Dipolos

$$V(\vec{r}) = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{\sqrt{r^2 + (d/2)^2 - dr\cos\theta}} - \frac{1}{\sqrt{r^2 + (d/2)^2 + dr\cos\theta}}\right]$$

$$V(\vec{r}) = \frac{q}{4\pi\varepsilon_0 r}\left[\frac{1}{\sqrt{1 + (d/2r)^2 - (d/r)\cos\theta}} - \frac{1}{\sqrt{1 + (d/2r)^2 + (d/r)\cos\theta}}\right]$$

Dipolos

$$V(\vec{r}) = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{\sqrt{r^2 + (d/2)^2 - dr\cos\theta}} - \frac{1}{\sqrt{r^2 + (d/2)^2 + dr\cos\theta}}\right]$$

$$V(\vec{r}) = \frac{q}{4\pi\varepsilon_0 r}\left[\frac{1}{\sqrt{1 - (d/r)\cos\theta}} - \frac{1}{\sqrt{1 + (d/r)\cos\theta}}\right]$$

$$V(\vec{r}) = \frac{q}{4\pi\varepsilon_0 r}\left[1 + \frac{d}{2r}\cos\theta - \left(1 - \frac{d}{2r}\cos\theta\right) + \cdots\cdots\right] \qquad \underline{d << r}$$

$$V(\vec{r}) = \frac{qd\cos\theta}{4\pi\varepsilon_0 r^2}$$

Definición: Momento dipolar $\vec{p} = q\vec{d}$

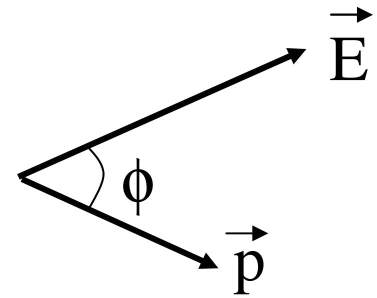
$$V(\vec{r}) = \frac{\vec{p}\cdot\hat{r}}{4\pi\varepsilon_0 r^2} = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2}$$

Campo eléctrico: $\vec{E} = -\nabla V$

Coordenadas esféricas: $E_r = -\frac{\partial V}{\partial r} = \frac{2p\cos\theta}{4\pi\varepsilon_0 r^3}$

$$E_\theta = -\frac{1}{r}\frac{\partial V}{\partial \theta} = \frac{p\,\mathrm{sen}\theta}{4\pi\varepsilon_0 r^3}$$

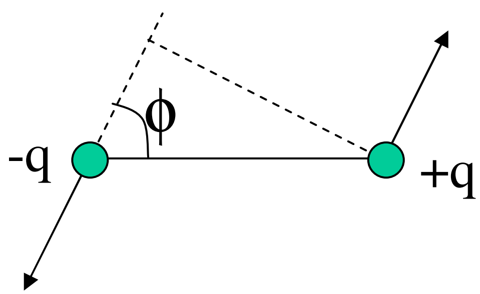
Energía de un dipolo en un campo eléctrico $\vec{E}$



$$W = q(V_+ - V_-) = -q\int_{-}^{+}\vec{E}\cdot d\vec{l} = -qdE\cos\phi = -\vec{p}\cdot\vec{E}$$

Fuerza = 0 si el campo E es uniforme.

Torque: $N = Eqd\,\mathrm{sen}\phi = pE\,\mathrm{sen}\phi$ $\vec{N} = \vec{p}\times\vec{E}$



Definición: Vector polarización $\vec{P}$

$$\vec{P} = \lim_{\Delta\tau \to 0} \left(\frac{1}{\Delta\tau} \sum_i \vec{p}_i \right)$$

Ej: N átomos/m^3, cada átomo tiene $\vec{p}$: $\vec{P} = N\vec{p}$

En un vol. d^3r, el momento dipolar es: $d\vec{p}(\vec{r}) = \vec{P}(\vec{r})\,d^3r$

Potencial producido en un punto definido por $\vec{r}$

$$dV(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \frac{\vec{P}(\vec{r}\,') \cdot (\vec{r} - \vec{r}\,')\,d^3r'}{|\vec{r} - \vec{r}\,'|^3}$$

Integrando sobre vol: defn: $\vec{R} = \vec{r} - \vec{r}'$

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int_V \frac{\vec{P}\cdot\vec{R}}{R^3}d^3r \quad = \frac{1}{4\pi\varepsilon_0}\int_V \vec{P}\cdot\nabla'\left(\frac{1}{R}\right)d^3r'$$

$$\nabla' = \hat{i}\frac{\partial}{\partial x'} + \hat{j}\frac{\partial}{\partial y'} + \hat{k}\frac{\partial}{\partial z'}$$

Demostrar que: $\nabla'\left(\frac{1}{R}\right) = \frac{\hat{R}}{R^2} = \frac{\vec{R}}{R^3}$

$$\text{Identidad: } \nabla\cdot\left(\vec{P}a\right)=\vec{P}\cdot\nabla a + a\nabla\cdot\vec{P}$$

$$a=\frac{1}{R}: \qquad \nabla\cdot\left(\frac{\vec{P}}{R}\right)=\vec{P}\cdot\nabla'\left(\frac{1}{R}\right)+\frac{1}{R}\nabla\cdot\vec{P}$$

$$\therefore\ \vec{P}\cdot\nabla'\left(\frac{1}{R}\right)=\nabla\cdot\left(\frac{\vec{P}}{R}\right)-\frac{1}{R}\nabla\cdot\vec{P}$$

$$\therefore\ V(\vec{r})=\frac{1}{4\pi\varepsilon_0}\int_{\mathbf{V}}\vec{\mathbf{P}}\cdot\nabla'\left(\frac{1}{\mathbf{R}}\right)\mathbf{d}^3\mathbf{r}'=\frac{1}{4\pi\varepsilon_0}\int_{\mathbf{V}}\nabla'\cdot\left(\frac{\vec{\mathbf{P}}}{R}\right)\mathbf{d}^3\mathbf{r}'-\frac{1}{4\pi\varepsilon_0}\int_{\mathbf{V}}\frac{1}{\mathbf{R}}\nabla'\cdot\vec{\mathbf{P}}\mathbf{d}^3\mathbf{r}'$$

Aplicar teorema de Gauss a la primera integral

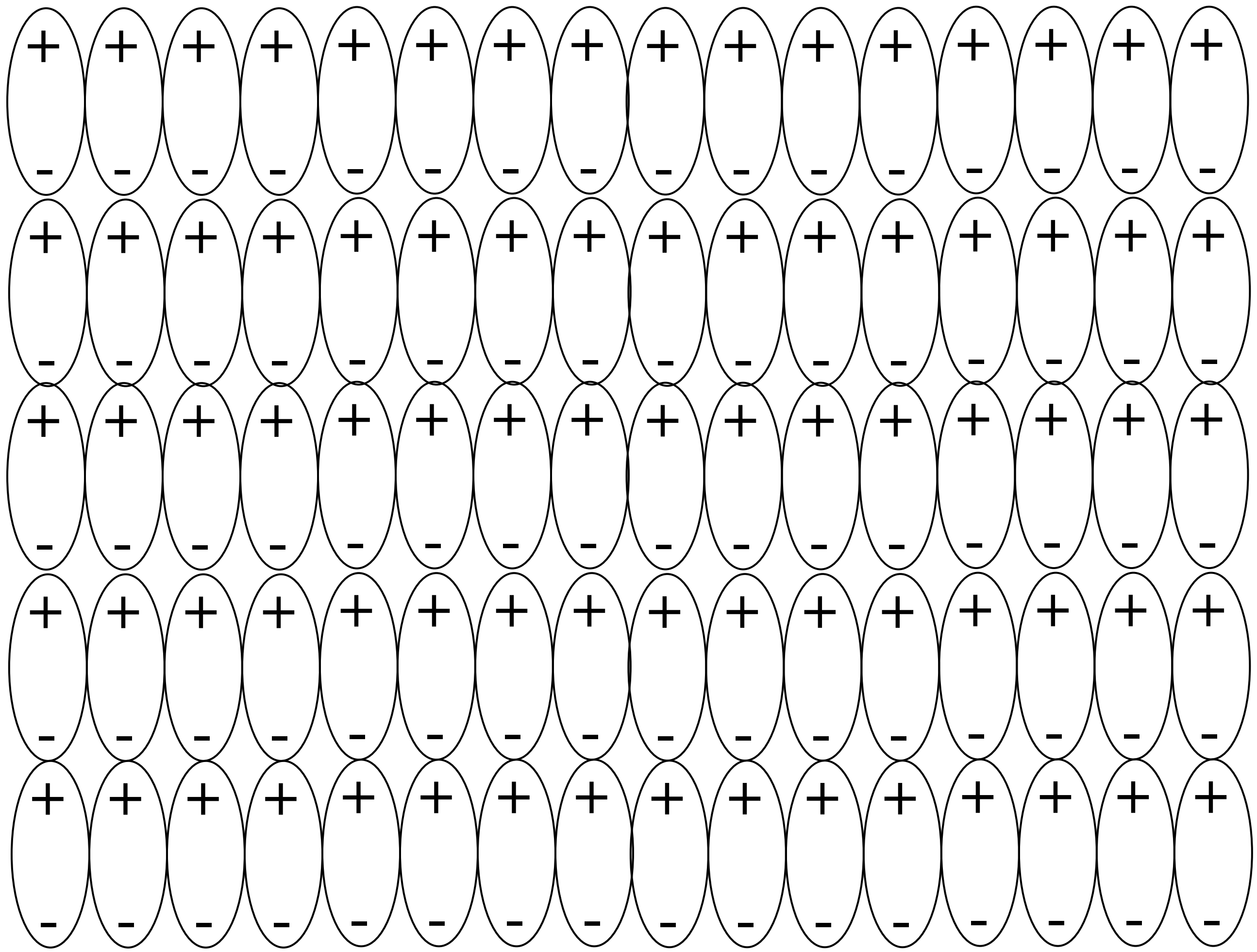
$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \oint_S \frac{\vec{P}\cdot\hat{n}}{R} dS' - \frac{1}{4\pi\varepsilon_0} \int_V \frac{\nabla'\cdot\vec{P}}{R} d^3r'$$

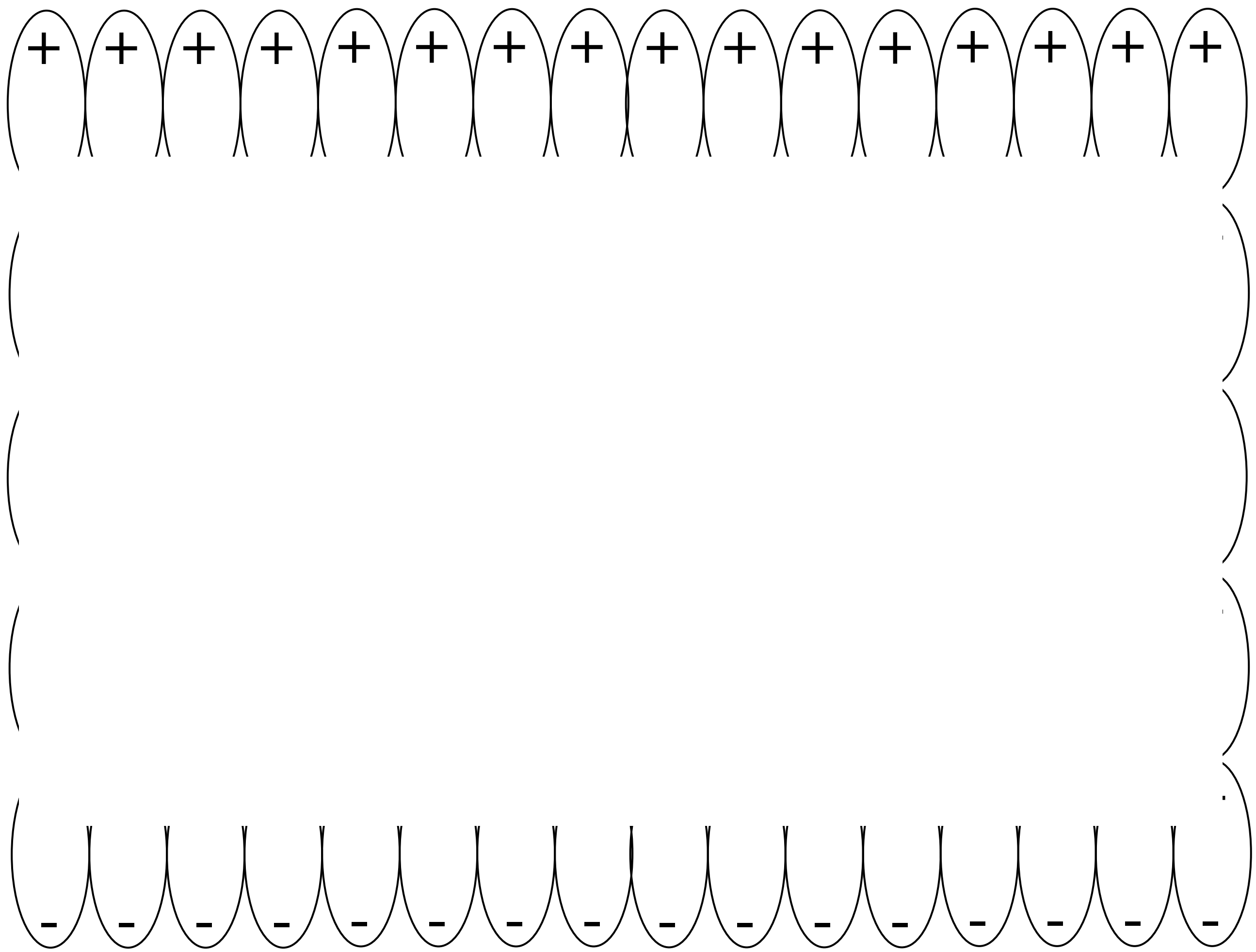
Comparar:

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \oint_S \frac{\sigma}{R} dS' + \frac{1}{4\pi\varepsilon_0} \int_V \frac{\rho}{R} d^3r'$$

Conclusión:

$$\sigma_P = \vec{P}\cdot\hat{n} \qquad \rho_P = -\nabla\cdot\vec{P}$$





Dieléctricos lineales: $\vec{P} = \chi \vec{E}$ $\chi = \text{Susceptibilidad eléctrica}$

$$\chi$$

En general, en los cristales, χ es un tensor:

$$P_i = \sum_{j=1}^{3} \chi_{ij} E_j$$

Teoría de Gauss en medios dieléctricos:

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0} \qquad \therefore \qquad \nabla \cdot \left(\varepsilon_0 \vec{E}\right) = \rho$$

Ahora: $\rho = \rho_l + \rho_P$

$$\nabla \cdot \left(\varepsilon_0 \vec{E}\right) = \rho_l + \rho_P = \rho_l - \nabla \cdot \vec{P}$$

$$\nabla \cdot \left(\varepsilon_0 \vec{E} + \vec{P}\right) = \rho_l$$

Definición: $\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$ = Desplazamiento eléctrico

$\nabla \cdot \vec{D} = \rho_l$ Ley de Gauss

Para deducir la forma integral de la ley de Gauss:

$$\int_V \nabla \cdot \vec{D}\, d^3r = \int_V \rho_l\, d^3r$$

$$\text{Pero:} \quad \int_V \nabla \cdot \vec{D}\, d^3r = \oint_S \vec{D} \cdot d\vec{S} \qquad \text{(teor. de Gauss)}$$

$$\oint_S \vec{D} \cdot d\vec{S} = q_l \qquad \text{Ley de Gauss}$$

$$[D] = [\text{Coulomb/m}^2]$$

Caso particular: Dieléctrico lineal, $\vec{P} = \chi \vec{E}$

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P} = (\varepsilon_0 + \chi)\vec{E}$$

$$\vec{D} = \varepsilon \vec{E} \qquad \varepsilon = \varepsilon_0 + \chi$$

$$\varepsilon = K\varepsilon_0$$

ε = permitividad del material

$$K = 1 + \frac{\chi}{\varepsilon_0} \qquad \therefore \quad \chi = (K - 1)\varepsilon_0$$

K = constante dieléctrica

Material	K	Rigidez dieléctrica
Aire	1.00059	3×10^6
Vidrio	5 a 10	9×10^6
Polietieleno	2.3	19×10^6
Al_2O_3	4.5	6×10^6
Agua	80	
SiO_2	4	
SiN	6	