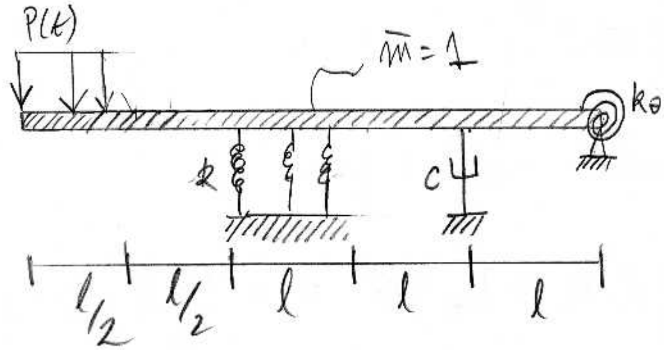
# Pauta P1 C1 CI42G

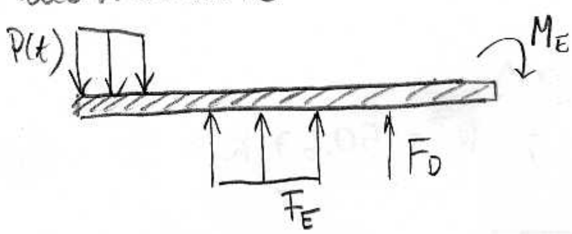



$l = 2$

$\bar{m} = 1$

$k_\theta = 50$ kgf · m

i) Ec. de movimiento:



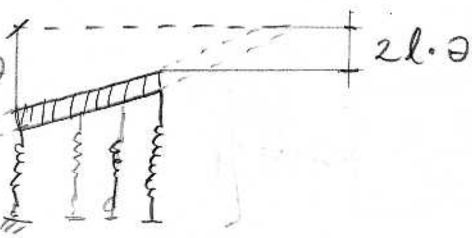

$$\sum M_o = I \ddot{\theta}$$

$$\sum M_o = -F_E \cdot L_{FE} - F_D \cdot l + P_o \frac{15 l^2}{8} - M_E$$

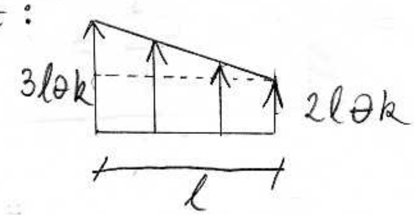
$$\Rightarrow I_o \cdot \ddot{\theta} + F_D \cdot l + F_E \cdot L_{FE} + M_E = -P_o \cdot \frac{15 l^2}{8}$$

$$F_D = \dot{\theta} \cdot l \cdot c$$

$F_E =$




$\Rightarrow F_E$:




$$F_E = \frac{(3+2)\, l\theta k}{2} \cdot l = \frac{5}{2} l^2 k \theta$$

Para calcular $L_{FE}$ tenemos que sacar el C.G de las fuerzas de la cama de resortes:

$$C.G = \frac{\frac{l}{2} \cdot 2 \cancel{l^2 \theta k} + \frac{2}{3} l \cdot \frac{1}{2} \cdot \cancel{l} \cdot \cancel{l\theta k}}{\frac{5}{2} \cancel{l^2 k \theta}} = \frac{l + \frac{2}{6} l}{5/2} = \frac{8}{15} l$$

$$\Rightarrow L_{FE} = \frac{8}{15} l + 2l = \frac{38}{15} l$$

Calculamos la inercia:

$$I_o = \frac{1}{3}(\bar{m}\,4\ell)(4\ell)^2 = \frac{64}{3}\bar{m}\,\ell^3$$

luego la ec. de movimiento resulta:

$$\underbrace{\frac{64}{3}\bar{m}\,\ell^3}_{m^*}\ddot{\theta} + \underbrace{c\,\ell^2}_{c^*}\dot{\theta} + \underbrace{\left(\frac{19}{3}\ell^3 k + k_\theta\right)}_{k^*}\theta = -\underbrace{\frac{15}{18}\ell^2 \cdot P_o}_{P_o^* = 225 \text{ kgf}\cdot\text{m}}$$

Evaluando:

$m^* = 170.67$ kgf$\cdot$s$^2\cdot$m ; $c^* = 4c$ kgf$\cdot$s$\cdot$m ; $k^* = 50.67\,k + 50$ kgf$\cdot$m

queremos $T = \frac{1}{2}$ seg. $\Rightarrow T = 2\pi\sqrt{\frac{m^*}{k^*}} = \frac{1}{2}$

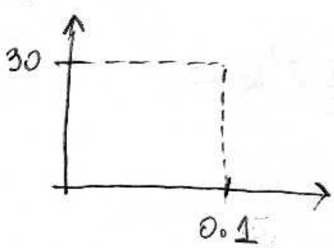
$$\Rightarrow \quad 2\pi\sqrt{\frac{170.67}{50.67k+50}} = 0.5 \quad \Longrightarrow \boxed{k = 530.91 \text{ kgf}/m^2} \Rightarrow k^* = 26951.21 \text{ kgf}\cdot\text{m}$$

Para encontrar $c$, sabemos $\beta = \frac{c^*}{2m^*\omega} = \frac{c^*}{2m^* \cdot \frac{2\pi}{T}} = \frac{c^*}{4\pi m^*} \cdot T$

como $\beta = 0.05 \Rightarrow c^* = 214.47$ kgf$\cdot$s$\cdot$m

$$\Longrightarrow \boxed{c = \frac{c^*}{4} = 53.62 \ \frac{\text{kgf}\cdot\text{s}}{m}}$$

Impacto:




$P_o = 30$
$t_d = 0.15$

$$\frac{t_d}{T} = \frac{0.1}{0.5} = \frac{1}{5} < \frac{1}{4} \Rightarrow$$ Impacto de corta duración

luego $$\theta(t) = e^{-\beta\omega t}\left[\frac{1}{m^*\omega_D}\int_0^{t_d} P^*(t)\,dt\right]\operatorname{sen}(\omega_D t)$$

Por otra parte, para impacto constante:

$$\theta_{max} = \begin{cases} \frac{P_0}{k} \cdot 2\,\text{sen}\left(\frac{\pi \cdot t_d}{T}\right) & \text{si } \frac{t_d}{T} < 1/2 \\ 2\frac{P_0}{k} & \text{si } \frac{t_d}{T} \geq 1/2 \end{cases}$$

Como $\frac{t_d}{T} = \frac{1}{5}$, $\theta_{max} = \frac{P_0^*}{k^*} \cdot 2\,\text{sen}\left(\frac{\pi \cdot t_d}{T}\right)$

$$\Rightarrow \theta_{max} = \frac{225}{26951.21} \cdot 2 \cdot \text{sen}\left(\frac{\pi \cdot 0.1}{0.5}\right)$$

$$\boxed{\begin{aligned} &\Rightarrow \theta_{max} = 0.0098 \text{ rad} \\ &\Rightarrow v_{max} = 0.078 \text{ m.} \end{aligned}}$$

Para el giro en $\frac{2T}{3}$:

$\omega_D \approx \omega$ ; $T_D \approx T$

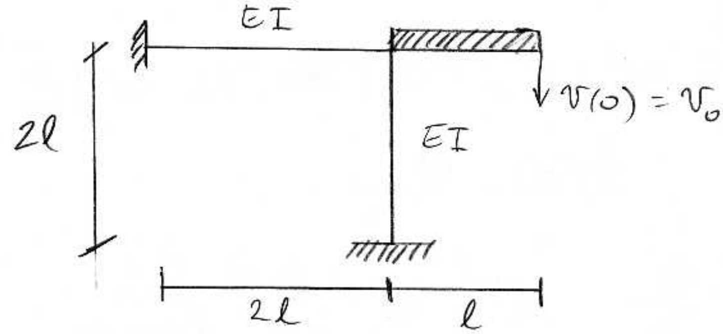
$$\theta\left(\frac{2T}{3}\right) = e^{-\beta \cdot \frac{2\pi}{T} \cdot \frac{2T}{3}} \left[\frac{T}{m^* 2\pi} \cdot P_0^* \cdot t_d\right] \text{sen}\left(\frac{2\pi}{T} \cdot \frac{2T}{3}\right)$$

$$\Rightarrow \theta\left(\frac{2T}{3}\right) = e^{-\frac{4}{3}\beta\pi} \left[\frac{P_0^* \cdot t_d \cdot T}{m^* \cdot 2\pi}\right] \text{sen}\left(\frac{4}{3}\pi\right)$$

$$\Rightarrow \theta\left(2T/3\right) = -0.0074 \text{ rad}$$

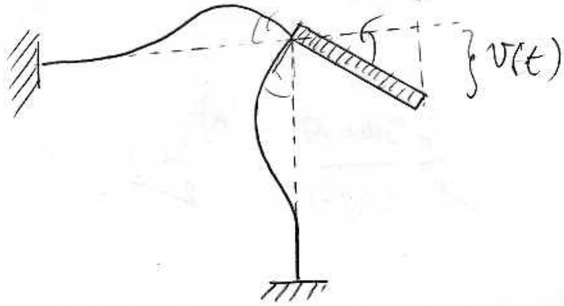
Pauta P2 C1 CI42G

P2




i) Ecuación de movimiento:

$$\theta = v/\ell \quad \Rightarrow \quad \theta_0 = \frac{v_0}{\ell}$$




$$\sum M_o = I_o \ddot{\theta}$$

$$\sum M_o = 2 \cdot \frac{4EI}{2\ell} \cdot \theta + c^* \cdot \dot{\theta}$$

$$\Rightarrow \quad \frac{1}{3}(\bar{m} L)\ell^2 \ddot{\theta} + c^* \cdot \dot{\theta} + \frac{4EI}{\ell}\theta = 0$$

$$\Rightarrow \quad \omega = \sqrt{\frac{k^*}{m^*}} = \sqrt{\frac{\frac{4EI}{\ell}}{\frac{1}{3}\bar{m}\ell^3}} = \sqrt{\frac{12EI}{\bar{m}\ell^4}} = \frac{2}{\ell^2}\sqrt{\frac{3EI}{\bar{m}}}$$

$$\Rightarrow \quad \boxed{\omega = \frac{2}{\ell^2}\sqrt{\frac{3EI}{\bar{m}}}} \; ; \; \boxed{T = \pi \cdot \ell^2 \sqrt{\frac{\bar{m}}{3EI}}}$$

$$T = 1 \text{ s.} \quad \Rightarrow \quad \pi \ell^2 \sqrt{\frac{\bar{m}}{3EI}} = 1 \quad \Rightarrow \quad \frac{1}{\pi^2 \ell^4} = \frac{\bar{m}}{3EI}$$

$$\Rightarrow \quad \boxed{EI = \frac{\bar{m}\pi^2\ell^4}{3}}$$

$$\theta(t) = \varsigma \cos(\omega t - \vartheta)\, e^{-\beta\omega t}$$

$$\dot{\theta}(t) = -\beta\omega \varsigma e^{-\beta\omega t} \cos(\omega t - \vartheta) - \omega \varsigma \,\text{sen}(\omega t - \vartheta)\, e^{-\beta\omega t}$$

En $t=0$ $\quad \theta(0) = v_0/l \qquad \dot{\theta}(0) = \dot{v}_0/l$

$$\Rightarrow \theta(0) = \varsigma \cos(\vartheta) = \frac{v_0}{l} \quad ① \quad \Rightarrow \quad \varsigma = \frac{v_0}{l\cos\vartheta}$$

$$\dot{\theta}(0) = -\beta\omega\varsigma\cos(\vartheta) + \omega\varsigma\,\text{sen}(\vartheta) = \dot{v}_0/l \quad ②$$

$$\Rightarrow ① \text{ en } ②: \; -\beta\omega \frac{v_0}{\cancel{l}} + \omega \frac{v_0}{\cancel{l}} \frac{\text{sen}\,\vartheta}{\cos\vartheta} = \frac{\dot{v}_0}{\cancel{l}}$$

$$\Rightarrow \quad \omega v_0 \,\text{tg}\,\vartheta = \dot{v}_0 + \beta\omega v_0$$

$$\Rightarrow \boxed{\vartheta = \text{arctg}\left(\frac{\dot{v}_0}{\omega v_0} + \beta\right)}$$

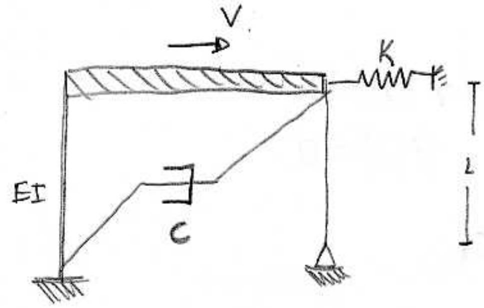
de ①:

$$\varsigma \cos(\vartheta) = \frac{v_0}{l} \quad \Rightarrow \boxed{\varsigma = \frac{v_0}{l\cos(\vartheta)} = \frac{v_0}{l\cos\left(\text{arctg}\left(\frac{\dot{v}_0}{\omega v_0} + \beta\right)\right)}}$$

Control N°1 : CI426

Primavera 2008.

**P3)**




$L = 3m$

$EI = 1\ tonf\text{-}m^2$

$K = 1{,}5\ tonf/m$

$W = 2\,tonf$

$\beta = 5\%$

$Q_{max\,cd} = 190\ kgf$

$M_{max\,cd} = 210\ kgf\text{-}m$

$$K^* = \frac{12EI}{L^3} + \frac{3EI}{L^3} + K = \frac{15\,EI}{L^3} + K = 2{,}056\ \frac{tonf}{m}. \qquad (0{,}5)$$

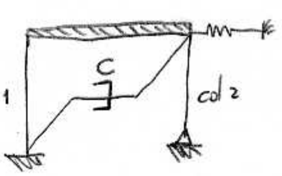
$$w = \sqrt{\frac{K^*}{M^*}} = \sqrt{\frac{2{,}056\cdot 9{,}8}{2}} = 3{,}174 \longrightarrow T = \frac{2\pi}{w} = 1{,}98\ [seg]. \qquad (0{,}5)$$

Del espectro: $S_a = 300\ [cm/seg^2] = 3\ [m/seg^2]$. $(0{,}5)$

$$PS_v = \frac{S_a}{w} = \frac{3}{3{,}174} = 0{,}9452\ [m/seg] \qquad (0{,}5)$$

$$PS_d = \frac{S_a}{w^2} = \frac{3}{3{,}174^2} = 0{,}2978\ [m] \qquad (0{,}5)$$

Corte y Momento por columnas: (2 ptos / 0,5 c/u)




$$Q_{col\,1} = \frac{12EI}{L^3}\cdot PS_d = 0{,}1324\ [tonf] = 132{,}4\ [kgf]. < Q_{max\,cd}\ \underline{\underline{ok}}$$

$$Q_{col\,2} = \frac{3EI}{L^3}\cdot PS_d = 0{,}0331\ [tonf] = 33{,}09\ [kgf]. < Q_{max\,cd}\ \underline{\underline{ok}}$$

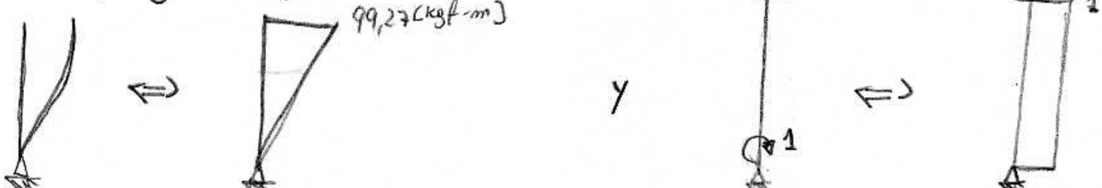
$$M_{col\,1} = \frac{6EI}{L^2}\,PS_d = 0{,}1985\ [tonf\text{-}m] = 198{,}5\ [kgf\text{-}m] < M_{max\,cd}\ \underline{\underline{ok}}$$

$$M_{col\,2} = \frac{3EI}{L^2}\,PS_d = 0{,}0993\ [tonf\text{-}m] = 99{,}27\ [kgf\text{-}m] < M_{max\,cd}\ \underline{\underline{ok}}$$

Fza del disipador: $F_D = C\cdot PS_v = 2M^* w\,\beta\cdot PS_v = 2\cdot\frac{2}{9{,}8}\cdot 3{,}174\cdot 0{,}05\cdot 0{,}9452 = 0{,}0612\ tonf$
(1,0).
$= 61{,}23\ [kgf]$.

Máximo giro en la base de col derecha (col 2): (0,5).

Alternativa 1 (Carga unitaria).



$$1\cdot\theta = \int_0^L M_R \phi^V = \int_0^L M\cdot\frac{m}{EI}\,dx = \frac{1}{EI}\int_0^L M = \frac{1}{1}\cdot\frac{1}{2}\cdot 3\cdot 99{,}27\cdot 10^{-3} = 0{,}1489 \,/\!/$$

Alternativa 2 (Ecuación de elástica).

$$\frac{d^2V}{dx^2} = \frac{M}{EI} \rightarrow V(x) = \iint \frac{M}{EI}\,dx + C_1X + C_2 \qquad \text{C.B} \quad V(0)=0,\ \mathring{V}(L)=0$$

$$V(x) = \frac{1}{EI}\cdot\iint 99{,}27\cdot 10^{-3}\cdot\frac{X}{3}\,dx + C_1X + C_2$$

$$V(x) = \frac{99{,}27\cdot 10^{-3}}{1\cdot 3\cdot 2\cdot 3}X^3 + C_1X + C_2 = 5{,}515\cdot 10^{-3}X^3 + C_1X + C_2 \qquad V(0)=0 \rightarrow C_2 = 0$$

$$\mathring{V}(X) = 16{,}545\cdot 10^{-3}X^2 + C_1 \qquad \mathring{V}(L)=0 \Rightarrow 0 = 16{,}545\cdot 10^{-3}\cdot 3^2 + C_1$$

$$C_1 = -0{,}1489$$

Luego $\theta = \mathring{V}(0) = 0{,}1489 \,/\!/$.