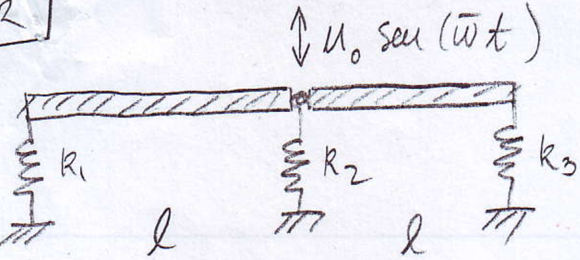
P2) **PAUTA**

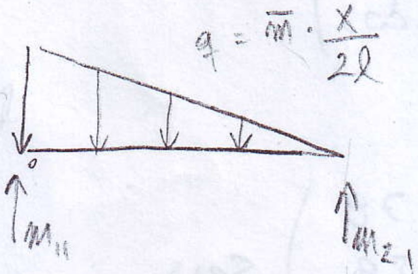


i) Matriz de Rigidez.

Por generación directa. $[K] = \begin{bmatrix} k_1 & 0 & 0 \\ 0 & k_2 & 0 \\ 0 & 0 & k_3 \end{bmatrix} = \begin{bmatrix} 300 & 0 & 0 \\ 0 & 1000 & 0 \\ 0 & 0 & 500 \end{bmatrix}$ kgf/m

ii) Matriz de Masa:

$\ddot{u}_1 = 1$ resto cero:



$\sum F_y = 0 \quad m_{11} + m_{21} = \frac{1}{2} \cdot l \ \bar{m} = \frac{\bar{m} l}{2}$

$\sum M_o = 0 \Rightarrow m_{21} \cdot l = \frac{1}{3} \cdot l \cdot \frac{1}{2} \ l \ \bar{m}$

$\therefore m_{21} = \frac{\bar{m} l}{6}$

$\Rightarrow m_{11} = \frac{\bar{m} l}{3}$

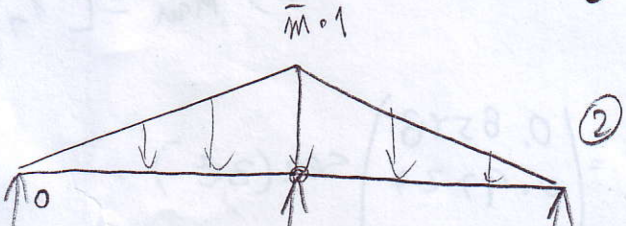
$m_{31} = 0$

Análogamente: $m_{33} = \frac{\bar{m} l}{3}$

$m_{23} = \frac{\bar{m} l}{6}$

$m_{13} = 0$

$\ddot{u}_2 = 1$ resto cero:



① $\sum F_y = 0 \Rightarrow m_{12} + m_{22} + m_{32} = \bar{m} \cdot l$

$\sum M_o = 0$

② $\Rightarrow m_{22} \cdot l + m_{32} \cdot 2l - \frac{1}{2} \bar{m} l \cdot \frac{2}{3} l - \frac{1}{2} \bar{m} l \cdot \frac{4}{3} l = 0$

③ $\sum M_{izq} = 0$

$\Rightarrow -m_{12} \cdot l + \frac{1}{2} \bar{m} l \cdot \frac{1}{3} l = 0$

$\Rightarrow m_{12} = \frac{\bar{m} l}{6}$

Por simetría $m_{32} = m_{12} = \frac{\bar{m}l}{6}$

$\Rightarrow m_{22} = \frac{4}{6}\bar{m}l \Rightarrow m_{22} = \frac{2}{3}\bar{m}l$

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luego $[M] = \begin{bmatrix} \frac{1}{3} & \frac{1}{6} & 0 \\ \frac{1}{6} & \frac{2}{3} & \frac{1}{6} \\ 0 & \frac{1}{6} & \frac{1}{3} \end{bmatrix} \bar{m}l = \begin{bmatrix} 8.50 & 4.25 & 0 \\ 4.25 & 17 & 4.25 \\ 0 & 4.25 & 8.5 \end{bmatrix} \frac{kgf \cdot s^2}{m}$

iii) ϕ, w, T

$[\phi] = \begin{bmatrix} 0.9417 & -0.5423 & -0.4476 \\ 0.2964 & 0.3064 & 0.6041 \\ 0.1595 & 0.7823 & -0.6594 \end{bmatrix}$ (ordenados)

$\omega = \begin{pmatrix} 5.5223 \\ 7.0136 \\ 10.4187 \end{pmatrix} \frac{rad}{s} \qquad T = \begin{pmatrix} 1.1378 \\ 0.8959 \\ 0.6031 \end{pmatrix}$ Segs.

iv) Normalizamos

$[M_m] = \begin{bmatrix} 12.0204 & & 0 \\ & 9.9231 & \\ 0 & & 5.9181 \end{bmatrix}$; $\{\tilde{\phi}_i\} = \frac{\{\phi_i\}}{\sqrt{M_{m_i}}} = \begin{bmatrix} 0.2716 & -0.1722 & -0.1840 \\ 0.0855 & 0.0973 & 0.2483 \\ 0.0460 & 0.2483 & -0.2710 \end{bmatrix}$

$\Rightarrow M_m = \begin{bmatrix} 1 & & \\ & 1 & \\ & & 1 \end{bmatrix}$

v) Vector de cargas

$\{P_0\} = [\tilde{\phi}]^T \cdot \{P\} = [\tilde{\phi}]^T \begin{Bmatrix} 0 \\ M_0 \operatorname{sen}(\bar{\omega}t) \\ 0 \end{Bmatrix} = \begin{pmatrix} 0.8548 \\ 0.9727 \\ 2.4832 \end{pmatrix} \operatorname{sen}(3t)$

$\downarrow$

P_0^*

vi) Ec. de movimiento:

$$M_m \ddot{y}_i + C_m \dot{y}_i + K_m y_i = \{\phi_i\}^T \{P\} \quad // \text{ divido x } M_{m_i} = 1$$

$$\iff \ddot{y}_i + 2\beta_i \omega_i \dot{y}_i + \omega_i^2 = P_{oi}^* \, \text{sen}(\bar{\omega} t)$$

<u>Régimen Permanente</u>!

$$\Rightarrow y(t) = \frac{P_{oi}}{K_{mi}} \cdot D \, \text{sen}(\bar{\omega} t - \theta_i) = \frac{P_{oi}}{\omega_i^2} \cdot D \, \text{sen}(\bar{\omega} t - \theta_i)$$

$$\theta = \text{arctg}\left(\frac{2\beta_i \gamma_i}{1-\gamma_i^2}\right) \quad ; \quad \gamma_i = \frac{\bar{\omega}}{\omega_i} = \begin{pmatrix} 0.5433 \\ 0.4277 \\ 0.2879 \end{pmatrix} \qquad \beta_i = 3\%, \forall i$$

$$\Rightarrow D_i = \frac{1}{\sqrt{(1-\gamma_i^2)^2 + (2\beta_i \gamma_i)^2}} = \begin{pmatrix} 1.4172 \\ 1.2233 \\ 1.0902 \end{pmatrix}$$

Así:

$$y_1(t) = 0.0397 \, \text{sen}(3t - 0.0462)$$

$$y_2(t) = 0.0242 \, \text{sen}(3t - 0.0314)$$

$$y_3(t) = 0.0249 \, \text{sen}(3t - 0.0188)$$

En el GDL 1:

$$v_1(t) = \phi_{11} \cdot y_1 + \phi_{21} \cdot y_2 + \phi_{31} \cdot y_3 \quad ; F_1 = K_1 \cdot v_1(t)$$

modo GDL

$$\boxed{F_1(t) = 3.2367 \cdot \text{sen}(3t - 0.0462) - 1.2493 \, \text{sen}(3t - 0.0314) - 1.3766 \cdot \text{sen}(3t - 0.0188)}$$