.49 \\ 0.47 & 0.004 & 0.201 & 0.236 & 0.141 & 0.304 \\ -0.002 & -0.193 & 0.201 & -0.19 & 0.007 & 0.194 \\ -0.004 & -0.474 & -0.195 & -0.237 & -0.303 & -0.137 \\ -0.481 & -0.802 & -0.196 & -0.643 & -0.495 & -0.37 \\ -0.806 & -0.403 & -0.199 & -0.678 & -0.296 & -0.492 \\ -0.783 & -0.791 & -0.555 & -0.794 & -0.673 & -0.691 \\ -0.791 & -0.247 & -0.555 & -0.488 & -0.375 & -0.673 \\ -0.156 & 0.206 & -0.259 & 0.337 & -0.027 & 0.174 \\ -0.386 & -0.048 & -0.322 & -0.177 & -0.132 & -0.365 \\ 0.389 & 0.097 & -0.112 & 0.615 & 0.073 & 0.219 \\ -0.095 & -0.61 & 0.167 & -0.646 & -0.178 & -0.06 \\ 0.075 & 0.603 & -0.059 & 0.278 & 0.264 & -0.014 \\ 0.576 & 0.334 & 0.089 & 0.455 & 0.212 & 0.333 \\ 0.629 & 0.88 & 0.309 & 0.745 & 0.594 & 0.492 \\ 0.32 & 0.232 & -0.008 & 0.296 & 0.173 & 0.156 \\ 0.224 & -0.222 & 0.033 & 0.007 & -0.015 & 0.052 \\ -0.24 & -0.214 & 0.06 & -0.254 & -0.077 & -0.159 \\ -0.214 & -0 & 0.06 & -0.107 & 0.03 & -0.077 \\ 0.03 & 0.237 & -0.048 & 0.134 & 0.094 & -0.009 \\ -0.222 & 0.094 & 0.106 & -0.064 & 0.1 & -0.058 \\ 0.142 & 0.32 & -0.008 & 0.231 & 0.156 & 0.067 \end{pmatrix}$$

Módulo de la tensión:

$$\tau_{i,e} := \sqrt{\left(Fix_{i,e}\right)^2 + \left(Fiy_{i,e}\right)^2}$$

$\tau^T =$

0.88	0.924	0.32	0.874	0.615	0.594
0.914	0.375	0.341	0.687	0.293	0.657
0.907	0.947	0.375	0.895	0.663	0.648
0.947	0.94	0.375	0.912	0.66	0.662
0.937	0.38	0.375	0.699	0.322	0.696
0.938	0.954	0.382	0.914	0.669	0.664
0.954	0.928	0.382	0.909	0.66	0.669
0.914	0.404	0.364	0.701	0.344	0.677
0.902	0.791	0.561	0.822	0.673	0.711
0.791	0.285	0.83	0.505	0.547	0.743
0.258	0.483	0.261	0.339	0.237	0.256
0.446	0.094	0.849	0.251	0.459	0.607
0.41	1.631	0.816	1.01	1.071	0.525
1.606	0.643	0.662	1.061	0.453	0.945
0.148	0.698	0.668	0.393	0.585	0.399
0.7	0.357	0.418	0.525	0.34	0.523
0.725	0.88	0.34	0.771	0.604	0.545
0.334	0.424	0.277	0.311	0.173	0.208
0.423	0.442	0.303	0.371	0.088	0.09
0.442	0.216	0.292	0.296	0.11	0.159
0.215	0.492	0.382	0.278	0.435	0.214
0.545	1.387	0.196	0.965	0.784	0.367
1.384	0.514	0.218	0.938	0.362	0.781
0.446	0.342	0.388	0.356	0.298	0.411

GRAFICO DE TENSIONES

$i := 1..5$

$y_i :=$	$x_i :=$
−1.502	−0.482
−0.5335	−0.241
−0.255	0
0.533	0.5
0.94	1.0

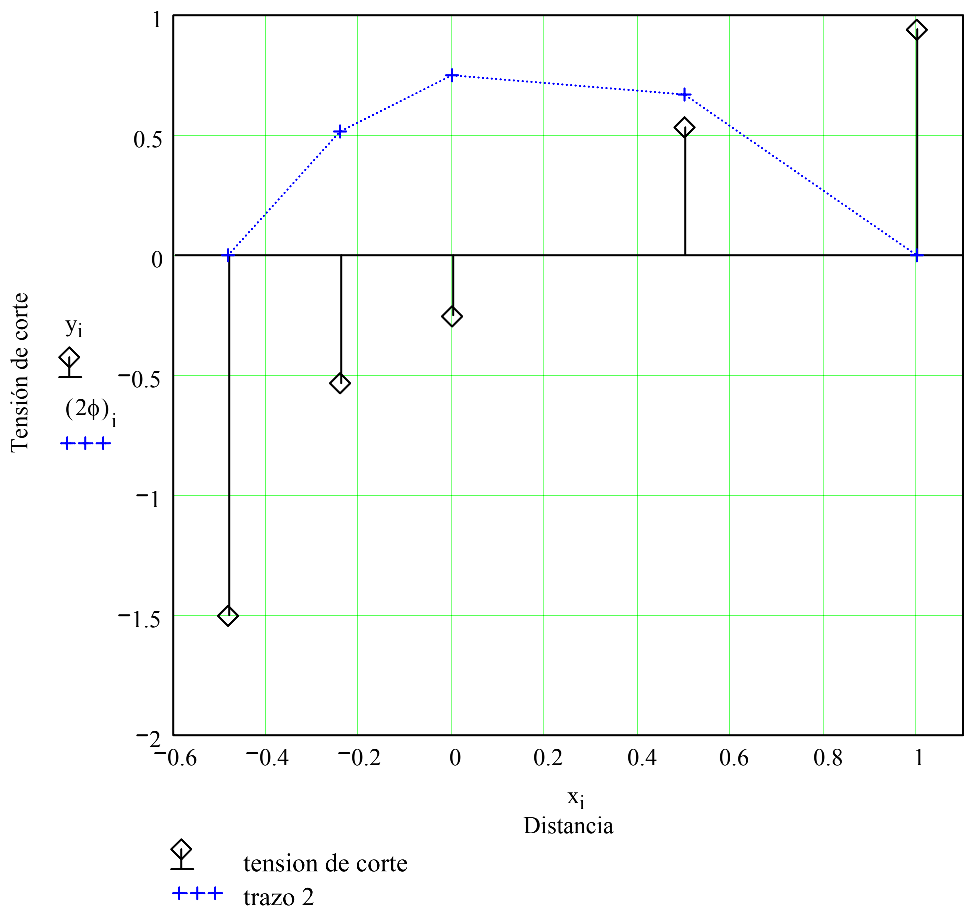
$\phi_i :=$
0
0.258
0.375
0.335
0

$$\frac{1606 + 1631 + 1387 + 1384}{4} = 1502$$

$$\frac{947 + 937 + 938}{3} = 940.667$$

$$\frac{696 + 371}{2} = 533.5$$

$$\frac{303 + 292 + 382 + 196 + 218 + 388}{6} = 296.5$$



1
0.5
0
−0.5
−1
−1.5
−2
−0.6
−0.4
−0.2
0
0.2
0.4
0.6
0.8
1
Tensión de corte
y_i
$(2\phi)_i$
x_i
Distancia
tension de corte
trazo 2