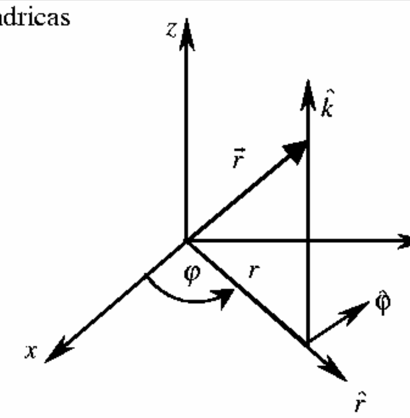
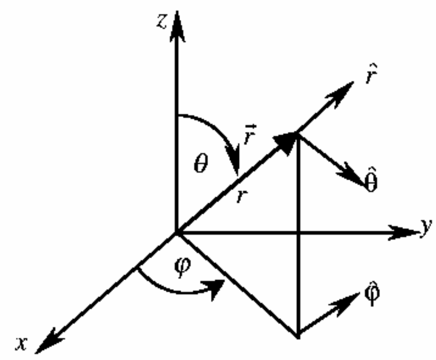
Formulario Matemático de Electromagnetismo

C.Cilíndricas



C. Esféricas



$\vec{r} = r\hat{r} + z\hat{k}$

$x = r\cos\varphi$

$y = r\,\mathrm{sen}\,\varphi$

$z = z$

$\vec{r} = r\hat{r}$

$x = r\,\mathrm{sen}\,\theta\cos\varphi$

$y = r\,\mathrm{sen}\,\theta\,\mathrm{sen}\,\varphi$

$z = r\cos\theta$

1. Gradientes

Cartesianas	Cilíndricas	Esféricas
$\nabla\phi = \frac{\partial\phi}{\partial x}\hat{i} + \frac{\partial\phi}{\partial y}\hat{j} + \frac{\partial\phi}{\partial z}\hat{k}$	$\nabla\phi = \frac{\partial\phi}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial\phi}{\partial\varphi}\hat{\varphi} + \frac{\partial\phi}{\partial z}\hat{k}$	$\nabla\phi = \frac{\partial\phi}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial\phi}{\partial\theta}\hat{\theta} + \frac{1}{r\,\mathrm{sen}\,\theta}\frac{\partial\phi}{\partial\varphi}\hat{\varphi}$

2. Divergencias

Cartesianas

$$\nabla\cdot\vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

Cilíndricas

$$\nabla\cdot\vec{A} = \frac{1}{r}\frac{\partial(rA_r)}{\partial r} + \frac{1}{r}\frac{\partial A_\varphi}{\partial\varphi} + \frac{\partial A_z}{\partial z}$$

Esféricas

$$\nabla\cdot\vec{A} = \frac{1}{r^2}\frac{\partial(r^2 A_r)}{\partial r} + \frac{1}{r\,\mathrm{sen}\,\theta}\frac{\partial(\mathrm{sen}\,\theta A_\theta)}{\partial\theta} + \frac{1}{r\,\mathrm{sen}\,\theta}\frac{\partial A_\varphi}{\partial\varphi}$$

3. Rotores

Cartesianas

$$\nabla\times\vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\hat{i} + \left(\frac{\partial A_z}{\partial x} - \frac{\partial A_x}{\partial z}\right)\hat{j} + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\hat{k}$$

Cilíndricas

$$\nabla\times\vec{A} = \frac{1}{r}\begin{vmatrix} \hat{r} & r\hat{\varphi} & \hat{k} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\varphi} & \frac{\partial}{\partial z} \\ A_r & rA_\varphi & A_z \end{vmatrix} = \frac{1}{r}\left\{\left(\frac{\partial A_z}{\partial\varphi} - \frac{\partial(rA_\varphi)}{\partial z}\right)\hat{r} + r\left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right)\hat{\varphi} + \left(\frac{\partial(rA_\varphi)}{\partial r} - \frac{\partial A_r}{\partial\varphi}\right)\hat{k}\right\}$$

Esféricas

$$\nabla\times\vec{A} = \frac{1}{r^2\,\mathrm{sen}\,\theta}\begin{vmatrix} \hat{r} & r\hat{\theta} & r\,\mathrm{sen}\,\theta\hat{\varphi} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\varphi} \\ A_r & rA_\theta & r\,\mathrm{sen}\,\theta A_\varphi \end{vmatrix}$$

$$= \frac{1}{r^2\,\mathrm{sen}\,\theta}\left\{\left(\frac{\partial(r\,\mathrm{sen}\,\theta A_\varphi)}{\partial\theta} - \frac{\partial(rA_\theta)}{\partial\varphi}\right)\hat{r} + \left(\frac{\partial A_r}{\partial\varphi} - \frac{\partial(r\,\mathrm{sen}\,\theta A_\varphi)}{\partial r}\right)\hat{\theta} + \left(\frac{\partial(rA_\theta)}{\partial r} - \frac{\partial A_r}{\partial\theta}\right)r\,\mathrm{sen}\,\theta\hat{\varphi}\right\}$$

4. Laplacianos

Cartesianas	Cilindricas
$\nabla^2\phi = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2}$	$\nabla^2\phi = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\phi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2\phi}{\partial\varphi^2} + \frac{\partial^2\phi}{\partial z^2}$
Esféricas $\nabla^2\phi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\phi}{\partial r}\right) + \frac{1}{r^2 \operatorname{sen}\theta}\frac{\partial}{\partial\theta}\left(\operatorname{sen}\theta\frac{\partial\phi}{\partial\theta}\right) + \frac{1}{r^2\operatorname{sen}^2\theta}\frac{\partial^2\phi}{\partial\varphi^2}$	

5. Elementos diferenciales

De línea		
Cartesianas $d\vec{l} = dx\hat{\imath} + dy\hat{\jmath} + dz\hat{k}$	Cilíndricas $d\vec{l} = dr\hat{r} + rd\varphi\hat{\varphi} + dz\hat{k}$	Esféricas $d\vec{l} = dr\hat{r} + rd\theta\hat{\theta} + r\operatorname{sen}\theta d\varphi\hat{\varphi}$
De superficie		
Cartesianas $d\vec{s} = dydz\hat{\imath} + dxdz\hat{\jmath} + dxdy\hat{k}$	Cilíndricas $d\vec{s} = rd\varphi dz\hat{r} + drdz\hat{\varphi} + rdrd\varphi\hat{k}$	Esféricas $d\vec{s} = r^2\operatorname{sen}\theta d\theta d\varphi\hat{r} + r\operatorname{sen}\theta drd\varphi\hat{\theta} + rd\theta dr\,\hat{\varphi}$
De volumen		
Cartesianas $dv = dxdydz$	Cilíndricas $dv = rdrd\varphi dz$	Esféricas $dv = r^2\operatorname{sen}\theta drd\varphi d\theta$

donde: en cartesianas $\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$

en cilíndricas $\vec{A} = A_r\hat{r} + A_\varphi\hat{\varphi} + A_z\hat{k}$

en esféricas $\vec{A} = A_r\hat{r} + A_\varphi\hat{\varphi} + A_\theta\hat{\theta}$

6. Identidades Vectoriales

$$\nabla\times(\nabla\phi) = 0$$
$$\nabla\cdot(\nabla\times\vec{A}) = 0$$
$$\nabla\times(\nabla\times\vec{A}) = \nabla(\nabla\cdot\vec{A}) - \nabla^2\vec{A}$$
$$\nabla(\phi\psi) = \phi\nabla\psi + \psi\nabla\phi$$
$$\nabla\cdot(\phi\vec{A}) = \phi\nabla\cdot\vec{A} + \vec{A}\cdot\nabla\phi$$
$$\nabla\cdot(\phi\nabla\psi) = \phi\nabla^2\psi + \nabla\psi\cdot\nabla\phi$$
$$\nabla\times(f(r)r) = 0$$
$$\nabla\left(\frac{1}{r}\right) = -\frac{\vec{r}}{r^3} \qquad \left(\text{con}\,|\vec{r}| = r\right)$$
$$\nabla\cdot(\vec{A}\times\vec{B}) = \vec{B}\cdot(\nabla\times\vec{A}) - \vec{A}\cdot(\nabla\times\vec{B})$$
$$\nabla\cdot\vec{r} = 3 \qquad \nabla\times\vec{r} = 0$$
$$\nabla(\vec{A}\cdot\vec{r}) = \vec{A}$$
$$\nabla\times(\phi\vec{A}) = \nabla\phi\times\vec{A} + \phi(\nabla\times\vec{A})$$
$$\nabla(r^n) = nr^{n-2}\vec{r}$$
$$\nabla^2\left(\frac{1}{r}\right) = \delta(\vec{r})$$
$$\nabla^2\left(\frac{1}{r}\right) = 0 \qquad (\text{para} \quad r\neq 0)$$
$$\nabla\times(\vec{A}\times\vec{B}) = (\vec{B}\cdot\nabla)\vec{A} - (\vec{A}\cdot\nabla)\vec{B} + \vec{A}\nabla\cdot\vec{B} - B\nabla\cdot\vec{A}$$
$$\nabla\times(\phi\vec{A}) = \phi\nabla\times\vec{A} - \vec{A}\times\nabla\phi$$