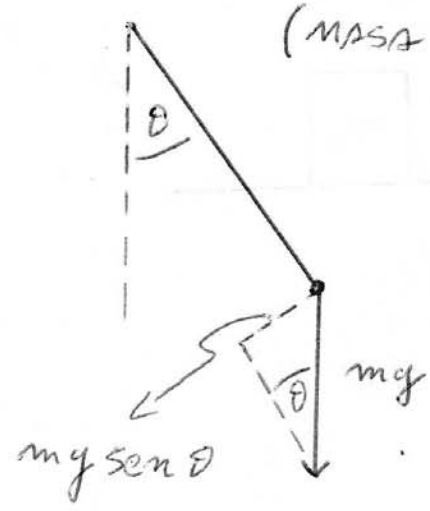
P.1

i)



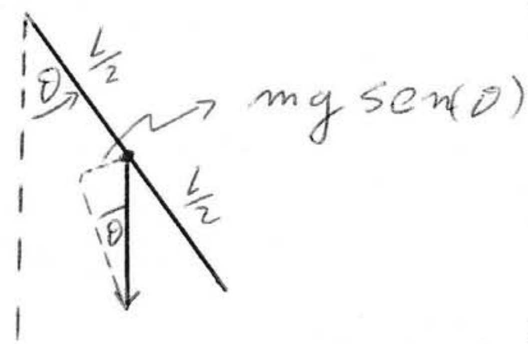

(MASA PUNTUAL)

$$-mg\,\mathrm{sen}(\theta) = m\ddot{\theta}L$$

$$\rightsquigarrow \ddot{\theta} + \frac{g}{L}\theta = 0$$

$$\omega_1 = \sqrt{g/L}$$

ii)




(barra)

$$\tau = -\frac{L}{2} mg\,\mathrm{sen}(\theta) \approx -\frac{mgL}{2}$$

$$\tau = I\ddot{\theta}, \quad I = \frac{1}{12} mL^2$$

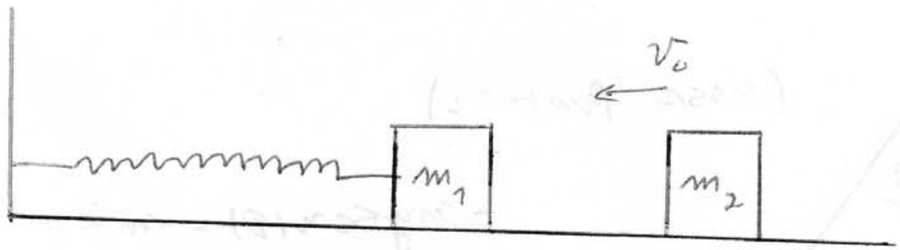
$$-\frac{mgL}{2}\theta = \frac{1}{12} mL^2\ddot{\theta} \to \ddot{\theta} + \frac{6g}{L}\theta$$

$$\omega_2 = \sqrt{\frac{6g}{L}}$$

$$\omega_2 > \omega_1$$

→ AL PARARSE AUMENTA LA Velocidad

P. 2




a)

Conservacion del momentum:

$$-m_2 v_0 = (m_1 + m_2) v_f$$

$$v_f = \frac{-m_2}{m_1 + m_2} v_0$$

Ecuacion de movimiento:

$$F = -kx \longrightarrow -kx = (m_1 + m_2)\ddot{x}$$

$$\ddot{x} + \frac{k}{m_1 + m_2} x = 0 \longrightarrow x(t) = A\cos\left(\sqrt{\frac{k}{m_1+m_2}}\, t + \phi\right)$$

$$\dot{x}(t) = -A\sqrt{\frac{k}{m_1+m_2}}\, \operatorname{sen}\left(\sqrt{\frac{k}{m_1+m_2}}\, t + \phi\right)$$

C.I

$$x(0) = A\cos(\phi) = 0 \longrightarrow \phi = \frac{\pi}{2}$$

$$\dot{x}(0) = -A\sqrt{\frac{k}{m_1+m_2}} = \frac{-m_2}{m_1+m_2} v_0 \longrightarrow A = \frac{m_2\sqrt{m_1+m_2}}{\sqrt{k}\,(m_1+m_2)} v_0$$

$$\omega = \sqrt{\frac{k}{m_1+m_2}} \quad ; \quad T = \frac{2\pi}{\omega} = \frac{2\pi\sqrt{m_1+m_2}}{\sqrt{k}}$$

) Conservacion del momentum:

$$-m_2 v_0 = m_1 v_f + m_2 v_2$$

Conservacion de energia:

$$\frac{1}{2} m_2 v_0^2 = \frac{1}{2} m_1 v_f^2 + \frac{1}{2} m_2 v_2^2$$

Resolviendo las ecuaciones se tiene:

$$v_2 = \frac{(m_1 - m_2)}{m_1 + m_2} v_0 \quad ; \quad v_f = \frac{-2m_2}{m_1 + m_2} v_0$$

nalogo al caso anterior y como solo oscila $m_1$ la olucion de la ecuacion de movimiento es:

$$X(t) = A\cos\left(\sqrt{\frac{K}{m_1}}\, t + \phi\right)$$

$$\dot{X}(t) = -A\sqrt{\frac{K}{m_1}}\, \mathrm{sen}\left(\sqrt{\frac{K}{m_1}}\, t + \phi\right)$$

$$X(0) = A\cos(\phi) = 0 \longrightarrow \phi = \frac{\pi}{2}$$

$$\dot{X}(0) = -A\sqrt{\frac{K}{m_1}} = \frac{-2m_2}{m_1 + m_2} v_0 \longrightarrow A = \frac{2 m_2 \sqrt{m_1}}{(m_1 + m_2)\sqrt{K}} v_0$$

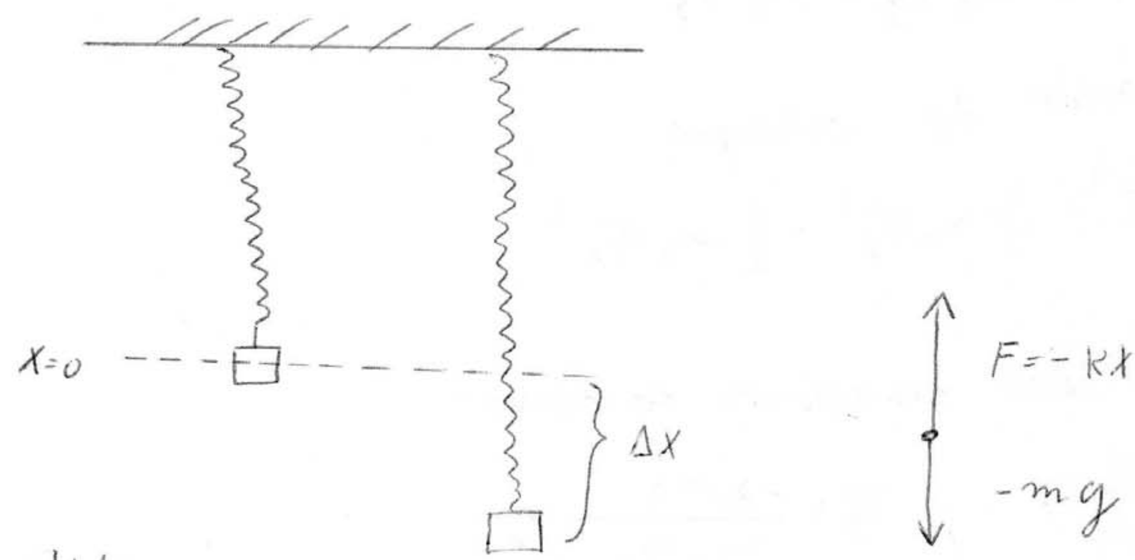
$$\omega = \sqrt{\frac{K}{m_1}} \longrightarrow T = \frac{2\pi}{\omega} = \frac{2\pi\sqrt{m_1}}{\sqrt{K}}$$

) $$X_a(t) = \frac{m_2\sqrt{m_1 + m_2}}{\sqrt{K}(m_1 + m_2)} v_0 \cos\left(\sqrt{\frac{K}{m_1 + m_2}}\, t + \frac{\pi}{2}\right)$$

$$X_b(t) = \frac{2 m_2 \sqrt{m_1}}{(m_1 + m_2)\sqrt{K}} v_0 \cos\left(\sqrt{\frac{K}{m_1}}\, t + \frac{\pi}{2}\right)$$

P3

a)




$$\sum F = -k\Delta x - mg = 0 \longrightarrow \Delta x = -\frac{mg}{k}$$

b)

$$\sum F = m\ddot{x} \longrightarrow -kx - mg = m\ddot{x} \longrightarrow \ddot{x} + \frac{k}{m}x + g = 0$$

Cambio de variable: $\frac{k}{m}y = \frac{k}{m}x + g \quad / \frac{d^2}{dt^2}$

$$\frac{k}{m}\ddot{y} = \frac{k}{m}\ddot{x} \longrightarrow \ddot{y} = \ddot{x}$$

$$y = x + \frac{mg}{k}$$

Se reemplaza en la EDO y queda.

$$\ddot{y} + \frac{k}{m}y = 0 \longrightarrow y(t) = A\cos\left(\sqrt{\frac{k}{m}}t + \phi\right)$$

; Volvemos con el cambio de variable $x = y - \frac{mg}{k}$

$$x = A\cos\left(\sqrt{\frac{k}{m}}t + \phi\right) - \frac{mg}{k}$$

$$\omega = \sqrt{\frac{k}{m}} \longrightarrow T = \frac{2\pi}{\omega} = \frac{2\pi\sqrt{m}}{\sqrt{k}}$$

$$\dot{X}(t) = -A\sqrt{\frac{K}{m}}\,\text{sen}\left(\sqrt{\frac{K}{m}}\,t+\phi\right)$$

$$X(0) = A\cos(\phi) - \frac{mg}{K} = 0$$

$$\dot{X}(0) = -A\sqrt{\frac{K}{m}}\,\text{sen}(\phi) = 0 \longrightarrow \phi = 0$$

$$X(0) = A - \frac{mg}{K} = 0 \longrightarrow A = \frac{mg}{K}$$

$$X(t) = \frac{mg}{K}\cos\left(\sqrt{\frac{K}{m}}\,t\right) - \frac{mg}{K}$$

$$\dot{X}(t) = -\frac{mg}{K}\sqrt{\frac{K}{m}}\,\text{sen}\left(\sqrt{\frac{K}{m}}\,t\right)$$

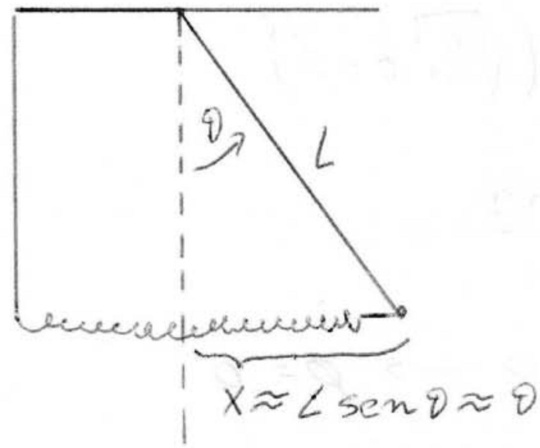
$$K(t) = \frac{1}{2} m \dot{X}(t)^2$$

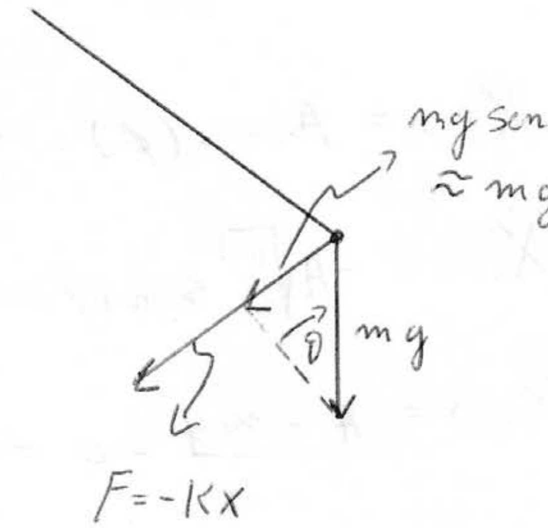
$$V(t) = \frac{1}{2} K X(t)^2$$

d) La velocidad maxima corresponde a la amplitud de la funcion de la velocidad $\dot{X}(t)$ es decir

$$V_{max} = \frac{mg}{K}\sqrt{\frac{K}{m}}$$

# P.4




$$\sum F = -KL\theta - mg\,\theta = mL\ddot{\theta}$$

$$\ddot{\theta} + \frac{(KL - mg)}{mL}\,\theta = 0$$

$$\omega = \sqrt{\frac{KL - mg}{mL}}$$

# P.5

Se consideran 2 cilindros uno de masa positiv
y uno de masa negativa las masas serian

$m_0 = -\rho\pi\left(\frac{R}{4}\right)^2 \cdot D$ D: largo cilindro

$m_1 = \rho\pi R^2 D$ ρ: densidad

El centro de masa desde el centro del cilindro grande
es:

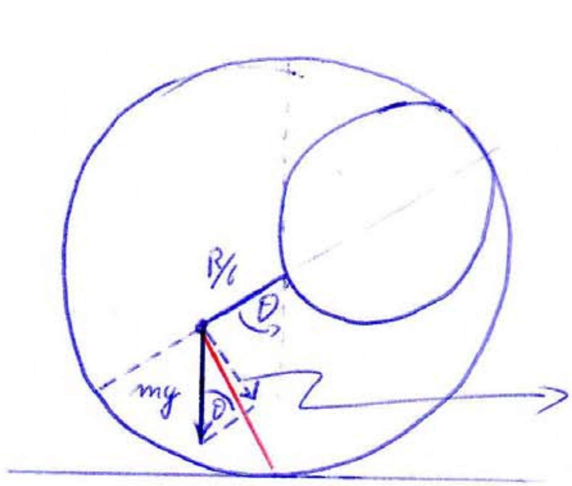
$$CM = \frac{(\rho\pi R^2 D)\,0 + (-\rho\pi \frac{R^2}{4} D)\,\frac{R}{2}}{\rho\pi R^2 D + (-\rho\pi\frac{R^2}{4} D)} = -\frac{R}{6}$$

Ahora se obtiene el momento de inercia desde el punto de contacto con el suelo y de esa forma al sumar torques no se considera el roce

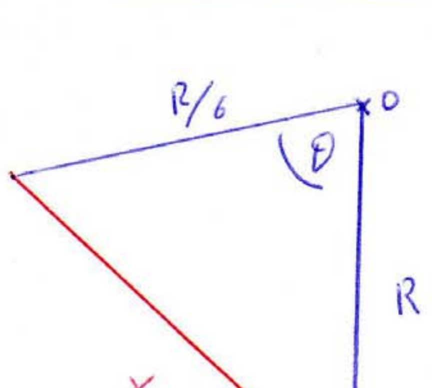
$$I_0 = \frac{1}{2}\left(-\rho\pi\frac{R^2}{4}D\right)\left(\frac{R}{4}\right)^2 + \left(-\rho\pi\frac{R^2}{4}D\right)\left(\frac{3R}{2}\right)^2 = \frac{-73(\rho\pi R^2 D)R^2}{128}$$

$$I_1 = \frac{1}{2}(\rho\pi R^2 D)R^2 + (\rho\pi R^2 D)R^2 = (\rho\pi R^2 D)R^2$$

$$I_{total} = I_0 + I_1 = (\rho\pi R^2 D)R^2 - \frac{73}{128}(\rho\pi R^2 D)R^2 = \frac{55}{128}(\rho\pi R^2 D)R^2$$




Para obtener el tor es necesario conocer la distancia del cm donde se aplica la f al punto de referencia, el de contacto




Para eso se tiene la relacion coseno:

$$x^2 = \left(\frac{R}{6}\right)^2 + R^2 + 2\frac{R}{6}R\cos\theta$$

$$= \left(\frac{\cos\theta}{3} + \frac{5}{4}\right)R^2$$

$$x = \sqrt{\frac{\cos\theta}{3} + \frac{5}{4}}\,R$$

tiene que el torque: $\tau = \sqrt{\frac{\cos\theta}{3} + \frac{5}{4}}\, R \cdot mg\, \mathrm{Sen}$

ma pequeños Angulos: $\cos\theta \approx 1$

$\mathrm{Sen}\,\theta \approx \theta$

$$\tau \approx \sqrt{\frac{1}{3} + \frac{5}{4}}\, R m g \theta = \sqrt{\frac{19}{12}}\, R m g \theta$$

$$\tau = I \ddot{\theta} \longrightarrow \sqrt{\frac{19}{12}}\, R m g \theta = \frac{55}{128} (\rho \pi R^2 D) R^2 \ddot{\theta}$$

ona se obtiene la masa m:

$$m = \rho \pi R^2 D - \rho \pi \frac{R^2}{4} D = \frac{3}{4} \rho \pi R^2 D$$

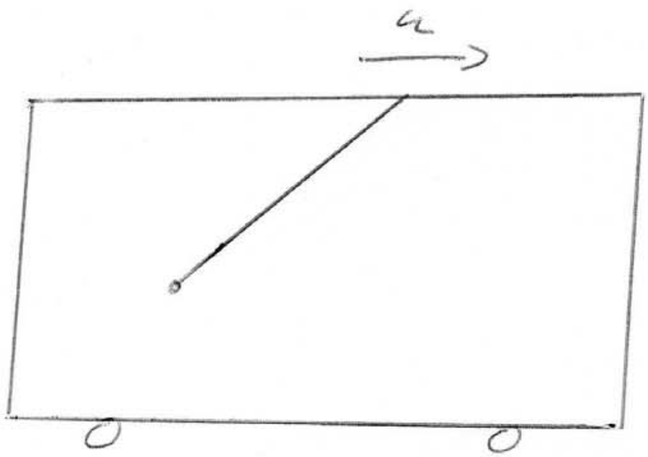
a ecuacion queda:

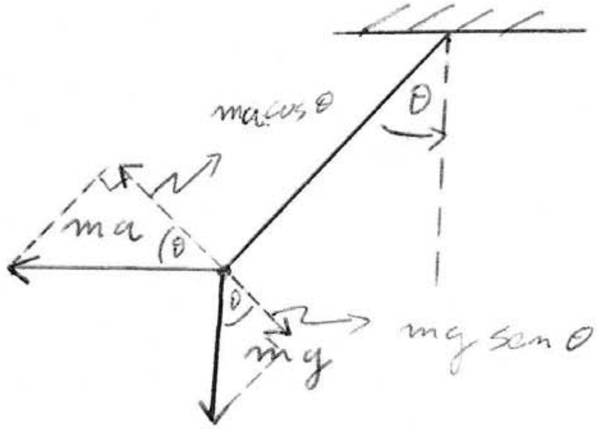
$$\sqrt{\frac{19}{12}}\, \frac{3}{4} \cancel{(\rho \pi R^2 D)}\, \cancel{R}\, g\, \theta = \frac{55}{128} \cancel{(\rho \pi R^2 D)}\, R^{\cancel{2}}\, \ddot{\theta}$$

$$\ddot{\theta} + \frac{3}{4} \frac{\sqrt{19}}{\sqrt{12}} \frac{128}{55} \frac{g}{R} \theta = 0$$

$$\omega^2 = \frac{16}{55} \sqrt{57}\, \frac{g}{R}$$

$\boxed{6}$




El movimiento acelerado puede considerarse como una fuerza constante y horizontal ma

$mg\,\text{sen}\,\theta \approx mg\theta$

$ma\cos\theta \approx ma$

$\sum F = -mg\theta + ma = ml\ddot{\theta}$

$\ddot{\theta} + \frac{g}{l}\theta - \frac{a}{l} = 0$

cambio de variable:

$\frac{g}{l}\phi = \frac{g}{l}\theta - \frac{a}{l}$

$\phi = \theta - \frac{a}{g}$

$\ddot{\phi} = \ddot{\theta}$

$\ddot{\phi} + \frac{g}{l}\phi = 0$

$\phi_{(t)} = A\cos\left(\sqrt{\frac{k}{l}}\,t + \phi_0\right)$

$\longrightarrow \quad \omega = \sqrt{\frac{k}{l}}$

$\theta_{(t)} = A\cos\left(\sqrt{\frac{k}{l}}\,t + \phi_0\right) + \frac{a}{g}$

La acelera
solo influye
el punto sob
que se oscil.
no en la f