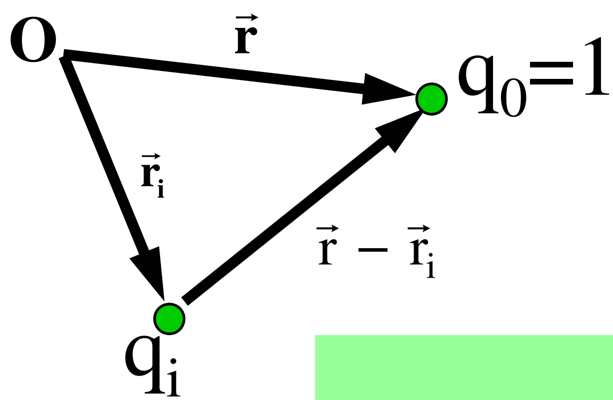
Campo eléctrico

Supong. q_0 y q_i, $i = 1 \ldots n$

$$\vec{\mathbf{F}} = \frac{\mathbf{q}_0}{4\pi\varepsilon_0} \sum_{\mathbf{i}=1}^{\mathbf{n}} \frac{\mathbf{q_i}\hat{\mathbf{R}}_\mathbf{i}}{\mathbf{R}_\mathbf{i}^2}$$

$$\vec{\mathbf{R}}_\mathbf{i} = \vec{\mathbf{r}} - \vec{\mathbf{r}}_\mathbf{i}$$



$$\hat{\mathbf{R}}_\mathbf{i} = \frac{\vec{\mathbf{r}} - \vec{\mathbf{r}}_\mathbf{i}}{\left|\vec{\mathbf{r}} - \vec{\mathbf{r}}_\mathbf{i}\right|}$$

$$\mathbf{R}_\mathbf{i} = \left|\vec{\mathbf{r}} - \vec{\mathbf{r}}_\mathbf{i}\right|$$

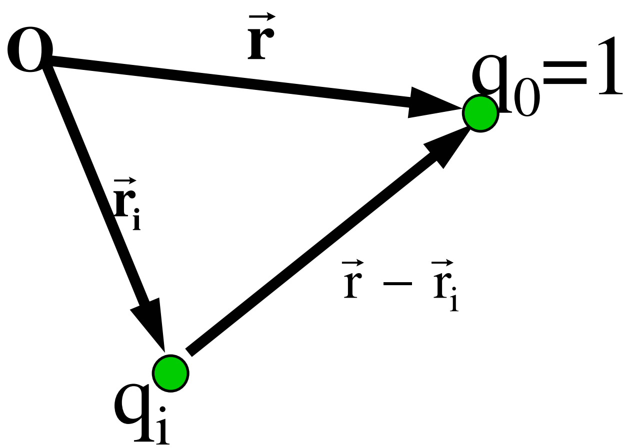
Definición de campo eléctrico: $\vec{E} = \dfrac{\vec{F}}{q_0}$

Mejor aún: $\vec{E} = \lim \dfrac{\vec{F}}{q_0}$

Campo eléctrico.

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i\hat{R}_i}{R_i^2}$$

$$\vec{\mathbf{R}}_i = \vec{\mathbf{r}} - \vec{\mathbf{r}}_i$$



$$\hat{\mathbf{R}}_i = \frac{\vec{\mathbf{r}} - \vec{\mathbf{r}}_i}{\left|\vec{\mathbf{r}} - \vec{\mathbf{r}}_i\right|}$$

$$\mathbf{R}_i = \left|\vec{\mathbf{r}} - \vec{\mathbf{r}}_i\right|$$

Distribución continua de carga en el volumen

Definición de densidad de carga $\rho(\vec{r})$: $\qquad \rho(\vec{r}) = \lim \dfrac{\sum q_i}{\Delta r'}$

Carga dq crea campo eléctrico dE

$$d\vec{E} = \frac{dq}{4\pi\varepsilon_0} \frac{(\vec{r} - \vec{r}\,')}{\left|\vec{r} - \vec{r}\,'\right|^3} = \frac{\rho d^3 r'}{4\pi\varepsilon_0} \frac{(\vec{r} - \vec{r}\,')}{\left|\vec{r} - \vec{r}\,'\right|^3}$$

$dq = \rho\ d^3r'$

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \int_V \frac{\rho(\vec{r}\,')(\vec{r} - \vec{r}\,')d^3r'}{\left|\vec{r} - \vec{r}\,'\right|^3}$$

Distribución continua de carga en una superficie

Definición de densidad de carga superficial $\sigma(\vec{r})$: $\qquad \sigma(\vec{r}) = \lim \dfrac{\sum q_i}{\Delta s'}$

$$dq = \sigma \cdot ds' \qquad\qquad \vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \int_S \frac{\sigma(\vec{r}\,')(\vec{r} - \vec{r}\,')ds'}{\left|\vec{r} - \vec{r}\,'\right|^3}$$

Distribución continua de carga en una línea

Definición de densidad de carga lineal $\lambda(\vec{r})$: $\lambda(\vec{r}) = \lim \dfrac{\sum q_i}{\Delta l'}$

$$dq = \lambda \cdot dl' \qquad \vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \int_L \frac{\lambda(\vec{r}\,')(\vec{r} - \vec{r}\,')dl'}{\left|\vec{r} - \vec{r}\,'\right|^3}$$

Ejemplo: Encontrar el campo eléctrico *fuera* de una esfera de radio R que tiene una distribución de carga uniforme σ.

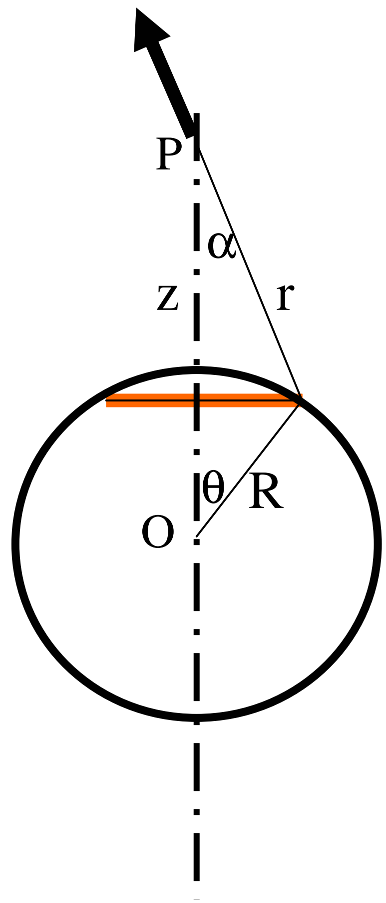
$$dq = \sigma dS$$

$$z = OP$$

$$dS = 2\pi R^2 \operatorname{sen}\theta \cdot d\theta$$

$$dq = 2\pi R^2 \sigma \operatorname{sen}\theta \cdot d\theta$$

$$\hat{z} \cdot d\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{2\pi R^2 \sigma \operatorname{sen}\theta \cdot d\theta \cdot \cos\alpha}{r^2}$$



Utilizando fórmula del coseno: $R^2 = z^2 + r^2 - 2\,z\,r\cos\alpha$

$$\cos\alpha = \frac{z^2 + r^2 - R^2}{2zr} \qquad \therefore \qquad E = \frac{\sigma 2\pi R^2}{4\pi\varepsilon_0}\int_0^{\pi}\frac{\operatorname{sen}\theta \cdot d\theta\left(z^2 + r^2 - R^2\right)}{r^2 \cdot 2zr}$$

Cambio de variable: $\theta \to r$

$r^2 = z^2 + R^2 - 2zR\cos\theta;$ $\quad r\,dr = z\,R\,\operatorname{sen}\theta\,d\theta$

$$\operatorname{sen}\theta \cdot d\theta = \frac{r \cdot dr}{z \cdot R}$$

$$\boxed{E = \frac{\sigma 2\pi R^2}{4\pi\varepsilon_0} \int_{z-R}^{z+R} \frac{\left(z^2 - R^2 + r^2\right) \cdot dr}{2z^2 R r^2}}$$

$$E = \frac{\sigma 2\pi R^2}{4\pi\varepsilon_0} \left[\int_{z-R}^{z+R} \frac{\left(z^2 - R^2\right) \cdot dr}{2z^2 R r^2} + \frac{2R}{2z^2 R} \right]$$

$$E = \frac{\sigma 2\pi R^2}{4\pi\varepsilon_0} \left\{ \frac{\left(z^2 - R^2\right)}{2z^2 R} \left[\frac{1}{z-R} - \frac{1}{z+R} \right] + \frac{1}{z^2} \right\}$$

$$E = \frac{\sigma 2\pi R^2}{4\pi\varepsilon_0} \left\{ \frac{2R}{2z^2 R} + \frac{1}{z^2} \right\} = \frac{\sigma 4\pi R^2}{4\pi\varepsilon_0} \cdot \frac{1}{z^2}$$

$$E = \frac{q}{4\pi\varepsilon_0 z^2} \qquad z \geq R$$

Ejemplo: Encontrar el campo eléctrico *dentro* de una esfera de radio R que tiene una distribución de carga uniforme σ.

$$E = \frac{\sigma 2\pi R^2}{4\pi\varepsilon_0} \int_{R-z}^{R+z} \frac{\left(z^2 - R^2 + r^2\right)\cdot dr}{2z^2 R r^2}$$

$$E = \frac{\sigma 2\pi R^2}{4\pi\varepsilon_0} \left[\int_{R-z}^{R+z} \frac{\left(z^2 - R^2\right)\cdot dr}{2z^2 R r^2} + \frac{2z}{2z^2 R} \right]$$

$$E = \frac{\sigma 2\pi R^2}{4\pi\varepsilon_0}\left\{\frac{\left(z^2 - R^2\right)}{2z^2 R}\left[\frac{1}{R-z} - \frac{1}{R+z}\right] + \frac{1}{zR}\right\}$$

$$E = \frac{\sigma 2\pi R^2}{4\pi\varepsilon_0}\left\{\frac{-2z}{2z^2 R} + \frac{1}{zR}\right\} = \frac{\sigma 2\pi R^2}{4\pi\varepsilon_0}\cdot\left\{\frac{-1}{zR} + \frac{1}{zR}\right\}$$

$$E = 0 \qquad z < R$$

Aplicaciones de las fuerzas eléctricas

Tubo de rayos catódicos: Osciloscopio, T.V. . . .

Aceleradores de partículas

Fotocopiadoras

Dispositivos purificadores del aire

Data show: DMD = Digital Micromirror Device

MEM = Micro Electro-Mechanical

Microscopio electrónico