Since the well is symmetric about $x = 0$, we need only match wave functions at $x = b$ and a. We look at $E < 0$, so that we introduce and $\alpha^2 = 2m|E|/\hbar$ and $q^2 = 2m(V_0-|E|)/\hbar$. We now write down
Even solutions:

$$\begin{aligned} u(x) &= \cosh\alpha x & 0 < x < b \\ &= A \sin qx + B \cos qx & b < x < a \\ &= C e^{-\alpha x} & a < x \end{aligned}$$

Matching $\frac{1}{u(x)}\frac{du(x)}{dx}$ at $x = b$ and at $x = a$ leads to the equations

$$\alpha \tanh \alpha b = q\frac{A\cos qb - B\sin qb}{A\sin qb + B\cos qb}$$

$$-\alpha = q\frac{A\cos qa - Bsnqa}{A\sin qa + B\cos qa}$$

From the first equation we get

$$\frac{B}{A} = \frac{q\cos qb - \alpha \tanh \alpha b \sin qb}{q\sin qb + \alpha \tanh \alpha b \cos qb}$$

and from the second

$$\frac{B}{A} = \frac{q\cos qa + \alpha \sin qa}{q\sin qa - \alpha \cos qa}$$

Equating these, cross-multiplying, we get after a little algebra

$$q^2 \sin q(a-b) - \alpha \cos q(a-b) = \alpha \tanh \alpha b[\alpha \sin q(a-b) + q\cos q(a-b)]$$

from which it immediately follows that

$$\frac{\sin q(a-b)}{\cos q(a-b)} = \frac{\alpha q(\tanh \alpha b + 1)}{q^2 - \alpha^2 \tanh \alpha b}$$

<u>Odd Solution</u>

Here the only difference is that the form for $u(x)$ for $0 < x < b$ is $\sinh\alpha x$. The result of this is that we get the same expresion as above, with $\tanh\alpha b$ replaced by $\coth\alpha b$.