More on consumption

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Taking stock

- Perfect foresight model
  - Consumption is linear in wealth

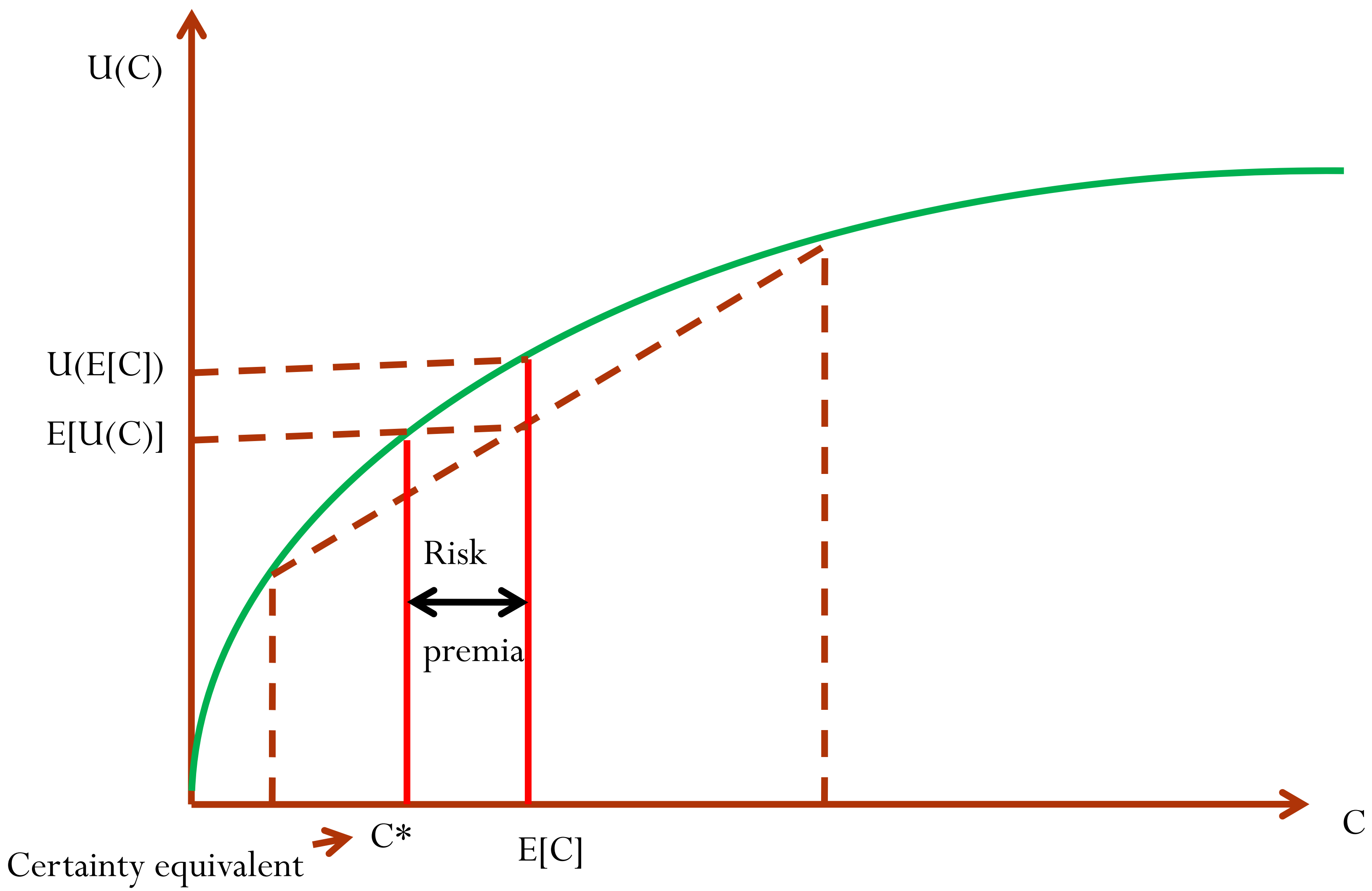
$$C_t = K_t Y_t$$

  - Consumption insensitive to fluctuations
  - Low marginal rate of consumption

- Drawbacks: we know from micro data
  - Consumption responds to anticipated income growth (Shea)
  - Marginal propensity to consume is higher in practice
  - Marginal propensity to consume decreases with wealth

Can uncertainty improve upon the perfect foresight model?

Risk aversion



Risk aversion

- Due to the concavity of the utility function
- Implication:
- Consumer wants to smooth consumption
  - over time (self insurance)

$$u'(C_t) = R\beta u'(C_{t+1})$$

  - over state of nature (insurance)

$$u'(C^i) = u'(C^j)$$

Quadratic utility

- When one adds uncertainty, the Euler equation becomes

$$u'(C_t) = R\beta E_t \left\{u'(C_{t+1})\right\}$$

- If utility function is quadratic,

$$u(C_t) = C_t - \frac{a}{2}C_t^2$$

- Expected marginal utility equals marginal expected utility

$$E_t \left\{u'(C_{t+1})\right\} = u'\left(E_t \left\{C_{t+1}\right\}\right)$$

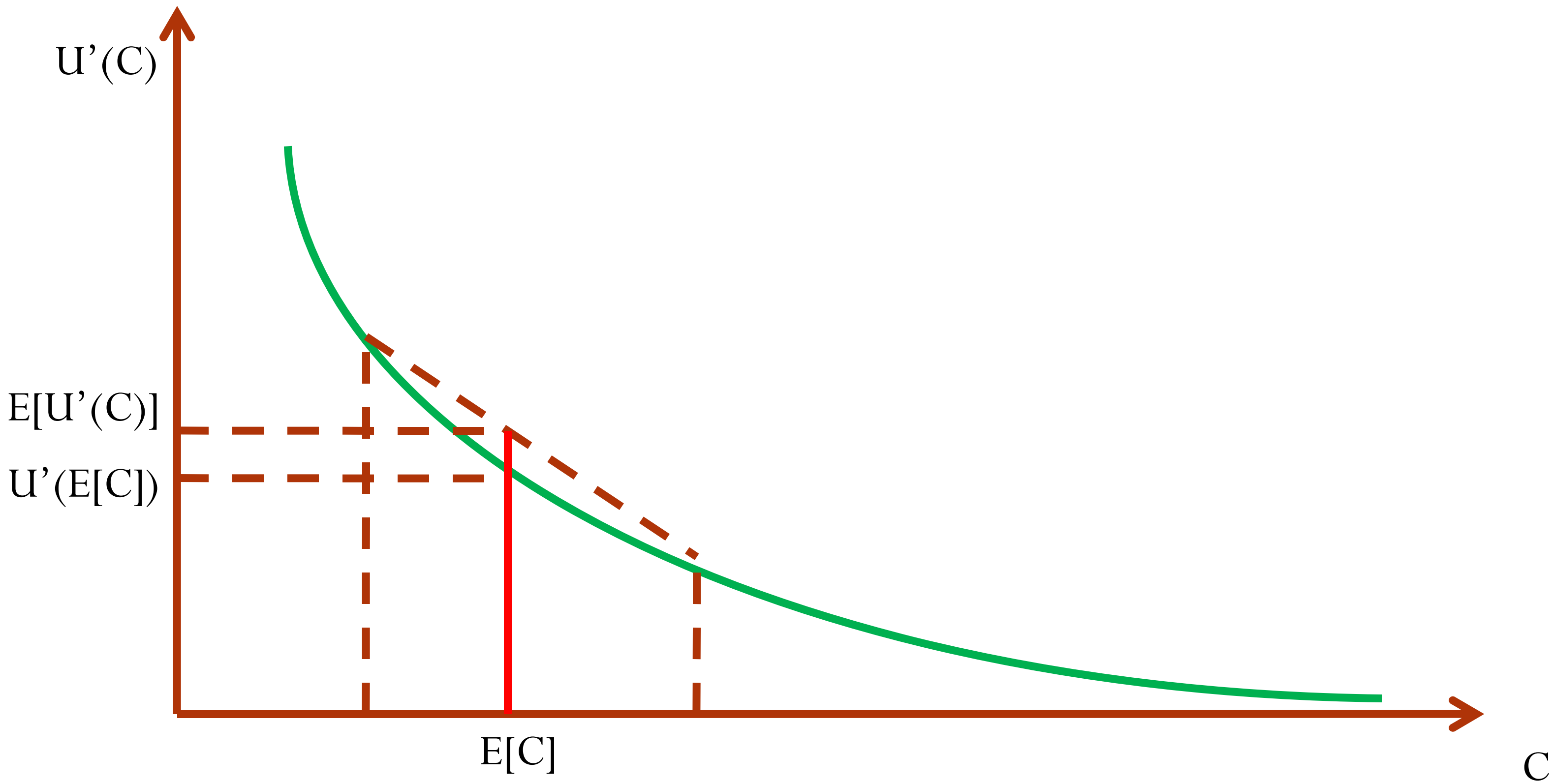
Quadratic utility

- With quadratic utility,
  - Because marginal utility is linear in consumption
  - the solution displays a certainty equivalent result
  - Random walk is the only departure from perf. foresight model
- Disadvantage of quadratic utility
  - Uncertainty plays no additional role
  - Finite marginal utility in zero
  - Increasing absolute/relative risk aversion

$$\left.\begin{aligned} u'(C_t) &= 1 - aC_t \\ u''(C_t) &= -a \end{aligned}\right\} \Rightarrow -\frac{u''(C_t)C_t}{u'(C_t)} = \frac{aC_t}{1 - aC_t}$$

Precautionary savings

- What if marginal utility is not linear in consumption?
- Assume it is convex



Precautionary savings

- If marginal utility is convex

$$u'''(C_t) > 0$$

$$E_t\{u'(C_{t+1})\} > u'(E_t\{C_{t+1}\})$$

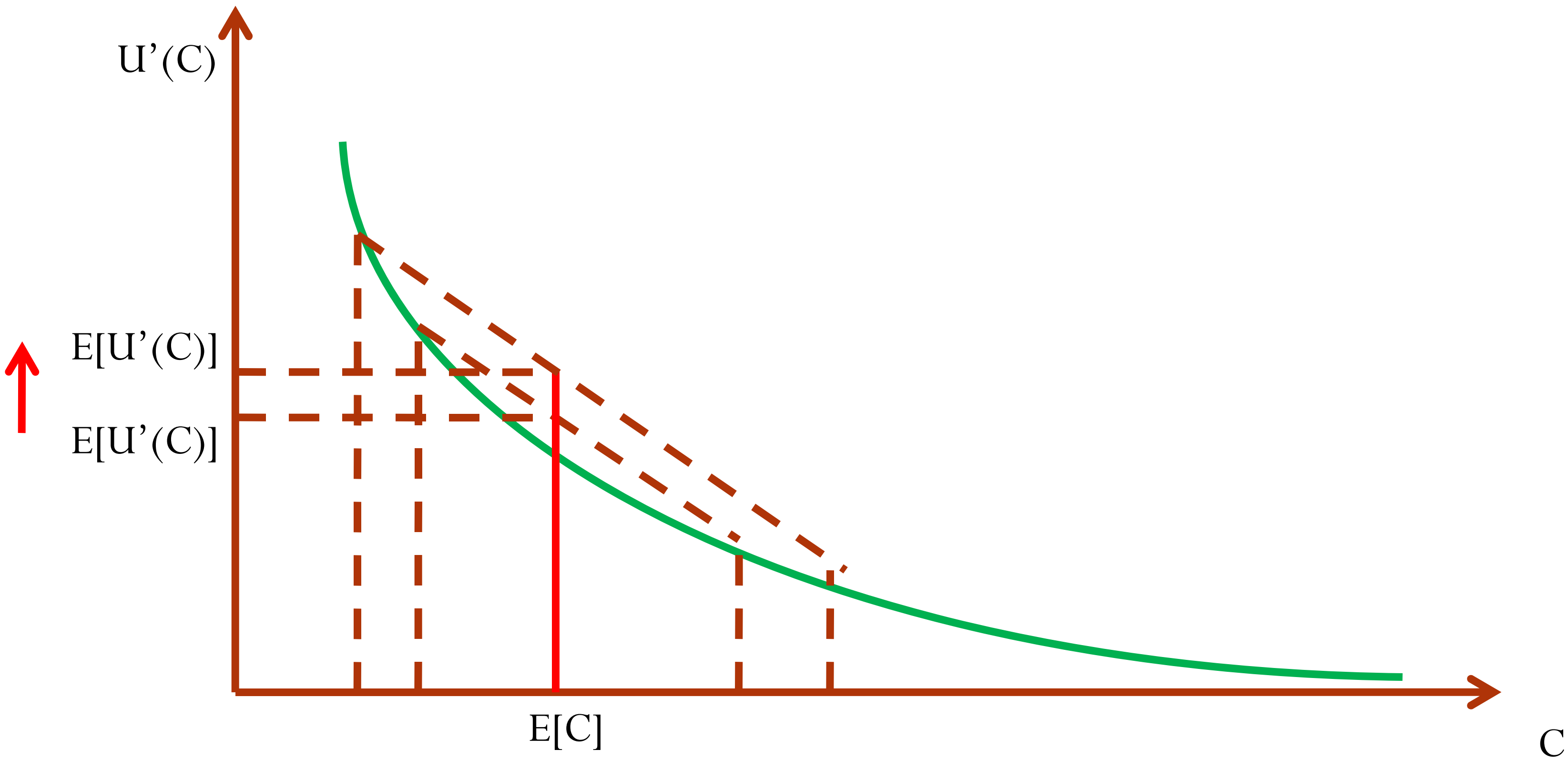
- Consumption is lower than in the quadratic case. Remember:

$$u'(C_t) = R\beta E_t\{u'(C_{t+1})\}$$

- The agent
  - is said to be prudent
  - and achieves precautionary savings
- Uncertainty provides additional insights

Precautionary savings

- An increase in uncertainty decreases consumption in the Euler equation



Practical issues

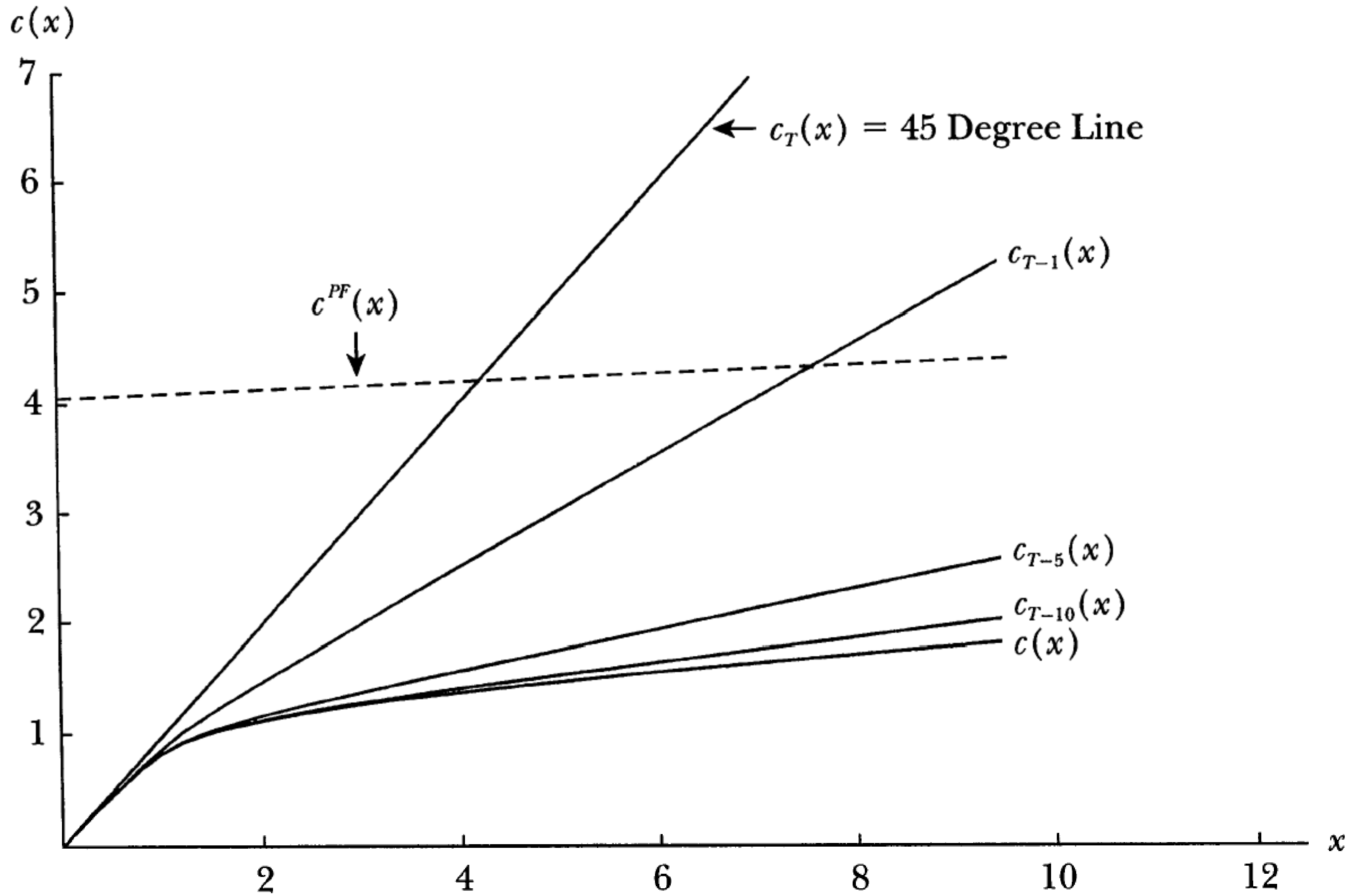
- Unfortunately,
  - if utility is not quadratic
  - the solution is computationally cumbersome/impossible
- Requires the use of simulations
- Remember, the consumption function

$$C_t = \tilde{C}\left(M_t\right)$$

solves

$$u'\left(\tilde{C}\left(M_t\right)\right) = \beta R U_{t+1}'\left(R\left(M_t - \tilde{C}\left(M_t\right)\right) + Y_{t+1}\right)$$

Convergence of Consumption Functions $c_{T-n}(x)$ as n Rises



Source: Carroll (2001)

Perfect foresight vs. uncertainty

- The converged consumption function is below the perfect foresight solution
  - Precautionary savings makes consumption to decrease

- As wealth goes up, MPC tends to perfect foresight
  - As wealth goes up, risk disappears

- The function is below the 45° line
  - Agent never borrows
  - He fears zero wealth as marginal utility is infinite in zero
  - Sel-imposed borrowing constrained

Borrowing constraints

- Assumption: agent cannot borrow more than a given quantity
- Even if utility is quadratic, one may have similar results

- Implications:
- When the restriction binds, consumption will be below optimum
- The fact that it may bind in the future reduces consumption today

Borrowing constraints

- Agent lives three periods
- Expected utilty over the last two periods

$$U = \left(C_2 - \frac{a}{2} C_2^2 \right) +$$

$$E_2 \left(A_1 + Y_2 + Y_3 - C_2 \right)$$

$$-E_2 \frac{a}{2} \left(A_1 + Y_2 + Y_3 - C_2 \right)$$

Borrowing constraints

- Derivative with respect to consumption

$$\frac{\partial U}{\partial C_2} = a\left(A_1 + Y_2 + E_2 Y_3 - 2C^2\right)$$

- Solution for consumption in second period

$$C_2 = \min\left\{\frac{A_1 + Y_2 + E_2 Y_3}{2}, A_1 + Y_2\right\}$$

Borrowing constraints

- If condition does not binds in the first period,we have

$$C_1 = E_1 \left\{ C_2 \right\}$$

- Given the constraint may be binding in period 2 makes expected consumption in period 2 to decrease

- In turn, agent chooses lower consumption in period 1