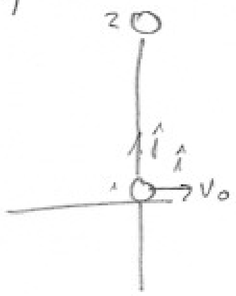
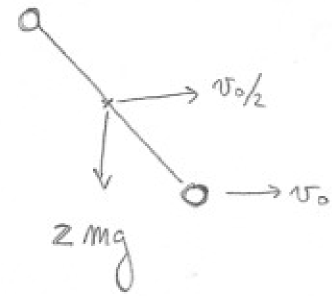
P1) a)



$$\vec{v}_{0\,CM} = \frac{m v_0 \hat{\imath} + m \cdot 0}{2m} = \frac{v_0}{2}\hat{\imath}$$

DCL



$\hat{\imath})\quad 0 = 2m\ddot{x}_{CM} = 0$

$\hat{\jmath})\quad -2mg = 2m\,\ddot{y}_{CM}$

$$\ddot{y}_{CM} = -g$$

$$\boxed{\dot{y}_{CM} = -gt}$$

Se tiene además que $\vec{\tau}_{CM} = 0 \Rightarrow I_{CM}\ddot{\theta} = 0$

$$\Rightarrow \dot{\theta} = cte$$
$$\vec{l}_{CM} = \vec{cte}$$

Además:

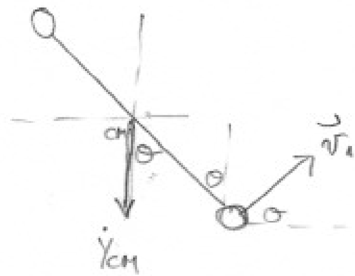
$$\vec{l}_{0_{CM}} = m\frac{L}{2} v_0 \hat{k}$$

$$\vec{l}(0)_{CM} = \left(m\frac{L}{2}v_1 + m\frac{L}{2}v_2\right)\hat{k}$$

$$v_1 = v_2 = \frac{L}{2}\dot{\theta}$$

$$\Rightarrow v_0 = L\dot{\theta} \Rightarrow \dot{\theta} = \frac{v_0}{L} \Rightarrow \theta(t) = \frac{v_0}{L}t$$

b) La velocidad vertical será



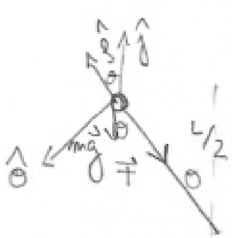
$$\dot{Y}_1(t) = -gt + \frac{L}{2}\dot{\theta}\sin\theta$$

$$\boxed{\dot{Y}_1(t) = -gt + \frac{v_0}{2}\sin\left(\frac{v_0}{L}t\right)}$$

$$\dot{Y}_1 > 0 \Rightarrow gt < \frac{v_0}{2}\sin\left(\frac{v_0}{L}t\right)$$

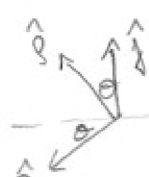
$$\Rightarrow \boxed{\sin\left(\frac{v_0}{L}t\right) \geq \frac{2gt}{v_0}}$$

c)



$$\vec{a}_0 = -g\hat{j} \qquad \vec{a}' = -\frac{L}{2}\dot{\theta}^2\hat{\rho}$$

$$\vec{F} = -T\hat{\rho} - mg\cos\theta\,\hat{\rho} + mg\sin\theta\,\hat{\theta}$$



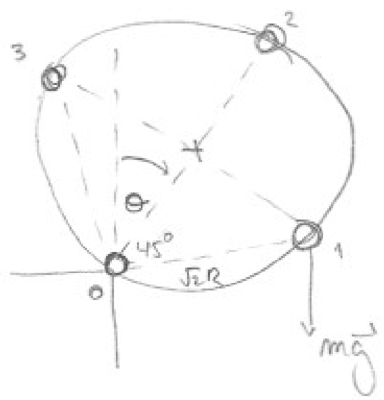
$$\hat{j} = \hat{\rho}\cos\theta - \hat{\theta}\sin\theta$$

$$\vec{a}_0 = -g\cos\theta\,\hat{\rho} + g\sin\theta\,\hat{\theta}$$

$$-T - \cancel{mg\cos\theta} = -\cancel{mg\cos\theta} - m\frac{L}{2}\dot{\theta}^2$$

$$T = m\frac{L}{2}\dot{\theta}^2 \Rightarrow \boxed{T = m\frac{v_0^2}{2L}}$$

P2

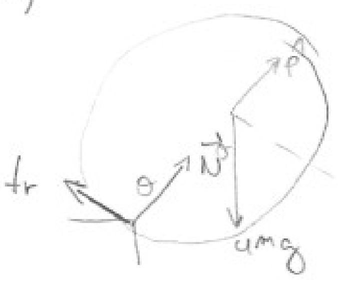


$$\vec{\tau}_1 = \sqrt{2}R \sin(\theta + 45^\circ)\, mg\, \hat{k}$$
$$\vec{\tau}_2 = 2R \sin\theta\, mg\, \hat{k}$$
$$\vec{\tau}_3 = -\sqrt{2}R \sin(45 - \theta)\, mg\, \hat{k}$$
$$\vec{l}_0 = \left(m r_1^2 \dot{\theta} + m r_2^2 \dot{\theta} + m r_3^2 \dot{\theta}\right)\hat{k}$$
$$= m\left(2R^2 + 4R^2 + 2R^2\right)\dot{\theta}\,\hat{k}$$
$$\vec{l}_0 = 8mR^2\dot{\theta}\,\hat{k}$$
$$\vec{\tau}_0 = 8mR^2\ddot{\theta}\,\hat{k}$$

$$\sin(\theta + 45) = \sin\theta\cos 45 + \cos\theta\sin 45 = \frac{(\sin\theta + \cos\theta)}{\sqrt{2}}$$
$$\sin(45 - \theta) = -(\sin\theta\cos 45 - \cos\theta\sin 45) = \frac{(-\sin\theta + \cos\theta)}{\sqrt{2}}$$

$$\Rightarrow \sum \vec{\tau} = \left(2Rmg\sin\theta + 2Rmg\sin\theta\right)\hat{k}$$
$$4Rmg\sin\theta = 8mR^2\ddot{\theta}$$
$$\Rightarrow \ddot{\theta} = \frac{g}{4R}\sin\theta$$
$$\Rightarrow \dot{\theta}^2 = \frac{g}{2R}(1 - \cos\theta)$$

b)



$$\sum \vec{F} = N\hat{\rho} - 4mg\cos\theta\,\hat{\rho} + 4mg\sin\theta\,\hat{\theta} - fr\,\hat{\theta}$$
$$N - 4mg\cos\theta = -4mR\dot{\theta}^2$$
$$N = 4mg\cos\theta - 2mg(1 - \cos\theta)$$
$$N = 2mg(1 + \cos\theta)$$

$\sin 30^\circ = \frac{1}{2}$

$\cos 30 = \frac{\sqrt{3}}{2}$

c) $4mg\sin\theta - fr = 4mR\ddot{\theta} \Rightarrow fr = \frac{4mg}{2} - \frac{mg}{2}$

$fr(\theta = 30^\circ) = \mu N = 2\mu mg(1 - \cos 30^\circ) \Rightarrow 2\mu mg\left(1 - \frac{\sqrt{3}}{2}\right) = \frac{3}{2}mg \Rightarrow \mu = \frac{3}{2(2 - \sqrt{3})}$