Pauta Ej N° 5 MESSA

9 Mayo 2003

[1] En un sist. 1° orden $y(t) = y_0(1 - e^{-t/\tau})$ donde y_0 es la amplitud del escalon.

$$x \longrightarrow \boxed{G} \longrightarrow y \qquad \text{con} \quad x = \frac{y_0}{s}$$

$$G = \frac{1}{\tau s + 1}$$

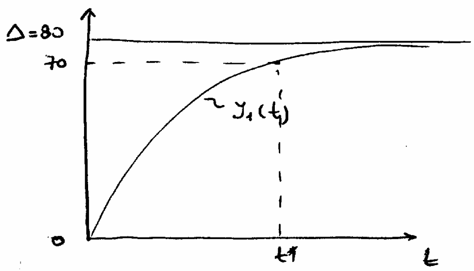
$$Y = \frac{y_0}{s}\,\frac{1}{\tau s + 1}.$$

Al sumergir la esfera, ésta es sometida a un escalon de altura $\Delta = 100 - 20$ °C. A los $t^* = 60$ segundos la temperatura es $T = 0.9 \cdot T_a = 90$ °C. Si referimos todo a $T_b = 20$ °C, tenemos.

$$y(t) \longrightarrow y_1(t) = \Delta(1 - e^{-t/\tau}) \qquad \text{con} \quad \Delta = T_a - T_b = 80.$$

Cuando $t = t^* = 60$ seg, $y(t^*) = 0.9\,T_a = 90$ °C $\rightarrow y_1(t^*) = 70$ °C.

$$y_1(t^*) = \Delta(1 - e^{-t^*/\tau}) \Rightarrow e^{-t^*/\tau} = 0.125. \Rightarrow \tau = \frac{60}{\ln(0.125)} = 28.9 \text{ Sec}$$



Ej N°5 . MESSA 9/5/08

2] $G_o(s) = \dfrac{0.4s+1}{s^2+0.6\cdot s}$ en lazo cerrado $G(s) = \dfrac{0.4s+1}{s^2+s+1}$

la respuesta al escalon unitario $\Rightarrow Y = \frac{1}{s}\cdot G(s) = \dfrac{1}{s}\,\dfrac{0.4\cdot s+1}{s^2+s+1}$.

$$Y = \frac{1}{s}\,\frac{0.4s+1}{s^2+s+1} = \frac{A}{s} + \frac{Bs+C}{s^2+s+1} \Rightarrow As^2+As+A+Bs^2+Cs = 0.4s+1$$

$A+B=0 \Rightarrow B=-1$

$A+C = 0.4 \Rightarrow C=-0.6$

$A=1$

$$\Rightarrow Y(s) = \frac{1}{s} - \frac{s}{s^2+s+1} - \frac{0.6}{s^2+s+1} = \frac{1}{s} - \frac{s+0.6}{s^2+s+1} = \underbrace{\frac{1}{s} - \frac{s+0.5}{(s+0.5)^2+0.75} - \frac{0.1}{(s+0.5)^2+0.75}}$$

$$Y(s) = \frac{1}{s} - \frac{s+0.5}{(s+0.5)^2+0.866^2} - 0.115\,\frac{0.866}{(s+0.5)^2+0.866^2}$$ la forma + simple.

$\therefore$ TL^{-1} $\Rightarrow$ $y(t) = 1 - e^{-0.5t}\cos(0.866\cdot t) - 0.115\,e^{-0.5t}\cdot\sin(0.866\cdot t)$

tiempo subida $\Rightarrow$ $y(t_r) = 1$

$t = t_r$

$$1 = 1 - e^{-0.5t_r}\cos(\,) - 0.115\,e^{-0.5t_r}\cdot\sin(0.866t_r)$$

$$tg(0.866\cdot t_r) = \frac{-1}{0.115} \Rightarrow \underline{t_r = 1.946\ [s]}$$

Sobrepaso o Overshoot : $\Rightarrow$ $\left.\dfrac{dy}{dt}\right|_{t=t_p} = 0$

$$0 = 0.5\cdot e^{-0.5\cdot t_p}\cdot\cos(0.866t_p) + 0.866\,e^{-0.5t_p}\cdot\sin(0.866\cdot t_p)$$
$$+\ 0.0575\,e^{-0.5t_p}\cdot\sin(0.866\cdot t_p) - 0.1\,e^{-0.5t_p}\cdot\cos(0.866\,t_p)$$

$\Rightarrow$ $0.4\cos(0.866\,t_p)\cdot 0.9235\,\sin(0.866\,t_p) = 0$

$\Leftrightarrow$ $tg(0.866\,t_p) = -0.433$

$\Rightarrow$ $\underline{t_p = 3.15}$

$y(t=t_p) = 1.17998$ $\Rightarrow$ $\underline{M_p = 17{,}998\ \%}$

Ej N°5 . MESSA 9/5/08 .

[3] Si el Sobrepaso (overshoot) es 0.25

$\Rightarrow 0.25 = e^{\left(\frac{-\pi\xi}{\sqrt{1-\xi^2}}\right)} \Rightarrow \boxed{\xi = 0.404}$ ¡ amortiguamiento !

tiempo de peak $t_p = 2 = \dfrac{\pi}{\omega_n\sqrt{1-\xi^2}} \Rightarrow \omega_n = \dfrac{\pi}{2\sqrt{1-\xi^2}} \Rightarrow \boxed{\omega_n = 1.717}$

frec. natural .

la funcion de transferencia de lazo abierto . es $\dfrac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} = \dfrac{2.948}{s^2 + 1.387 s + 2.948}$

la funcion de transf. de lazo cerrado es $= H = \dfrac{D G_0}{1 + D G_0 G_1}$

asi $\dfrac{\frac{k}{s^2}}{1 + \frac{k(1+k_0 s)}{s^2}} = \dfrac{k}{s^2 + k_0 k s + k} = H .$

homologando $\Rightarrow$ $k = 2.948$

$\underline{k_0 = 0.47 .}$