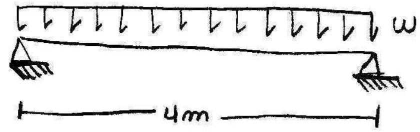
Auxiliar CI42B (11/09/08).

P1



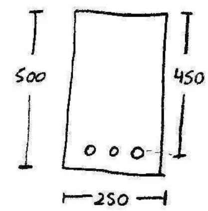
$w_d = 1{,}5$ kN/m

$w_\ell = 5$ kN/m

$f'_c = 25$ [MPa]

$f_y = 420$ [MPa]

$\gamma_h = 25$ kN/m³



Pregunta: Diseñe el refuerzo requerido por la sección para resistir las cargas solicitantes dadas.

→ Peso propio

$$w_0 = 25 \cdot 0{,}5 \cdot 0{,}25 = 3{,}125 \text{ [kN/m]}$$

$$w_u = 1{,}2\,(1{,}5 + 3{,}125) + 1{,}6 \cdot 5 = 13{,}55 \text{ [kN/m]} \rightarrow M_u = w_u \frac{\ell^2}{8} = 13{,}55 \cdot \frac{4^2}{8} = 27{,}1 \text{ [kN-m]}$$

- Diseño de Armadura:

3 Alternativas para calculo de As requerido:

i) $$a = \frac{f_y \cdot A_s}{0{,}85 f'_c \cdot b} = \frac{420}{0{,}85 \cdot 25 \cdot 250} \cdot A_s = 0{,}0791\,A_s. \quad M_n = \frac{M_u}{\phi} = f_y \cdot A_s\,(d - a/2) \rightarrow \frac{27{,}1 \cdot 10^6}{420 \cdot \underset{\text{Supuesto}}{0{,}9}} = A_s\left(450 - \frac{0{,}0791\,A_s}{2}\right)$$

$$0{,}0396\,A_s^2 - 450\,A_s + 7{,}169 \cdot 10^4 = 0 \rightarrow A_s = \frac{450 - \sqrt{450^2 - 4 \cdot 0{,}0396 \cdot 7{,}169 \cdot 10^4}}{2 \cdot 0{,}0396} = 161{,}6 \text{ [mm}^2\text{]}$$

ii) Sea $a \approx 20\%\,h = 100$ (mm)

$$A_s = \frac{M_u}{\phi f_y (d - a/2)} = \frac{27{,}1 \cdot 10^6}{0{,}9 \cdot 420 \underset{\text{Supuesto}}{(450 - \frac{100}{2})}} = 179{,}2 \text{ [mm}^2\text{]} \Rightarrow a = \frac{f_y \cdot A_s}{0{,}85 f'_c b} = \frac{420 \cdot 179{,}2}{0{,}85 \cdot 25 \cdot 250} = 14{,}17 \text{ [mm]}$$

$$A_s = \frac{27{,}1 \cdot 10^6}{0{,}9 \cdot 420\,(450 - \frac{14{,}17}{2})} = 161{,}9 \text{ [mm}^2\text{]} \Rightarrow a = \frac{420 \cdot 161{,}9}{0{,}85 \cdot 25 \cdot 250} = 12{,}8 \text{ [mm]}$$

$$A_s = \frac{27{,}1 \cdot 10^6}{0{,}9 \cdot 420\,(450 - \frac{12{,}8}{2})} = 161{,}6 \text{ [mm}^2\text{]} \rightarrow \text{Convergió}$$

iii) $$M_n = \frac{M_u}{\phi} = f_y \cdot A_s\,\underbrace{(d - a/2)}_{\approx 0{,}9 d} \Rightarrow A_s = \frac{M_u}{\phi f_y \cdot 0{,}9 d} = \frac{27{,}1 \cdot 10^6}{0{,}9^2 \cdot 420 \cdot 450} = 177 \text{ [mm}^2\text{]}$$

— o —

Verifico si $A_{req} = 161{,}6$ (mm²) comprobando supuesto $\phi = 0{,}9$

$$a = \frac{f_y \cdot A_s}{0{,}85 f'_c\,b} = \frac{420 \cdot 161{,}6}{0{,}85 \cdot 25 \cdot 250} = 12{,}78 \text{ [mm]} \Rightarrow c = \frac{a}{\beta_1} = \frac{12{,}78}{0{,}85} = 15{,}04 \text{ [mm]}$$

$$\Rightarrow \varepsilon_s = \varepsilon_u \frac{(d - c)}{c} = 0{,}003\left(\frac{450 - 15{,}04}{15{,}04}\right) = 0{,}0868 > 0{,}005 \rightarrow \phi = 0{,}9 \text{ ok}$$

Dado $A_s^{req} = 161,6\ [mm^2] \longrightarrow A_s = 2\phi12 = 226\ [mm^2]$. $(d = 500 - 40 - 10 - 12/2 = 444)$

- Verificación de $A_{s,min}$; $A_{s,max}$ ($\varepsilon_s \geq 0,004$)

$$A_{s,min} = \max\left\{\frac{1,4}{f_y} b_w \cdot d\ ;\ \frac{\sqrt{f'_c}}{4 f_y} b_w \cdot d\right\} = \left\{\frac{1,4}{420} \cdot 250 \cdot 444\ ;\ \frac{\sqrt{25}}{4 \cdot 420} \cdot 250 \cdot 444\right\} = \{370\ ;\ 330,4\} \qquad (5.10.5.1)$$

$A_{s,min} = 370\ [mm^2]$

Pero de 5.10.5.3 : Si $\frac{A_s}{A_s^{req}} \geq 1,33 \Rightarrow$ Condiciones 5.10.5.1 y 5.10.5.2 no Aplican

$$\frac{A_s}{A_s^{req}} = \frac{226}{161,6} = 1,399 \geq 1,33 \Longrightarrow A_s = 2\phi12 = 226\ [mm^2]$$

$A_{s,max}$:

$$a = \frac{f_y \cdot A_s}{0,85 \cdot f'_c\, b} = \frac{420 \cdot 226}{0,85 \cdot 25 \cdot 250} = 17,87\ [mm] \rightarrow c = \frac{a}{\beta_1} = \frac{17,87}{0,85} = 21,02\ [mm]$$

$$\varepsilon_s = 0,003\frac{(444 - 21,02)}{21,02} = 0,0604 \geq 0,004 \quad \underline{OK}\ A_{s,max}$$

$$\geq 0,005 \Longrightarrow \phi = 0,9$$

- Espaciamiento:

$$S = \frac{b - 2 \cdot R_{ec} - 2\phi_e - n_{barras} \cdot d_b}{n_{barras} - 1} = \frac{250 - 2 \cdot 40 - 2 \cdot 10 - 2 \cdot 12}{1} = 126\ [mm]$$

$$S = 126\ [mm] \geq \begin{cases} d_b = 12\ mm & \underline{OK} \\ 25\ mm & \underline{OK} \end{cases}$$

- Verificación de $\phi M_n \geq M_u$:

$$\phi M_n = \phi f_y \cdot A_s (d - a/2) = 0,9 \cdot 420 \cdot 226 \left(444 - \frac{17,87}{2}\right)$$

$$= 37,17\ [kN\text{-}m]$$

$$\phi M_n = 37,17\ [kN\text{-}m] \geq 27,1\ [kN\text{-}m] = M_u \ //$$

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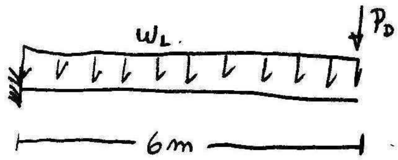
P2.



$w_L = 12$ [kN/m]

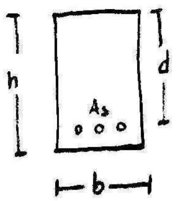
$P_D = 15$ [kN]

$f'_c = 30$ [MPa]

$f_y = 420$ [MPa]

$\gamma_h = 25$ [kN/m³]

Pregunta: Diseñe la viga (dimensiones y refuerzo) para resistir las cargas solicitantes dadas (debe incluirse además el peso propio).



$$M_u = 1{,}2 \cdot P_D \cdot L + 1{,}6 \cdot \frac{w_L \cdot L^2}{2} = 1{,}2 \cdot 15 \cdot 6 + 1{,}6 \cdot 12 \cdot \frac{6^2}{2} = 453{,}6 \text{ [kN-m]} \rightarrow$$ Sin considerar peso propio.

o Diseño: Supuesto: $\rho = 0{,}5\,\rho_b \Rightarrow \rho = 0{,}5 \cdot 0{,}85 \frac{f'_c}{f_y} \beta_1 \cdot \frac{\varepsilon_u}{\varepsilon_u + \varepsilon_y} = 0{,}5 \cdot 0{,}85 \cdot \frac{30}{420} \cdot 0{,}85 \cdot \frac{0{,}003}{0{,}003+0{,}0021} = 0{,}0152$

$$\rho = 0{,}0152$$

$$a = \frac{f_y \cdot \rho \cdot \cancel{b} \cdot d}{0{,}85 \cdot f'_c \cancel{b}} = \frac{420}{0{,}85 \cdot 30} \cdot \rho \cdot d = 16{,}47 \cdot \rho \cdot d \quad \wedge \quad \frac{M_u}{\phi} = f_y \cdot \rho \cdot b \cdot d\,(d - a/2)$$

$$d = \sqrt{\frac{M_u}{\phi f_y \cdot \rho \cdot b \cdot \left(1 - \frac{16{,}47\rho}{2}\right)}}$$

Supuesto: $b = 300$ mm $\Rightarrow$ $$d = \sqrt{\frac{453{,}6 \cdot 10^6}{\underset{\text{Supuesto}}{\underset{\downarrow}{0{,}9}} \cdot 420 \cdot 0{,}0152 \cdot 300 \left(1 - \frac{16{,}47 \cdot 0{,}0152}{2}\right)}} = \sqrt{\frac{453{,}6 \cdot 10^6}{1508}} = 548{,}4 \text{ [mm]}$$

Luego, asumo mi sección con $b = 300$ [mm] y $h = 548{,}4 + 70 = 618{,}4$ [mm] $\Rightarrow W_0 = 25 \cdot 0{,}3 \cdot 0{,}6184 = 4{,}638$ [kN/m]

Recalculo M_u considerando peso propio: $$M_u = 453{,}6 + 1{,}2 \cdot 4{,}638 \cdot \frac{6^2}{2} = 553{,}8 \text{ [kN-m]}$$

$$d = \sqrt{\frac{553{,}8 \cdot 10^6}{1508}} = 606 \text{ [mm]}$$ $\Rightarrow$ Asumo $d = 630$ [mm.] $\Rightarrow$ $h = 700$ [mm.]

$b = 300$ [mm.]

Se puede desde ahora resolver el problema como la P1 o de la forma sgte.

$A_s = \rho \cdot b_w \cdot d = 0{,}0152 \cdot 300 \cdot 630 = 2873$ [mm²]

Sea $A_s = 3\phi 36 = 3054$ [mm²] $\Rightarrow d = 700 - 40 - 10 - 36/2 = 632$ [mm.]

○ Verificación de $A_{s,min}$ y $A_{s,max}$ ($\varepsilon_s \geq 0{,}004$):

$$A_{s,min} = \max\left\{\frac{1{,}4}{f_y} b_w d \;;\; \frac{\sqrt{f'_c}}{4 f_y} b_w d\right\} = \max\left\{\frac{1{,}4}{420}\cdot 300\cdot 632 \;;\; \frac{\sqrt{30}}{4\cdot 420}\cdot 300\cdot 632\right\} = \max\{632 \;;\; 618{,}1\} = 632\,[mm^2] \checkmark$$

$A_{s,max}$?

$$a = \frac{f_y\cdot A_s}{0{,}85\cdot f'_c\, b} = \frac{420\cdot 3054}{0{,}85\cdot 30\cdot 300} = 167{,}7\,[mm.] \rightarrow c = \frac{a}{\beta_1} = \frac{167{,}7}{0{,}85} = 197{,}3\,[mm.]$$

$$\varepsilon_s = \varepsilon_u \frac{(d-c)}{c} = 0{,}003\frac{(632-197{,}3)}{197{,}3} = 0{,}0066 \geq 0{,}004 \;\underline{\underline{ok}}\;(A_{s,max})$$

$$\geq 0{,}005 \quad (\phi = 0{,}9)$$

○ Espaciamiento:

$$S = \frac{300-2\cdot 40-2\cdot 10-3\cdot 36}{2} = 46\,[mm] \geq \begin{cases} d_b = 36\,[mm.] & \underline{\underline{ok}} \\ 25\,[mm.] & \underline{\underline{ok}} \end{cases}$$

○ $\phi M_n \geq M_u$?

$$\phi M_n = \phi f_y\cdot A_s\,(d - a/2) = 0{,}9\cdot 420\cdot 3054\left(632 - \frac{167{,}7}{2}\right) = 632{,}8\,[kN\text{-}m]$$

$$M_u = 453{,}6 + 1{,}2\cdot 25\cdot 0{,}3\cdot 0{,}7\cdot\frac{6^2}{2} = 567\,[kN\text{-}m]$$

$$\phi M_n = 632{,}8\,[kN\text{-}m] \geq 567\,[kN\text{-}m] = M_u \;\underline{\underline{ok}}$$

$$\left(F.U = \frac{M_u}{\phi M_n} = \frac{567}{632{,}8} = 0{,}9 < 1 \;\underline{\underline{ok}}\right)$$

Factor de Utilización
(Nro que sirve como guía de un buen o mal diseño).

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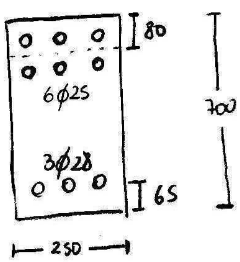
P3) Calcule el momento nominal minorado (ϕM_n) para la sección dada para el momento en ambas direcciones (flexión positiva y negativa), considere el caso de ambas armaduras trabajando y en el que solo la armadura traccionada trabaja. Compare resultados.



$f_c' = 25$ [MPa]

$f_y = 280$ [MPa]

$A_{s1} = 3\phi 28 = 1847$ [mm²]

$A_{s2} = 6\phi 25 = 2945$ [mm²]

a) Flexión Positiva:

i) Ambas armaduras:

$$0{,}85 f_c' \cdot a \cdot b + f_s' \cdot A_{s2} = f_s \cdot A_{s1}$$

Supuestos: $f_s = f_y$ $\quad f_s' < f_y$

$$0{,}85 \cdot f_c' \cdot b \cdot \beta_1 \cdot c + E_s \cdot \varepsilon_u \frac{(c-d')}{c} \cdot A_{s2} = f_y \cdot A_{s1}$$

$* \ \varepsilon_s' = \varepsilon_u \dfrac{(c-d')}{c}$

$$0{,}85 \cdot 25 \cdot 250 \cdot 0{,}85 \cdot c^2 + (200.000 \cdot 0{,}003 \cdot 2945 - 280 \cdot 1847)c - 200.000 \cdot 0{,}003 \cdot 2945 \cdot 80 = 0$$

$$4516\, c^2 + 1{,}25 \cdot 10^6 c - 1{,}414 \cdot 10^8 = 0 \rightarrow c = \frac{-1{,}25 \cdot 10^6 + \sqrt{(1{,}25 \cdot 10^6)^2 + 4 \cdot 4516 \cdot 1{,}414 \cdot 10^8}}{2 \cdot 4516}$$

$c = 86{,}25$ [mm]

Luego compruebo supuestos:

$(f_y = 280\text{[MPa]})$

$$\varepsilon_s = \varepsilon_u \frac{(d-c)}{c} = 0{,}003 \frac{(700-65-86{,}25)}{86{,}25} = 0{,}0191 \geq 0{,}0014 = \varepsilon_y \ \checkmark$$

$$\varepsilon_s' = \varepsilon_u \frac{(c-d')}{c} = 0{,}003 \frac{(86{,}25-80)}{86{,}25} = 0{,}0002 < 0{,}0014 = \varepsilon_y \ \checkmark$$

$$f_s' = E_s \varepsilon_s' = 43{,}48 \text{ [MPa]}$$

$$\phi M_n = \phi \left(f_y \cdot A_{s1} (d - a/2) + f_s' \cdot A_{s2} \, (a/2 - d') \right) = 0{,}9 \left(280 \cdot 1847 \left(635 - \frac{0{,}85 \cdot 86{,}25}{2}\right) + 43{,}48 \cdot 2945 \left(\frac{0{,}85 \cdot 86{,}25}{2} - 80\right) \right)$$

$\hookrightarrow \varepsilon_s \geq 0{,}005$

$\phi M_n = 0{,}9 \cdot 303{,}9$ [kN-m] $= 273{,}5$ [kN-m] //

(ii) Solo armadura traccionada:

$$a = \frac{f_y \cdot A_{s1}}{0{,}85 f_c' b} = \frac{280 \cdot 1847}{0{,}85 \cdot 25 \cdot 250} = 97{,}35 \text{ [mm]} \rightarrow c = 114{,}5 \text{ [mm]}$$

$$\rightarrow \varepsilon_s = 0{,}003 \frac{(700-65-114{,}5)}{114{,}5} = 0{,}0136 \geq 0{,}0014 = \varepsilon_y \ (f_s = f_y)$$

$$\geq 0{,}005 \ (\phi = 0{,}9).$$

$$\phi M_n = \phi f_y \cdot A_{s1} (d - a/2) = 0{,}9 \cdot 280 \cdot 1847 \cdot \left(700 - 65 - \frac{97{,}35}{2}\right) = 0{,}9 \cdot 303{,}2 \text{ [kN-m]} = 272{,}9 \text{ [kN-m]}$$

b) Flexión Negativa

i) Ambas Armaduras: $0{,}85 f_c' \cdot a \cdot b + f_s' \cdot A_{s1} = f_s A_{s2}$ Supuesto: $f_s = f_s' = f_y$

$$a = \frac{f_y (A_{s2} - A_{s1})}{0{,}85 f_c' \cdot b} = \frac{280(2945 - 1847)}{0{,}85 \cdot 25 \cdot 250} = 57{,}87 \text{ [mm]} \rightarrow c = 68{,}08 \text{ [mm]}$$

$$\varepsilon_s' = 0{,}003 \frac{(68{,}08 - 65)}{68{,}08} = 0{,}0001 \leq 0{,}0014 = \varepsilon_y \rightarrow \text{Supuesto } f_s' = f_y \ \underline{\text{MALO}}.$$

Supuestos: $f_s = f_y$, $f_s' < f_y$

$$0{,}85 \cdot f_c' \cdot b \cdot \beta_1 \cdot c + E_s \cdot \varepsilon_u \frac{(c - d')}{c} \cdot A_{s1} = f_y \cdot A_{s2}$$

$$0{,}85 \cdot f_c' \cdot b \cdot \beta_1 c^2 + (E_s \cdot \varepsilon_u \cdot A_{s1} - f_y \cdot A_{s2})\, c - E_s \cdot \varepsilon_u A_{s1} \cdot d' = 0$$

$$0{,}85 \cdot 25 \cdot 250 \cdot 0{,}85 c^2 + (200.000 \cdot 0{,}003 \cdot 1847 - 280 \cdot 2945)\, c - 200.000 \cdot 0{,}003 \cdot 1847 \cdot 65 = 0.$$

$$4516 c^2 + 2{,}836 \cdot 10^5 c - 7{,}203 \cdot 10^7 = 0 \rightarrow c = \frac{-2{,}836 \cdot 10^5 + \sqrt{(2{,}836 \cdot 10^5)^2 + 4 \cdot 4516 \cdot 7{,}203 \cdot 10^7}}{2 \cdot 4516}$$

$$c = 98{,}74 \text{ [mm]} \Rightarrow a = 83{,}93 \text{ [mm]}$$

Compruebo supuestos: $\varepsilon_s' = 0{,}003 \frac{(98{,}74 - 65)}{98{,}74} = 0{,}001 \leq 0{,}0014 = \varepsilon_y$ $\underline{\text{ok}}$ $(f_s' = 205 \text{ [MPa]})$

$$\varepsilon_s = 0{,}003 \frac{(700 - 80 - 98{,}74)}{98{,}74} = 0{,}0158 \geq 0{,}0014 = \varepsilon_y \ \underline{\text{ok}}$$
$$\geq 0{,}005 \ (\phi = 0{,}9).$$

$$\phi M_n = \phi \left(f_y \cdot A_{s2} (d - a/2) + f_s' \cdot A_{s1} (a/2 - d') \right) = 0{,}9 \left(280 \cdot 2945 \left(620 - \frac{83{,}93}{2}\right) + 205 \cdot 1847 \left(\frac{83{,}93}{2} - 65\right)\right)$$

$$\phi M_n = 0{,}9 \cdot 467{,}9 \text{ [kN-m]} = 421{,}1 \text{ [kN-m]}.$$

ii) Solo Armadura traccionada: $a = \frac{f_y \cdot A_{s2}}{0{,}85 f_c' \, b} = \frac{280 \cdot 2945}{0{,}85 \cdot 25 \cdot 250} = 155{,}2$ [mm.] $c = 182{,}6$ [mm.]

$$\varepsilon_s = 0{,}003 \frac{(620 - 182{,}6)}{182{,}6} = 0{,}0072 \geq 0{,}0014 = \varepsilon_y \ \underline{\text{ok}}$$
$$\geq 0{,}005 \ (\phi = 0{,}9)$$

$$\phi M_n = \phi \cdot f_y \cdot A_s (d - a/2) = 0{,}9 \cdot 280 \cdot 2945 \left(620 - \frac{155{,}2}{2}\right) = 0{,}9 \cdot 447{,}3 \text{ [kN-m]} = 402{,}6 \text{ [kN-m]}.$$

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Comparación de Resultados:

ϕM_n	Ambas Armadura	Solo Armadura traccionada	Diferencia (%)
flexión Positiva	273,5 [kN-m]	272,9 [kN-m]	0,22 %
flexión Negativa	421,1 [kN-m]	402,6 [kN-m]	4,4 %