Generación de triangulaciones para métodos de elementos finitos, problemas, algoritmos y aplicaciones

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Outline

- Motivación: MEF, terrenos, CG
- Problemas de Triangulaciones y algoritmos en 2D / 3D
- Algoritmos longest-edge
    - Lepp-bisección
    - Lepp-Delaunay
- Aplicaciones y objetivos:
    - Refinamiento de triangulaciones para software MEF.
    - Generación de triangulaciones de buena calidad
    - Modelación de terrenos
    - Mallas paralelas
    - Aplicaciones de computación gráfica
- Conclusiones

Motivación

- Aplicaciones del Método de Elementos Finitos (MEF) para análisis en ingeniería y ciencias aplicadas.
    - Requieren triangulaciones 2D / 3D / superficie de buena calidad de objetos complejos.
    - Refinamiento / Desrefinamiento adaptativo de la malla a través de los cálculos de MEF.
- Aproximaciones 2D / 3D / superficie de objetos complejos que considere los detalles relevantes de éstos.
- Modelos de terrenos.
- Aplicaciones médicas.
- Aplicaciones de computación gráfica.

Algoritmos longest-edge / Lepp son técnicas apropiadas para manejar de manera integrada estos problemas.

Problemas sobre triangulaciones y algoritmos (I)

- Triangularizar conjuntos de puntos en 2D / 3D.
    - Algoritmo de Delaunay
- Triangularizar objetos 2D / 3D.
    - Algoritmo de Delaunay + postproceso
- Generación automática de triangulación Delaunay de buena calidad.
    - Algoritmos Lepp / Delaunay
    - Algoritmosd basados en circumcentro
- Mejoramiento de triangulanes

(continúa)

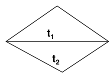
Problemas sobre triangulaciones y algoritmos (II)

- Refinamiento / Desrefinamiento de triangulaciones para MEF / MEF adaptivo (software para EDP).
    - Algoritmos Lepp-bisección.
- Simplificación de triangulaciones para datos enormes (grid data de terrenos / datos escaneados).
    - Triangulaciones bintree de triángulos rectángulos.
    - Algoritmos Lepp-surface para terrenos.
- Triangulaciones que evolucionan:
    - Algoritmos Lepp-Delaunay para fracturas
    - Algoritmos para fronteras / bordes móviles

Algoritmos longest-edge basados en conceptos de Lepp y aristas terminales

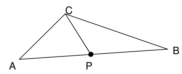
Basic concepts: valid triangulations

- In 2D, T is a valid triangulation if the intersection of pairs of adjacent triangles t_1, t_2 in T is either a common vertex or a common edge .
- In 3D, T is valid 3D triangulation if the intersection of pairs of adjacent tetrahedra t_1, t_2 is either a common vertex, or a common edge, or a common face



Basic concepts: longest-edge refinement algorithms

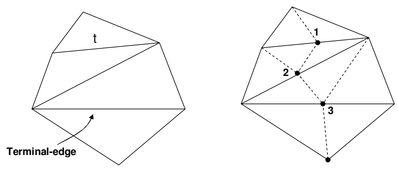
- Previous longest edge refinement algorithms (Rivara 84, 87, 92)
  - developed for adaptive fem
  - they allow the local refinement of the triangulations by maintaining the quality of the input mesh
  - based on the longest-edge bisection of triangles
  - they guarantee that the triangulation quality is maintained



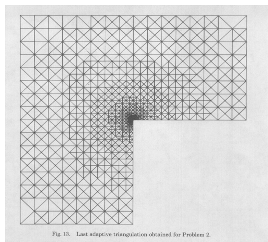
Remark: Terminal-edges are special longest-edges in the triangulation

Longest-edge refinement algorithm for adaptive finite element methods

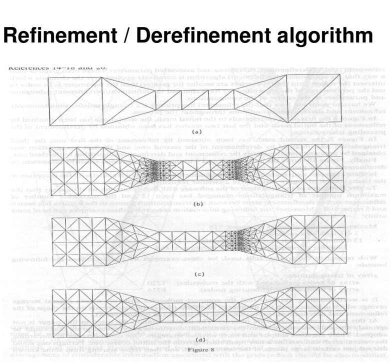
- Given a target triangle t, this triangle and a set of its neighbors are refined in order to maintain a valid triangulation



Example: adaptive triangulation obtained with the refinement algorithm



Refinement / Derefinement algorithm



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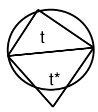
Construction of the initial triangulation Delaunay triangulation (DT)

- Problem: Given a point set P, construct a triangulation of P

P = { points }
T is a DT of P if the circuncircle of every triangle t in T does not include any vertex of P in its interior

t and t* are locally Delaunay triangles



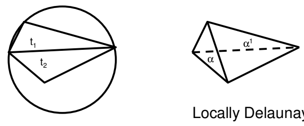
Optimality properties of the Delaunay triangulation 2D

- Constructs the most equilateral triangulation
- Maximizes the minimum angle
- Minimizes the maximum angle
- Maximizes the maximum radius of an enclosed circle
- Others

Bern 97
In Handbook of Discrete and
Computational Geometry
Goodman and O'Rourke (eds)

Diagonal swapping

t, t*: a pair of non-Delaunay triangles
They are transformed in a pair of locally
Delaunay triangles by diagonal swapping
This operation optimizes the angles



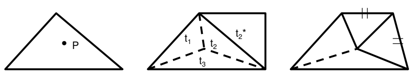
$\alpha > \alpha^1$

Locally Delaunay triangles

An incremental Delaunay algorithm based on edge swapping can be stated

Sketch

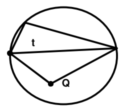
- P: point to be inserted
- The triangle that contains P is found
- P is joined with the vertices of P forming triangles t_1, t_2, t_3
- Recursively, if the pairs t_i, t_i* are not locally Delaunay, edge swapping is performed.



Constrained Delaunay triangulation CDT in 2D

- A CDT includes fixed edges to be respected
- A relaxation of the Delaunay triangulation conditions is used
- A kind of visibility problem
- If AB is a constrained edge, then the circuncircle of t doesn´t see the point Q inside the circle.

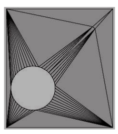
This concept allows the triangulation of general polygons (PSLG)



Over the geometric quality of the triangulations

- Delaunay algorithm constructs the most equilateral triangulation for the given data
- Constrained Delaunay algorithm constructs the most equilateral triangulation for the given data, considering the constrained edges

The quality of the triangulation depends on the point distribution



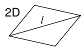
Automatic Quality triangulation problem

Problem:
Given a 2D or 3D bounded object, construct a quality triangulation of this object

- Additional points need to be inserted in order to improve the point distribution
- A point selection criteria + a point insertion strategy are needed

Terminal-Edge: key concept for mesh improvement and mesh refinement

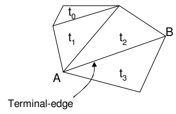
• Terminal-Edge *l* in triangulation τ
Special edge in τ such that *l* is longest-edge of every element that shares edge *l* in τ



• Terminal-star: set of elements (triangles in 2D, tetrahedra in 3D) that share a common terminal edge

Lepp(t): a key concept for finding terminal-edges

• **Longest-Edge Propagation Path (t) Lepp(t)**



Lepp(t0) = {t_0, t_1, t_2, t_3}

Lepp(t_0) identifies terminal-edge AB associated to t0 in τ

Longest-edge (t_i) > longest-edge (t_{i-1})

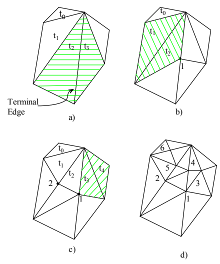
Improvement / Refinement Algorithms

Common basic idea: bisection of Terminal-Edges

• **Input: Set S of target elements to be improved or refined in mesh τ**
• **For each t in S do**
  While t remains in the mesh do
  - Find Lepp(t) and associated terminal-edge
  - Select point P to be inserted in τ as the midpoint of a terminal-edge
  - Insert point P in mesh (*)

(*) Refinement $\Rightarrow$ longest-edge bisection
(*) Improvement $\Rightarrow$ Delaunay point insertion

Lepp-bisection Refinement Algorithm



Algorithm maintains mesh quality!

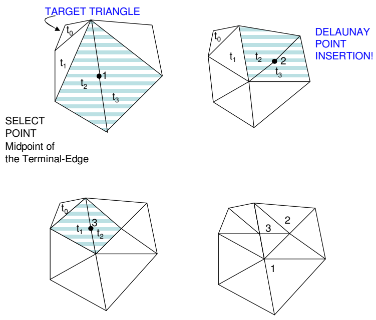
Derefinement Algorithm

- Based on the derefinement of previous terminal-edges
- It applies over triangulations produced by the refinement algorithm

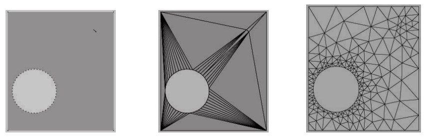
Automatic construction of quality triangulations and Improvement of triangulations

• We work over Delaunay triangulations
•Terminal-edges associated to bad quality elements (triangles, tetrahedra) are the best places for point insertion
• Lepp(t): for bad quality triangle t, allows to find associated terminal-edges

Lepp- Delaunay algorithm



Lepp- Delaunay
Automatic quality triangulation algorithm



Min angle ≥ 30º

More examples