Pauta Control 1 Fecha / Date: 21/04/08

1) Estado inicial:

Energía $W_1 = \frac{1}{2} Q V = \frac{Q^2}{2C_1}$

Estado final, cargas Q_1 y Q_2

$$Q_1 + Q_2 = Q$$

$$\frac{Q_1}{C_1} = V = \frac{Q_2}{C_2} \quad \therefore \quad Q_2 = Q_1 \cdot \frac{C_2}{C_1}$$

$$\therefore Q_1 + Q_1 \frac{C_2}{C_1} = Q$$

$$Q_1 \left(1 + \frac{C_2}{C_1}\right) = Q$$

$$Q_1 = \frac{Q}{1 + \frac{C_2}{C_1}} = \frac{Q\, C_1}{C_1 + C_2}$$

$$Q_2 = Q - Q_1 = Q\left(1 - \frac{C_1}{C_1 + C_2}\right)$$

$$Q_2 = \frac{Q \cdot C_2}{C_1 + C_2}$$

Energía: $W_2 = \frac{Q_1^2}{2C_1} + \frac{Q_2^2}{2C_2} = \frac{Q^2 C_1^2}{2C_1 (C_1 + C_2)^2} + \frac{Q^2 C_2^2}{2C_2 (C_1 + C_2)^2}$

$$W_2 = \frac{Q^2 (C_1 + C_2)}{2 (C_1 + C_2)^2} = \frac{Q^2}{2 (C_1 + C_2)}$$

$$\frac{W_2}{W_1} = \frac{\cancel{Q^2} \cdot \cancel{2} C_1}{\cancel{2} (C_1 + C_2) \cdot \cancel{Q^2}} = \frac{C_1}{C_1 + C_2} < 1$$

2) Sea $\sigma = \frac{Q}{a^2}$ = dens. de carga en las placas:

$$E_1 = \frac{\sigma}{\varepsilon_0} = \frac{Q}{a^2 \varepsilon_0}$$

$$D_1 = \sigma = \frac{Q}{a^2}$$

$$E_2 = \frac{\sigma}{K\varepsilon_0} = \frac{Q}{a^2 K \varepsilon_0}$$

$$D_2 = D_1 = \sigma = \frac{Q}{a^2}$$

Polarización: $P = \chi E_2 = (K-1)\varepsilon_0 E_2$

$$P = \frac{(K-1)}{K} \frac{\varepsilon_0 Q}{a^2 \varepsilon_0}$$

$$P = \frac{(K-1)}{K} \cdot \frac{Q}{a^2}$$

En los conductores: $\sigma = \frac{Q}{a^2}$

En el dieléctrico: $\sigma_p = P = \frac{(K-1)}{K} \frac{Q}{a^2}$

Calcular capacidad:

$$V = E_1 (d_1 - d_2) + E_2 \cdot d_2$$

$$= \frac{Q}{a^2 \varepsilon_0} \left[(d_1 - d_2) + \frac{1}{K} d_2 \right]$$

$$C = \frac{Q}{V} = \frac{\varepsilon_0 a^2}{(d_1 - d_2) + \frac{d_2}{K}} = \frac{K \varepsilon_0 a^2}{d_2 + K(d_1 - d_2)}$$

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3)

W_i = energía inicial; 1 gota de radio a
W_f = energía final; 2 gotas de radio b c/u

$$W_i = \frac{\varepsilon_0}{2}\int |E|^2 d^3r$$
$$= \frac{\varepsilon_0}{2} 4\pi \int_a^\infty r^2 dr \cdot \left(\frac{Q}{4\pi\varepsilon_0 r^2}\right)^2$$

$$W_i = \frac{Q^2}{2\cdot 4\pi\varepsilon_0}\int_a^\infty \frac{dr}{r^2} = \frac{1}{2}\frac{Q^2}{4\pi\varepsilon_0 a}$$

para W_f: $Q \to \frac{Q}{2}$, $a \to b$

calc b: $\frac{4}{3}\pi b^3 \times 2 = \frac{4}{3}\pi a^3 \therefore b = 2^{-\frac{1}{3}} \cdot a$

$$W_f = 2\times\frac{1}{2}\times\frac{(Q/2)^2\times 2^{+1/3}}{4\pi\varepsilon_0\, a} = \frac{1}{2}\times\frac{1}{2}\times\frac{Q^2}{4\pi\varepsilon_0 a}\cdot 2^{1/3}$$

$$W_f = 2^{-2/3}\, W_i = W_i / 2^{2/3} = \frac{W_i}{\sqrt[3]{4}} \approx \frac{W_i}{1,587} =$$
$$\approx 0.62996\, W_i$$

$$\Delta W = W_f - W_i = W_i\left(\frac{1}{\sqrt[3]{4}} - 1\right)$$
$$\approx -0.37\, W_i$$

$$\therefore\ W_f < W_i$$