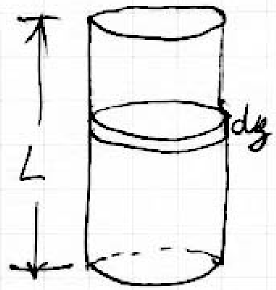
Pauta Ejercicio N° 3 Fecha / Date: 8-05-08

1.-



$$dR = \frac{1}{g(z)} \cdot \frac{dz}{\pi a^2} = \frac{1}{\pi a^2 g_0} \frac{dz}{(1 + z/L)}$$

$$R = \frac{1}{\pi a^2 g_0} \int_0^L \frac{dz}{1 + z/L}$$

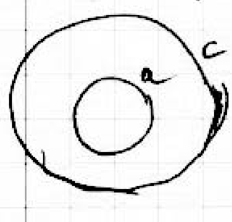
Cambio de variables: $x = z/L$

$$R = \frac{L}{\pi a^2 g_0} \int_0^1 \frac{dx}{1 + x}$$

$$R = \frac{L}{\pi a^2 g_0} \left| \ln(1+x) \right|_0^1 = \frac{L \ln(2)}{\pi a^2 g_0}$$

Corriente: $I = V/R = \dfrac{\pi a^2 g_0 V}{L \ln(2)}$

2.- Dos esferas concéntricas de radio a y c:



Determinar el potencial con las condic. de borde: $V(a) = V$, $V(c) = 0$

Ec. Laplace: $\dfrac{1}{r^2} \dfrac{\partial}{\partial r}\left(r^2 \dfrac{\partial V}{\partial r} \right) = 0$

Integrando: $r^2 \dfrac{\partial V}{\partial r} = C_1$

$$\frac{\partial V}{\partial r} = \frac{C_1}{r^2}$$

Integrando: $V = \dfrac{-C_1}{r} + C_2$

Condic. de borde:

$$\left. \begin{aligned} -\frac{C_1}{a} + C_2 &= V \\ -\frac{C_1}{c} + C_2 &= 0 \end{aligned} \right|$$

$$C_2 = \frac{C_1}{c}$$

$$C_1\left(\frac{1}{c} - \frac{1}{a}\right) = V$$

$$C_1 = \frac{V}{\frac{1}{c} - \frac{1}{a}} \; ; \quad C_2 = \frac{V}{c\left(\frac{1}{c} - \frac{1}{a}\right)}$$

$$V(r) = \frac{V}{\left(\frac{1}{a} - \frac{1}{c}\right)}\left(\frac{1}{r} - \frac{1}{c}\right)$$

$$\lim_{c\to\infty} V(r) = \frac{Va}{r}$$

$$\vec{E} = -\nabla V = -\frac{\partial V}{\partial r}\hat{r} = \frac{Va}{r^2} \quad (c \to \infty)$$

$$= \frac{Va\,\hat{r}}{\left(1 - \frac{a}{c}\right) r^2} \quad (c \text{ finito})$$

$$J = g E$$

en $r = a$: $J = \frac{gVa}{a^2} = \frac{gV}{a}$

$$I = J\, 4\pi a^2 = 4\pi a g V$$

$$R_a = \frac{V}{I} = \frac{1}{4\pi g a}$$

Dos esferas de radios a y b : $R = R_a + R_b$

$$= \frac{1}{4\pi g}\left(\frac{1}{a} + \frac{1}{b}\right)$$