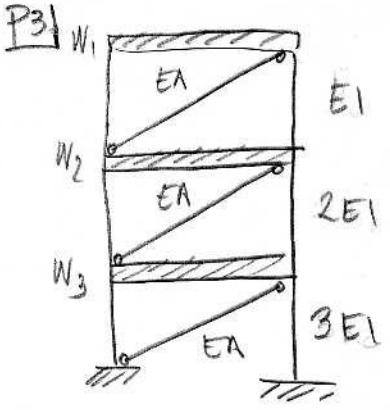
P3)



$EA = \frac{EI}{4}$

$\ell = 2\ m$

$W_1 = 200\ kgf$

$W_2 = 400\ kgf$

$W_3 = 500\ kgf$

i) Matriz de rigidez:

$$k_1 = \frac{24EI}{\ell^3} + \frac{EA}{2\sqrt{2}\ell} = \frac{24EI}{\ell^3} + \frac{EI}{8\sqrt{2}\ell} = \left(\frac{24}{\ell^3} + \frac{1}{8\sqrt{2}\ell}\right)EI$$

$$k_2 = \left(\frac{48}{\ell^3} + \frac{1}{8\sqrt{2}\ell}\right)EI \quad ; \quad k_3 = \left(\frac{72}{\ell^3} + \frac{1}{8\sqrt{2}\ell}\right)EI$$

luego $[K] = \begin{bmatrix} k_1 & -k_1 & 0 \\ -k_1 & k_1+k_2 & -k_2 \\ 0 & -k_2 & k_2+k_3 \end{bmatrix} = \begin{bmatrix} 3.0442 & -3.0442 & 0 \\ -3.0442 & 9.0884 & -6.0442 \\ 0 & -6.0442 & 15.0884 \end{bmatrix} EI$

ii) Matriz de Masa:

$$[M] = \begin{bmatrix} W_1/g & 0 & 0 \\ 0 & W_2/g & 0 \\ 0 & 0 & W_3/g \end{bmatrix} = \begin{bmatrix} 20.4082 & 0 & 0 \\ 0 & 40.8163 & 0 \\ 0 & 0 & 51.0204 \end{bmatrix}$$

iii) ϕ, EI, ω, T:

$$[\phi] = \begin{bmatrix} 0.7925 & -0.8299 & 0.3697 \\ 0.5513 & 0.3306 & -0.6553 \\ 0.2609 & 0.4494 & 0.6587 \end{bmatrix}, \quad \omega = \begin{pmatrix} 0.2130 \\ 0.4567 \\ 0.6431 \end{pmatrix} \cdot \sqrt{EI}$$

$$\Rightarrow T = \frac{2\pi}{\omega} = \begin{pmatrix} 29.4923 \\ 13.7574 \\ 9.7700 \end{pmatrix} \sqrt{\frac{1}{EI}}$$

Queremos $T_1 = 1$ seg $\Rightarrow 29.4923 \cdot \frac{1}{\sqrt{EI}} = 1$

$\Rightarrow EI = (29.4923)^2 \Rightarrow EI = 869.793 \text{ kgf.m}^2$

iv) Recalculamos ω y T ahora que tenemos EI:

$$\omega = \begin{pmatrix} 6.2832 \\ 13.4695 \\ 18.9667 \end{pmatrix} \frac{rad}{s} \quad T = \begin{pmatrix} 1 \\ 0.4665 \\ 0.3313 \end{pmatrix} \text{ segs.}$$

v) Normalizamos:

$$\{\tilde{\phi}_i\} = \frac{\{\phi_i\}}{\sqrt{M_m}} \quad ; \quad M_m = [\phi]^T [M] [\phi] = \begin{bmatrix} 28.6950 & 0 & \\ & 28.8215 & \\ 0 & & 42.4549 \end{bmatrix}$$

$$\Rightarrow \{\tilde{\phi}_i\} = \begin{bmatrix} 0.1479 & -0.1546 & 0.0567 \\ 0.1029 & 0.0616 & -0.1006 \\ 0.0487 & 0.0837 & 0.1011 \end{bmatrix} \Rightarrow M_m = \begin{bmatrix} 1 & 0 & \\ & 1 & \\ 0 & & 1 \end{bmatrix}$$

vi) Factor de Participación

$$\{L\} = [\tilde{\phi}]^T [M] \{r\}, \quad \{r\} = \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix} \Rightarrow \{L\} = \begin{pmatrix} 9.7048 \\ 3.6297 \\ 2.2107 \end{pmatrix}$$

vii) Evaluamos Sa:

$$Sa_i = 0.5\,g \; \frac{1 + 4.5 \left(\frac{T_i}{0.3}\right)^{1.5}}{1 + \left(\frac{T_i}{0.3}\right)^3} = \begin{pmatrix} 3.6568 \\ 10.0124 \\ 12.9924 \end{pmatrix} m/s^2$$

$$Sa_i = 0.5\,g\,\frac{1+4.5\left(\frac{T_i}{0.3}\right)^{1.5}}{1+\left(\frac{T_i}{0.3}\right)^{3}} = \begin{pmatrix} 3.6568 \\ 10.0124 \\ 12.9924 \end{pmatrix} m/s^2$$

viii) Obtenemos V_{max}:

$$V_{max_i} = \{\phi_i\}\,\frac{L_{mi}}{M_{mi}}\,\frac{Sa_i}{\omega_i^2} \Rightarrow \quad V_1 = \begin{pmatrix} 0.1330 \\ 0.0925 \\ 0.0438 \end{pmatrix} m$$

$$V_2 = \begin{pmatrix} -0.0310 \\ 0.0123 \\ 0.0168 \end{pmatrix} m \qquad V_3 = \begin{pmatrix} 0.0045 \\ -0.0080 \\ 0.0081 \end{pmatrix}$$

Combinando:

$$V_{max} = \sqrt{V_1^2 + V_2^2 + V_3^2} = \begin{pmatrix} 0.1366 \\ 0.0937 \\ 0.0476 \end{pmatrix} m$$