feature article

Alternatives to cosmological inflation

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Inflationary cosmology is elegant and can claim significant predictive successes. But its attendant difficulties have motivated competing scenarios that also agree with observation.

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The inflationary universe scenario is the current paradigm of early-universe cosmology. Inflation has been successful phenomenologically: It not only provides answers to several questions that cannot be addressed in the standard Big Bang (SBB) cosmology, but it also led to the first quantitative and predictive theory for the origin of structure in the universe, a theory whose predictions have been verified to great accuracy by cosmic microwave background (CMB) anisotropy experiments. At present, however, models of inflation suffer from serious conceptual problems that have driven attempts to find alternative paradigms.

Inflation's triumphs and troubles

Inflationary cosmology is based on the assumption that there was a period in the very early universe during which space underwent accelerated, exponential expansion.[1] The paradigm provides explanations for several puzzles of SBB cosmology. For example, during the period of accelerated expansion, the size of the region that could have been in causal contact increased exponentially. (Causal contact means that a single earlier event could, in principle, affect all the points in the region.) That enormous growth can explain the observed homogeneity and isotropy of the universe on large scales and thereby solve the so-called horizon problem of standard cosmology, illustrated in figure 1. Accelerated expansion reduces the spatial curvature and thus solves the flatness problem—that is, it eliminates the mystery from the observation that space now appears almost perfectly flat on cosmological scales. Without accelerated expansion of space, it appears impossible to explain the large size of the current universe, given its known age of about 14 billion years and its origin at a microscopic size. Inflation also provides a mechanism by which the entropy of the universe can increase by a large factor. Thus inflation solves two additional problems of standard cosmology, the size and entropy problems.

Given the theory of general relativity, inflation requires that matter be dominated by a component with a sufficiently large and negative pressure. Most models of inflation take that matter component to be a scalar field that is very weakly coupled to itself and to other matter. The dynamics of the scalar field are determined by its potential energy function, much as in the case of a Newtonian particle. If the scalar field is initially displaced from the minimum of its potential energy, then as with a particle, the field will slowly roll toward the potential minimum. But unlike in the particle case, the potential leads to a negative pressure whose absolute value is comparable to the field's energy density, and that pressure generates an almost exponential expansion of space.

Inflationary cosmology's most important success is to provide a causal scheme for generating primordial density fluctuations;[2] figure 2 illustrates the mechanism. Because of the accelerated expansion of space, the physical wavelengths of cosmological structures in the universe start out much smaller than the Hubble length that sets the scale beyond

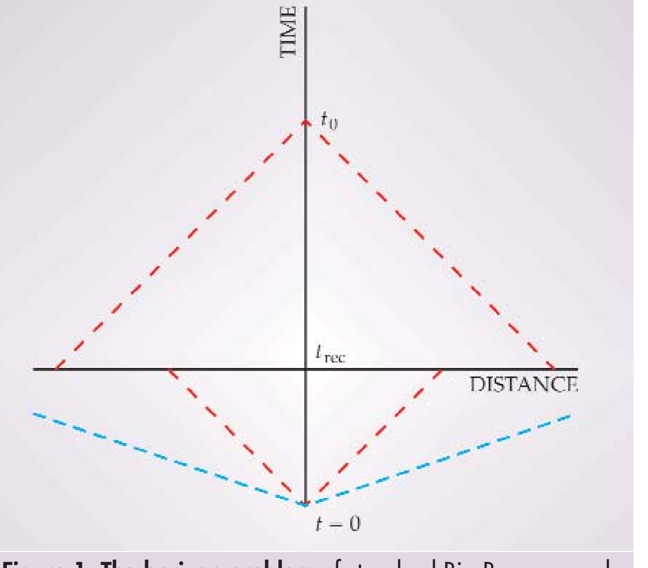

Figure 1. The horizon problem of standard Big Bang cosmology. In this spacetime sketch, t_0 denotes the present time, and the recombination time t_{rec} labels the last scattering surface, the surface astronomers probe when they look at the cosmic microwave background (CMB). In SBB cosmology, light paths are at 45° to the horizontal. Thus the points where the lower set of dashed red lines intersects the last scattering surface bound a region that can be in causal contact, that is, accessible by a common past event. The isotropic CMB is emitted from the points where the upper set of dashed red lines intersects the last scattering surface. Evidently, the photons of the CMB come from points that are not in causal contact, and so it is impossible to explain the overall isotropy of the CMB by causal physics. The dotted blue lines indicate light paths for a varying-speed-of-light theory in which the speed of light at early times is much greater than at present. In such a theory, the part of the last scattering surface in causal contact is much enhanced. The accelerated expansion of inflationary cosmologies also increases the size of the region in causal contact and so solves the horizon problem.

which causal physics cannot generate fluctuations. (The Hubble length is c/H, where c is the speed of light and H is the celebrated Hubble expansion rate. Cosmologists invariably set the speed of light to unity and denote the Hubble length by H^{-1}.) In SBB cosmology it is not possible for a causal mechanism to generate fluctuations on cosmological scales; the physical scales are always bigger than the Hubble length.

Any classical fluctuations initially present in an inflationary cosmology have a minimum size and are rapidly stretched during the accelerated expansion of space. Thus inflation relies on quantum vacuum fluctuations that are continuously created on all scales and at all times during the period of inflation. When their wavelengths are smaller than the Hubble length, the fluctuations oscillate. But once their wavelengths grow beyond the Hubble length, the fluctuations become standing waves.

Inflation, coupled with the assumption that most of the matter in the universe is nonbaryonic and nonrelativistic, is the starting point for the successful cold-dark-matter model of cosmological structure formation. Crucial to the success of the model is that the power spectrum of the induced curvature fluctuations be almost scale invariant and that the perturbations be standing waves while their wavelength is bigger than the Hubble length. Those two features lead to characteristic peaks at specific angular values in the CMB anisotropy maps, peaks that were predicted more than 15 years before they were observed (see PHYSICS TODAY, April 2003, page 21).

The inflationary universe scenario, however, suffers from some important conceptual problems.[3] First, although many toy models of inflation have been proposed, embedding inflation into a realistic particle-physics model has proven to be a challenge. It is difficult to obtain a potential for a scalar field that is sufficiently flat to allow for inflation, and even more difficult to adjust the model so that one obtains a sufficiently small amplitude of the primordial fluctuations. If inflation is generated by a scalar matter field, a spacetime singularity must have preceded the period of inflation.[4] This singularity problem demonstrates that inflationary cosmology is incomplete and cannot provide the ultimate theory of the early universe.

The so-called trans-Planckian problem concerns the most important success of inflationary cosmology: the structure-formation scenario. As figure 2 shows, if the period of inflation was long, then at the beginning of the accelerated expansion, the physical wavelengths of all scales currently observed in cosmology were incredibly small. Indeed, they would be smaller than the Planck length—the length scale built from Planck's constant, Newton's gravitational constant, and the speed of light. The theory of cosmological fluctuations, however, is based on general relativity. And that gravitational theory is without doubt inapplicable at the sub-Planckian scales present during the early phases of inflation. Moreover, it is easy to construct toy models of sub-Planckian physics that lead to a fluctuation spectrum with big deviations from scale invariance.[5] Such constructions show that the predictions of inflation are not robust; rather they rely on implicit assumptions about Planck-scale physics.

In a similar vein, the energy scale of inflation, determined by the observed amplitude of the CMB anisotropies and related to the rate of exponential expansion, is usually close to the scale at which general relativity breaks down. Can we really trust predictions derived in such a context?

The above-mentioned conceptual difficulties are main motivations for searching for alternatives to inflation. Is there a new theory of the very early universe that will resolve some of the problems of inflation while maintaining its phenomenological successes? If such an alternative exists, can one distinguish its predictions from those of inflationary cosmology?

Defects and varying light speed

In the mid-1980s, topological defects received a lot of attention as providing a noninflationary mechanism that explains the origin of inhomogeneities.[6] Topological defects arise generally in models of elementary-particle physics. In fact, such defects arise in many laboratory physical systems and include crystal defects that form when a metal solidifies and vortex lines in superconductors and superfluids (see the article by Tom Kibble, PHYSICS TODAY, September 2007, page 47). Defects of interest for cosmological structure formation are cosmic strings, which are linear defects, and textures, by which cosmologists denote point defects in spacetime. If formed in the early universe, the cosmic strings and textures that arise in many particle-physics models lead to a scale-invariant spectrum of cosmological fluctuations whose amplitudes roughly agree with observations. On the other hand, topological-defect models do not change the spacetime

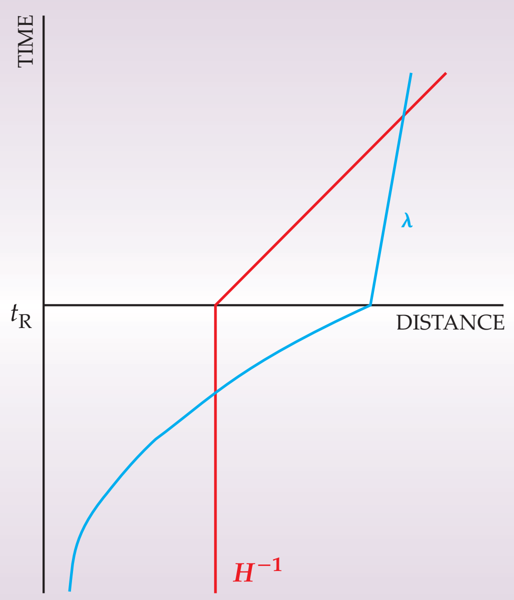

Figure 2. Fluctuation evolution in inflationary cosmology. The period of accelerated expansion ends at the so-called reheating time t_R, when the inflation field decays and regular matter is generated. After reheating comes the radiation-dominated phase of standard cosmology. The red line indicates the Hubble length, which is constant during the exponential expansion but increases linearly after t_R. The blue line denotes the evolution of a structure with wavelength λ. Initially, the wavelength is less than the Hubble length. Eventually, it exceeds the Hubble length and then evolves for a long time on super-Hubble scales. At late times the Hubble length catches up to the fluctuation wavelength. After that final crossing, the fluctuations oscillate. The evolution of fluctuations from sub-Hubble to super-Hubble and back is a common feature of alternatives to inflation that replicate inflation's success in generating fluctuations on cosmological scales.

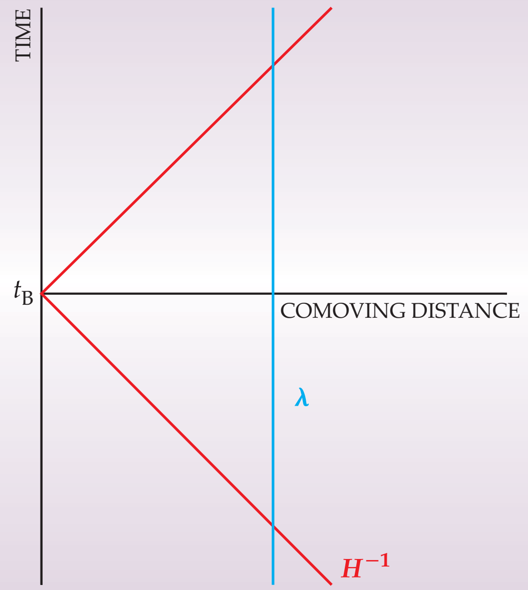

Figure 3. Pre–Big Bang cosmology. Before the bounce time t_B, the universe is contracting. The bounce itself is a singular transition between the contracting phase and the usual radiation phase of standard Big Bang cosmology. As in inflationary cosmology, the scale of a fluctuation (blue) starts out sub-Hubble, crosses the Hubble length (red), and evolves for a long time on super-Hubble scales before recrossing the Hubble length. The behavior of the Hubble length and a fixed wavelength appear particularly simple in terms of "comoving" distance, for which the expansion or contraction of space is factored out.

evolution of SBB cosmology and hence cannot provide explanations for the horizon, flatness, size, and entropy problems. Thus they never did provide a complete alternative to inflationary cosmology.

Moreover, detailed analysis of the CMB spectrum shows that topological-defect models cannot provide the dominant mechanism of structure formation. In such models, the cosmological fluctuations on super-Hubble scales are continuously perturbed by the network of defects. They are not frozen and coherent as in inflationary cosmology. Hence, topological-defect models do not predict the characteristic oscillations that have now been detected and mapped out with great precision in the angular power spectrum of CMB anisotropies.

The varying-speed-of-light scenario is a proposed alternative to inflationary cosmology that assumes the speed of light in the very early universe was much larger than it is today.[7] Relative to SBB cosmology, the enhanced speed of light increases the size of a region that can be in causal contact. Thus, as illustrated in figure 1, the varying-speed-of-light scenario provides a solution for the horizon problem. It also solves the flatness problem of SBB cosmology.

It is a high cost to throw out a sacred principle of successful relativistic theory—that the speed of light is constant—simply to solve problems of cosmology. Some theorists have worked to implement the varying-speed-of-light scenario, but no clear model has emerged. On the other hand, it is precisely the uncritical extrapolation of the sacred principles of current physics that has led to unsolved basic problems like the absence of a consistent theory of quantum gravity and the cosmological-constant problem. So one might argue that the right way to make true progress in early-universe cosmology is to go beyond the established framework.

Bouncing cosmologies

Superstring theory is the best candidate for a quantum theory of gravity that unifies all forces of nature. If superstring theory is indeed true, one of its primary domains of applicability will be to the physics of the early universe. The three alternatives to inflation on which the rest of this article will focus are all motivated by superstring theory. But in recent years, theorists have also constructed several explicit realizations of inflation from string theory.[8]

In 1993 Maurizio Gasperini and Gabriele Veneziano proposed the pre–Big Bang scenario as a possible alternative to cosmological inflation.[9] The starting point of the PBB scenario is so-called dilaton gravity, a background for string theory that includes a scalar field in addition to the usual general-relativity graviton, the mediator of the gravitational force. The modified theory includes a new symmetry not conventionally present, and as a result, the usual expanding phase of SBB cosmology must be preceded by a mirror-symmetric contracting phase. Gasperini and Veneziano assumed that new physics going beyond dilaton gravity could lead to a singularity-free connection between the contracting and expanding phases and thus realize a bouncing cosmology.

Figure 3 sketches the evolution of the key cosmological length scales in PBB cosmology. In the PBB scenario, the universe starts out cold, large, and flat; thus the model solves the horizon and size problems of SBB cosmology. Before the bounce between contracting and expanding phases, the Hubble length contracts relative to the wavelength of a fluctuation; afterward, it expands. As a result, the PBB scenario allows one to construct a causal mechanism for the formation of primordial cosmological fluctuations. Gasperini, Veneziano, and coworkers initially considered quantum fluctuations in the dilaton field as the source for cosmological perturbations. However, a careful analysis showed that, in conflict with observations, the perturbations are not scale invariant. Instead, one obtains a spectrum that has too much power at short wavelengths and not enough at the long wavelengths relevant to cosmology. The problem can be overcome by inducing curvature fluctuations via the so-called curvaton mechanism. That is, one sets up fluctuations in other matter fields so as to induce curvature perturbations after the bounce.

The ekpyrotic scenario, proposed in 2001 by Justin Khoury, Burt Ovrut, Paul Steinhardt, and Neil Turok,[10] has some elements in common with the PBB model. An important new ingredient, which comes from string theory, is extra spatial dimensions. In particular, the scenario assumes that one of the extra spatial dimensions is bounded by two branes (analogues of planes in higher dimensions), one of which confines the matter of our universe; figure 4 illustrates the idea. The separation of the branes decreases as a consequence of an attractive potential between them. Once the branes get to within one string length of each other, some of the energy is transferred to the brane on which we live and creates what appears to be a big bang. From our point of view, however, the bang would not appear singular. The ekpyrotic scenario can be described in terms of an effective field theory formulated in four spacetime dimensions. In that theory, spacetime evolution

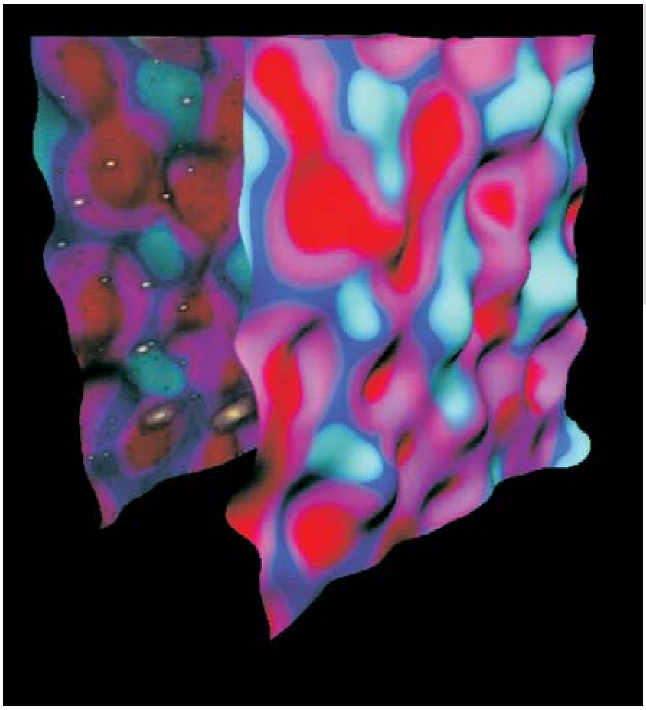

Figure 4. The ekpyrotic scenario. In this alternative to inflationary cosmology, the universe we inhabit is confined to one of two sheets that represent four-dimensional branes bounding a five-dimensional spacetime. The branes initially approach each other, then bounce and separate; in some models the process repeats in cycles. The colors on the branes indicate that the matter density is not uniform. (Courtesy of Stewart Dickson and Paul J. Steinhardt.)

looks quite similar to that in the PBB scenario. In particular, the initial phase during which the two boundary branes are approaching one another appears as a contracting period.

At the bounce that separates contraction from expansion, the effective 4D field theory develops a singularity. A singularity is also present in the underlying 5D gravitational background. However, in contrast to PBB cosmology, the effective string coupling in the ekpyrotic scenario becomes very weak near the singularity. That property raises the hope that string theory will offer a controlled way to resolve the singularity—that is, to show it as absent or to render it harmless.

For several years theoretical cosmologists vigorously debated about the spectrum of fluctuations in the ekpyrotic scenario. The dust on the issue has now settled: The spectrum of fluctuations seeded by initial quantum vacuum perturbations is scale invariant after the bounce and thus is in agreement with current data. To obtain that result, however, one needs to be careful and work with the 5D theory or, if working with the 4D effective theory, to take into account entropy modes that arise when one reduces the theory from five to four dimensions. An epoch in which the pressure of matter is much greater than its energy density is the key ingredient that leads to the scale invariance of the spectrum. A distinctive prediction of ekpyrotic models is an extremely low amplitude for primordial gravitational waves. Thus the detection of a large background of gravitational waves would provide a way to observationally falsify the ekpyrotic scenario.

The original ekpyrotic scenario considered a single bounce of the boundary planes. A variant of the scenario, the cyclic universe, considers multiple bounces.[11] To obtain the transition from expansion to contraction at large plane separations, one introduces a potential energy function for the planes that, at large distances, increases slowly with separation and becomes positive. Near the turnaround point, the universe is accelerating. In other words, the cyclic scenario has built into it a period of dark-energy domination.

String gas

Many theorists hope that string theory, once it is formulated nonperturbatively, will be free of singularities. One way to see how string theory should be able to avoid physical cosmological singularities is to consider a d-dimensional torus containing a gas of closed strings in thermal equilibrium and to follow the temperature of that gas as the volume is reduced. A d-dimensional torus is equivalent to a d-dimensional cube with periodic boundary conditions imposed on opposite faces. A 2D torus is usually just called a torus; it looks like the surface of a doughnut.

String theorists established early on that the temperature of a gas of closed strings could never exceed a maximal value, the Hagedorn temperature. Thus, as the size of the torus decreases while thermal equilibrium is maintained, the temperature of the string gas first rises, then levels off at a maximal value close to the limiting temperature, and finally starts to decrease as the size further decreases. Thus it appears that string-based cosmology should be able to avoid a temperature singularity.

String gas cosmology is a cosmological model motivated by the above considerations.[12] The philosophy behind it is to focus on the symmetries and degrees of freedom contained in string theory but not present in point-particle theories, and to construct a model of cosmology consistent with those symmetries. The new degrees of freedom that string theory provides are the strings' oscillatory modes and winding modes. The existence of the Hagedorn temperature is deduced from the exponential divergence of the number of string oscillatory modes as a function of their energy.

The presence of winding modes leads to a new symmetry of string theory, target-space duality (T-duality). As figure 5 shows, a string can wind around the 2D torus any integer number of times in either of two different ways. On any torus, the energy of momentum modes is quantized, taking on the values n/R, where the radius R of the torus in the appropriate dimension is measured relative to the string length and n is an integer. The energy of the winding modes is also quantized, but it takes on values mR, where m is an integer. Thus the mass spectrum of string states is unchanged if R goes to $1/R$ and the winding and momentum numbers m and n are interchanged. The second duality that string gas cosmology should obey is S-duality, a duality between strong and weak string coupling.

In string gas cosmology, cosmic evolution begins with a Hagedorn phase—a gas of strings with all possible modes excited and thus at a temperature just below the Hagedorn temperature. The background metric and dilaton are chosen to be invariant under the T- and S-dualities. As a consequence of that choice, spacetime is quasi-static: The physical wavelength of a fluctuation is nearly constant, and the Hubble length tends to infinity. The annihilation of closed strings

Figure 5. Windings around a torus. Some of the string on this skein of string winds around the equator three times to give a belt-like appearance. Other portions of the string wind through the openings on the top and bottom of the skein. (Courtesy of Jean-Christophe Hamilton.)

with opposite windings creates radiation and leads to a smooth transition from the initial Hagedorn phase to the usual radiation phase of SBB cosmology. Incidentally, an annihilation of string winding modes is possible in no more than three spatial dimensions, so string gas cosmology provides a possible scenario for explaining why there are exactly three large spatial dimensions. In following the post-annihilation evolution of the three large dimensions, one assumes that the Hagedorn phase had lasted sufficiently long to establish thermal equilibrium at scales of up to 1 mm, the physical wavelength corresponding to the current Hubble radius if it is evolved back in time to the beginning of the radiation phase.

Figure 6 shows a spacetime sketch of string gas cosmology. As in inflationary cosmology and in the PBB and ekpyrotic scenarios, fluctuations are initially inside the Hubble radius, and hence a causal generation mechanism for fluctuations is possible. The fluctuations are produced in the Hagedorn phase. Since that phase is quasi-static and dominated by a hot gas of strings, the fluctuations produced are string thermal fluctuations, not quantum vacuum fluctuations as in the case of the inflationary, PBB, and ekpyrotic scenarios.

In the past two years, theorists have shown that string thermal fluctuations of a gas of closed strings on a 3D torus lead to an almost scale-invariant spectrum of cosmological perturbations at late times.[13] The spectrum actually has a slight red tilt, an excess of power at long wavelengths. But such a red tilt is also obtained in most inflationary-universe models. Inflationary models also usually yield a spectrum of gravitational waves with a slight red tilt. By contrast, the string gas structure formation scenario predicts a small blue tilt, with more power at short wavelengths. Thus, gravitational-wave spectroscopy provides a potential way to discriminate between the predictions of inflation and string gas cosmology.

Alternatives have difficulties too

None of the alternatives to cosmological inflation discussed in this article are free from serious problems. The most important problem facing the PBB and the ekpyrotic scenarios is the singularity at the bounce point. It is true that inflationary cosmology also has a cosmological singularity, but that singularity is before the period of inflation and does not affect the physics between the onset of inflation and later times. In contrast, the singularity in both the PBB and the ekpyrotic scenarios is at the transition between the contracting phase when fluctuations are set up and the expanding, radiation phase of SBB cosmology. Without resolving the singularity, it is impossible to reliably compute the transition of the spectrum of fluctuations from the contracting phase where they are produced to the expanding phase where they are measured. In addition, from the 4D point of view the physics that leads to the transition between contraction and expansion implies violations of various energy theorems that regular matter obeys.

A large number of recent works in the context of the ekpyrotic model have used new string-theory techniques to resolve the singularity at the bounce point. Progress on that issue is encouraging. Along another line, a new version of the ekpyrotic scenario has recently been proposed in which a nonsingular bounce is obtained by making use of so-called ghost condensates.[14]

Without a final resolution of the singularity issue, the evolution of fluctuations in PBB and ekpyrotic cosmology is not uniquely determined. One may call this the trans-Planckian problem for PBB and ekpyrotic fluctuations. However, whereas in inflationary cosmology the trans-Planckian problem affects only the initial conditions for the fluctuations, in PBB and ekpyrotic cosmology the trans-Planckian domain is hit in the middle of the fluctuations' evolution time, at the transition point between when the background cosmology is contracting and when it is expanding. That mid-evolution hit, however, suggests that Planck-scale physics is more likely to have observable imprints on PBB and ekpyrotic cosmology than on the inflationary universe.

Inflationary cosmology not only provides a causal mechanism of structure formation but also, as discussed earlier, successfully solves several famous puzzles of SBB cosmology. How do PBB and ekpyrotic cosmology address those problems? First of all, the horizon problem of SBB cosmology is automatically solved provided the period of contraction is long. Assuming that the universe starts cold and large, the size and entropy problems of SBB cosmology are absent.

A key problem, however, is flatness. Early versions of both the PBB and ekpyrotic scenarios assumed that at the beginning of the contracting phase, classical fluctuations were absent—all fluctuations were of the quantum vacuum. Without that assumption, there would be a serious puzzle: Initial classical fluctuations would grow and lead to copious black hole formation around the bounce point, invalidating the cosmological scenario.[15] The authors of the PBB and ekpyrotic scenarios have since addressed the flatness problem and responded to other criticisms. For example, Alessandra Buonanno, Thibault Damour, and Veneziano have studied the emergence of PBB cosmology from a chaotic initial state through a process resembling a gravitational collapse.[16] In the cyclic scenario, the period of acceleration at large brane separation can provide appropriate initial conditions for the next contracting phase. Even without that phase of acceleration, the wavelength dependence of the classical fluctuation spectra implies that the universe will be smoothed, flattened, and isotropized during the period of ekpyrotic contraction.[17]

The main problem string gas cosmology currently faces is that no one has yet formulated an effective action that gives the required dynamics for the cosmological background. In particular, neither general relativity nor dilaton gravity can correctly describe the Hagedorn phase. However, one should not expect those theories to be applicable, since they do not include all the required symmetries of string theory.

In the absence of a suitable action, string gas cosmology should perhaps not yet be considered as a real alternative to inflation.[18] But it is a model based on new symmetries and new degrees of freedom that are known to be key ingredients of nonperturbative string theory. One might argue that such a model will give more reliable insight than one based on a well-defined effective field theory known to break down at the minuscule length scales of the very early universe.

Another key challenge for string gas cosmology is to explain the current size of the universe. If the Hagedorn phase can be engineered to take the place of the midpoint of a bouncing universe scenario, then the size and entropy problems are solved as they are in the PBB and ekpyrotic models, and the horizon problem is solved as well. Since the Hagedorn phase is nonsingular, nearly static, and in thermal equilibrium, the flatness problem also may be solved. The trans-Planckian problem for fluctuations does not arise: The physical wavelength of fluctuations currently measured in cosmological observations is always larger than 1 mm and thus never enters the trans-Planckian regime.

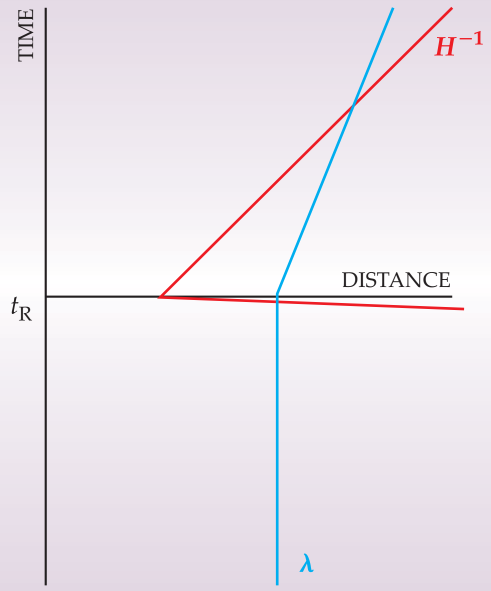

Figure 6. String gas cosmology. At time t_R, the universe undergoes a smooth transition from a quasi-static Hagedorn phase to the usual radiation phase of standard Big Bang cosmology. That smooth transition is in many ways analogous to the reheating transition in inflation. As one moves deeper into the Hagedorn phase, the Hubble radius (red) tends to infinity but the physical wavelength of a fluctuation (blue) is essentially constant. As in inflationary cosmology, the scale of a fluctuation starts out sub-Hubble, crosses the Hubble length, and evolves for a long time on super-Hubble scales before recrossing the Hubble length.

Time will tell

Inflationary cosmology, although extremely successful phenomenologically, is faced with serious conceptual problems that have motivated alternative models. At present, though, none of the proposed alternatives is as well developed as inflationary cosmology; all of them lack a formulation in terms of an effective field theory that can describe the entire period of cosmological evolution from when the cosmological perturbations are produced until when they are measured. Nor do any of the proposed alternatives solve the famous problems of SBB cosmology as elegantly as inflation does.

Nevertheless, it is now clear that alternative cosmological scenarios exist that can generate a nearly scale-invariant spectrum of cosmological perturbations to explain the details of the CMB anisotropy spectrum. Most important, the alternative models can be distinguished from inflation by observational means. And cosmologists expect a lot of new observational data over the next few years. New ground-based telescopes will produce maps of the CMB with much better angular resolution than that of the currently available maps. The European Space Agency's *Planck* satellite, due to be launched this year, will give full sky maps of the CMB with similarly improved resolution. Data from those and future measurements may eventually determine whether one of the alternative models is correct or whether inflation passes further challenges. It will surely reveal a lot about physics close to the Planck scale.

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