Problema # 2.

(a) $f(x) = \cos^3 x + \operatorname{sen}^3 x - 1 + \frac{\operatorname{sen}(2x)}{2}$.

notemos que

$\cos x = \cos(-x)$ $\quad$ $\cos(\cdot)$ par

$\operatorname{sen} x = -\operatorname{sen}(-x)$ $\quad$ $\operatorname{sen}(\cdot)$ impar

luego.

$$\cos^3 x + \operatorname{sen}^3 x = \cos^3(-x) - \operatorname{sen}^3(-x)$$

por indicacion :

$$\cos^3(-x) - \operatorname{sen}^3(-x) = (\cos(-x) - \operatorname{sen}(-x))(\operatorname{sen}^2(-x) + \operatorname{sen}(-x)\cos(-x) + \cos^2(-x))$$

$$= (\cos x + \operatorname{sen} x)(1 - \operatorname{sen} x \cos x)$$

entonces

$$f(x) = (\cos x + \operatorname{sen} x)(1 - \operatorname{sen} x \cos x) - 1 + \frac{\operatorname{sen}(2x)}{2}$$

$$= (\cos x + \operatorname{sen} x)\left(1 - \frac{\operatorname{sen} 2x}{2}\right) - \left(1 - \frac{\operatorname{sen}(2x)}{2}\right)$$

$$= (\cos x + \operatorname{sen} x - 1)\left(1 - \frac{\operatorname{sen} 2x}{2}\right)$$

· $f(x) = 0 \Rightarrow (\cos x + \operatorname{sen} x - 1)\left(1 - \frac{\operatorname{sen} 2x}{2}\right) = 0$

$\Rightarrow \cos x + \operatorname{sen} x - 1 = 0$ o $1 - \frac{\operatorname{sen} 2x}{2} = 0$

· pero $1 - \frac{\operatorname{sen} 2x}{2} = 0$ $\quad$ $\operatorname{sen} 2x = 2$ no hay solución

(pues 2 está fuera del recorrido de la función seno)

por lo tanto

$f(x) = 0$ ssi $(\cos x + \operatorname{sen} x - 1) = 0$

resolvamos

$$\cos x + \operatorname{sen} x = 1. \iff \cos x \pm \sqrt{1-\cos^2 x} = 1.$$

sea $\quad \alpha = \cos x \quad \Rightarrow$

$$\alpha \pm \sqrt{1-\alpha^2} = 1.$$

$$\pm\sqrt{1-\alpha^2} = 1-\alpha \qquad /(\;)^2$$

$$1-\alpha^2 = 1-2\alpha+\alpha^2$$

$$\Rightarrow \quad \alpha^2 - \alpha = 0 \quad \Rightarrow$$

$$\alpha = 0 \qquad \alpha = 1$$

$\Rightarrow \quad \cos x = 0 \quad$ o $\quad \cos x = 1$

$\Rightarrow \quad x = 2m\pi \pm \frac{\pi}{2},\ m\in\mathbb{Z} \quad$ o $\quad x = 2m\pi \pm 0,\ m\in\mathbb{Z}.$

$\quad = k\pi + \frac{\pi}{2},\ k\in\mathbb{Z}$

$\therefore \quad f(x) = 0 \quad$ si $\quad x = 2n\pi \quad$ o $\quad x = \frac{\pi}{2} + m\pi, \quad n\in\mathbb{Z}.$

(b) Demostrar

$$\frac{1}{tg(3x) - tg\,x} - \frac{1}{cotg(3x) - cotg\,x} = cotg(2x).$$

Sol:

$$\frac{1}{tg3x - tg\,x} - \frac{1}{cotg3x - cotg\,x} = \frac{1}{tg3x - tg\,x} - \frac{1}{\frac{1}{tg3x} - \frac{1}{tg\,x}}$$

$$= \frac{1}{tg3x - tg\,x} + \frac{tg3x\,tg\,x}{tg3x - tg\,x} = \frac{1 + tg3x\,tg\,x}{tg3x - tg\,x}.$$

notemos que:

$$tg(\alpha+\beta) = \frac{tg\alpha + tg\beta}{1 - tg\beta\,tg\alpha} \Rightarrow tg(\alpha-\beta) = \frac{tg\alpha - tg\beta}{1 + tg\beta\,tg\alpha}.$$

$$\Rightarrow \frac{1}{tg(\alpha-\beta)} = \frac{1 + tg\beta\,tg\alpha}{tg\alpha - tg\beta}$$ si $\alpha = 3x$, $\beta = 2x$

$$\Rightarrow \frac{1}{tg(3x-x)} = cotg(2x) = \frac{1 + tg3x\,tg\,x}{tg3x - tg\,x}.$$