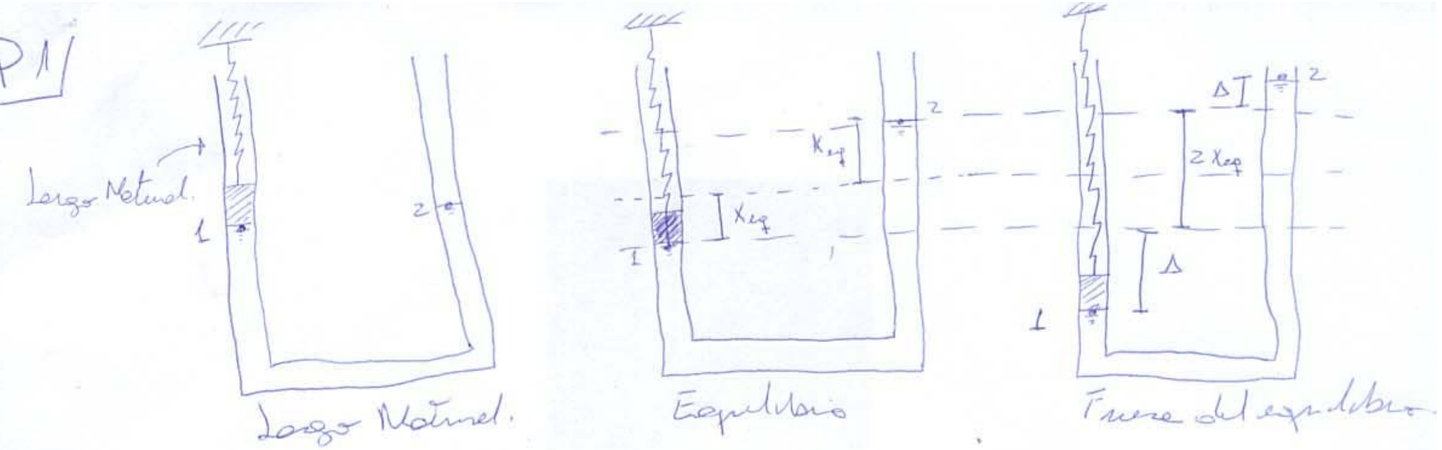
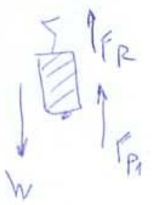
P1



Equilibrio



$$W - F_R - F_{P1} = 0 \qquad P_1 = \gamma\, 2 X_{eq}$$

$$W - P_1 A - k X_{eq} = 0$$

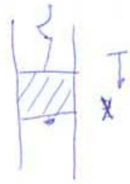
$$W - \gamma 2 X_{eq} A - k X_{eq} = 0$$

$$\boxed{X_{eq} = \frac{W}{2\gamma A + k}}$$

Fuera de la situacion de equilibrio

Ec. de Euler entre 1 y 2

$$\frac{L}{g}\frac{dv}{dt} + B_2 - B_1 + f_0 \frac{L}{D}\frac{v^2}{2g} = 0$$



$$m\ddot{x} = W - P_1 A - (\overbrace{\cancel{X_{eq}+\Delta}}^{X})k$$

$$P_1 = \frac{W - (\overset{X}{\cancel{X_{eq}+\Delta}})k - W/g}{A}$$

$$B_2 = \alpha \frac{v^2}{2g} + \Delta$$

$$B_1 = \frac{P_1}{\gamma} + \alpha\frac{v^2}{2g} - (2X_{eq} + \Delta)$$

$$v = \frac{d\Delta}{dt} = \dot{\Delta} \qquad \frac{dv}{dt} = \ddot{\Delta}$$

$$B_2 - B_1 = 2(X_{eq} + \Delta) - \frac{P_1}{\gamma}$$

$$x = X_{eq} + \Delta$$

$$\ddot{x} = \ddot{\Delta}$$

$$\frac{L}{g}\ddot{\Delta} + 2(X_{eq}+\Delta) - \frac{P_1}{\gamma} + f_0 \frac{L}{D}\frac{\dot{\Delta}^2}{2g} = 0$$

$$\frac{L}{g}\ddot{\Delta} + 2(X_{eq}+\Delta) - \frac{W}{\gamma A} + \frac{(\overset{X}{\cancel{X_{eq}+\Delta}})k}{\gamma A} + \frac{W}{gA\gamma}\ddot{X} + f_0 \frac{L}{D}\frac{\dot{\Delta}^2}{2g} = 0$$

$$\frac{L}{g}\ddot{\Delta} + 2(X_{eq}+\Delta) - \frac{W}{\gamma A} + \frac{(X_{eq}+\Delta)k}{\gamma A} + \frac{W}{gA\gamma}\ddot{\Delta} + f_0 \frac{L}{D}\frac{\dot{\Delta}^2}{2g}$$

$$\left(\frac{L}{g} + \frac{W}{gA\gamma}\right)\ddot{\Delta} + \left(2 + \frac{k}{\gamma A}\right)\Delta + \left(2 + \frac{k}{\gamma A}\right)X_{eq} - \frac{W}{\gamma A} + f_0 \frac{L}{D}\frac{\dot{\Delta}^2}{2g} = 0$$

Esta ecuacion no es homogenea, con el siguiente cambio de variable se deja la ecuacion homogenea:

$$Z = \Delta\left(2 + \frac{k}{\gamma A}\right) + X_{eq}\left(2 + \frac{k}{\gamma A}\right) - \frac{W}{\gamma A}$$

$$\dot{Z} = \dot{\Delta}\left(2 + \frac{k}{\gamma A}\right) \qquad \dot{\Delta}^2 = \frac{1}{(2 + k/\gamma A)^2}\dot{Z}^2$$

$$\ddot{Z} = \ddot{\Delta}\,(2 + k/\gamma A)$$

$$\left(\frac{L/g + W/gA\gamma}{2 + k/\gamma A}\right)\ddot{Z} + Z + f_0 \frac{L}{D}\frac{1}{(2 + k/\gamma A)^2}\frac{\dot{Z}^2}{2g} = 0$$

$$\ddot{Z} + \left(\frac{2 + k/\gamma A}{L/g + W/gA\gamma}\right)Z + f_0 \frac{L}{D}\frac{1}{(2 + k/\gamma A)(L/g + W/gA\gamma)}\frac{\dot{Z}^2}{2g} = 0$$

$$\left(\frac{2 + k/\gamma A}{L/g + W/gA\gamma}\right) = 2\frac{g}{L}\left(\frac{1 + k/2\gamma A}{1 + W/AL\gamma}\right) = 2\frac{g'}{L}$$

$$f_0 \frac{L}{(2 + k/\gamma A)(L/g + W/gA\gamma)2g} = f_0'$$

$$\ddot{Z} + 2\frac{g'}{L}Z + f_0' \frac{\cancel{L}}{D}\frac{\dot{Z}^2}{\cancel{2g}} = 0 \quad \text{(Resulta en clases)}$$

$$\left(1+f\frac{Z_m}{D}\right)e^{-f\frac{Z_m}{D}} = \left(1+f\frac{Z_{m+1}}{D}\right)e^{-f\frac{Z_{m+1}}{D}}$$

$$f_0' = \frac{1}{\left(2+3/1000\times 0{,}002\right)} \frac{0{,}035}{\left(2/9{,}8+0{,}5/9{,}8\times 1000\times 0{,}002\right)}$$

$$A = \frac{\pi\ 0{,}05^2}{4} = 0{,}002 \qquad f_0' = 0{,}044$$

i) La posicion mas baja es 0,301 m.

ii) A la segunda oscilacion el desplazamiento maximo de la columna sera inferior a 30% de

Z_m sera para $\Delta_0 = 1$

$$Z_m = \Delta_0\left(2+\frac{k}{\gamma A}\right) + X_{eq}\left(2+\frac{k}{\gamma A}\right) - \frac{W}{\gamma A}$$

$$= \Delta_0\left(2+\frac{k}{\gamma A}\right) + \frac{W}{\cancel{2\gamma A+k}}\frac{\cancel{2\gamma A+k}}{\gamma A} - \frac{W}{\gamma A} \Rightarrow Z_m = \Delta_0\left(2+\frac{k}{\gamma A}\right)$$

$$Z_m = 1\left(2+\frac{3}{1000\times 0{,}002}\right) = 3{,}5\ m. \qquad f_0' = \frac{Z_m}{\Delta} = \frac{0{,}044\times 3{,}5}{0{,}05} = 3{,}08$$

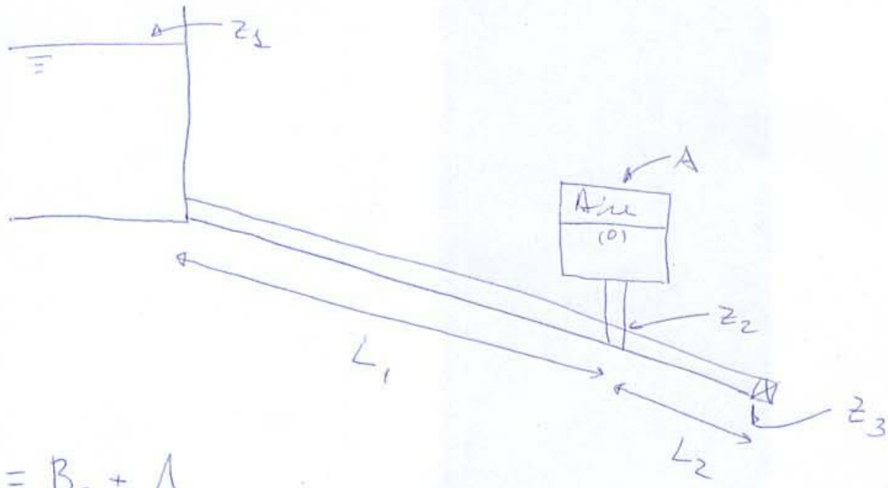
$$\left(1+f_0'\frac{Z_m}{D}\right)e^{-f_0'\frac{Z_m}{D}} = (1+3{,}08)e^{-3{,}08} = 0{,}188$$

$$\left(1+f_0'\frac{Z_{m+1}}{D}\right)e^{-f_0'\frac{Z_{m+1}}{D}} = 0{,}188 = F(Z_{m+1})$$

Z_{m+1}	$F(Z_{m+1})$
-1	0,289
-0,9	0,459
-1,1	0,084
-1,05	0,191
-1,055	0,181 → $\Delta = -0{,}301$

P2

Primero calculamos el nivel inicial al interior de la camara.



$$B_1 = B_3 + \Lambda_f$$

$$B_1 = z_1$$

$$B_3 = z_3 + \frac{V^2}{2g}$$

$$\Lambda_f = f\frac{(L_1+L_2)}{D}\frac{V^2}{2g}$$

$$\Rightarrow \quad z_1 = z_3 + \frac{V^2}{2g} + f\frac{(L_1+L_2)}{D}\frac{V^2}{2g}$$

$$V^2\left(\frac{1}{2g} + \frac{f(L_1+L_2)}{D\,2g}\right) = z_1 - z_3$$

$$\boxed{V = \sqrt{\frac{2g(z_1-z_3)}{1+f(L_1+L_2)/D}}}$$

$$B_1 = B_2 + \Lambda_f$$

$$z_1 = z_2 + h_2 + \frac{V_2^2}{2g} + f\frac{L_1}{D}\frac{V^2}{2g} \Rightarrow h_2 = z_1 - z_2 - \frac{V^2}{2g}\left(1 + f\frac{L_1}{D}\frac{V^2}{2g}\right)$$

cota nivel de camara

$$z_0 = h_2 + z_2 \quad (B_0 = B_{2'})$$

$$z_0 = z_1 - \frac{V^2}{2g}\left(1 + f\frac{L_1}{D}\right) \Rightarrow z_0 = z_1 - \frac{z_1 - z_3}{1+f(L_1+L_2)/D}\left(1 + f\frac{L_1}{D}\right)$$

$$\boxed{z_0 = z_1 - \frac{z_1 - z_3(D + fL_1)}{D + f(L_1+L_2)}}$$

Ahora pasamos al caso impermanente, la valvula se cierra en forma instantánea. No se puede aplicar la ecs. de Euler entre (1) y (0) ya que hay cambio de diametro.

Entre 1 y 2 $\frac{L_1}{g}\frac{du}{dt} + B_2 - B_1 + f\frac{L_1}{D}\frac{u^2}{2g} = 0$ (1)

u: Velocidad tubería
V: Velocidad en camara
Z: cota de nivel en camara.

Entre 2 y 0 $\frac{(Z-Z_2)}{g}\frac{dV}{dt} + B_0 - B_2 = 0$ (2) ← despreciamos perdidas

Sumando (1) y (2)

$$\cancel{\frac{L_1}{g}\frac{du}{dt} + B_0 - B_1 + f\frac{L_1}{D}} \quad \frac{L_1}{g}\frac{du}{dt} + \frac{(Z-Z_2)}{g}\frac{dV}{dt} + B_0 - B_1 + f\frac{L_1}{D}\frac{u^2}{2g} = 0$$

Continuidad $u\,\pi\frac{D^2}{4} = V\pi\frac{D'^2}{4} \Rightarrow \boxed{u = \left(\frac{D'}{D}\right)^2 V}$

$B_1 = Z_1$; $B_0 = Z + \frac{P_0}{\gamma} + \frac{V^2}{2g}$

$\boxed{Z = Z_0 + Y}$ Cambiamos la variable para tener la fluctuacion

$$\boxed{B_0 = Z_0 + Y + \frac{P_0}{\gamma} + \frac{V^2}{2g}}$$

Calculamos P_0

Proceso Isotermico $P_{at}\,V_0 = P_0'(t)\,V(t)$

En termodinamica se trabaja con presiones absolutas $P_0' = P_0 + P_{at}$

$P_0'(t) = \frac{P_{at}V_0}{V(t)}$ $\quad V = V_0 - AY \Rightarrow P_0 = \frac{P_{at}V_0}{V_0 - AY} - P_{at}$

$$P_0 = P_{at}\left(\frac{V_0}{V_0 - AY} - 1\right)$$

Se puede reescribir como: $P_0 = P_{at}\left(\frac{1}{1-\frac{AY}{V_0}} - 1\right)$

Si suponemos que $AY \ll V_0 \Rightarrow \frac{AY}{V_0} \ll 1 \Rightarrow P_0 = P_{at}\left(\cancel{1} + \frac{AY}{V_0} - \cancel{1}\right)$

$$\Rightarrow \boxed{P_0 = \frac{AY}{V_0}P_{at}}$$

$$\frac{L_1}{g}\left(\frac{D'}{D}\right)^2\frac{dV}{dt} + \frac{(z_0+Y-z_2)}{g}\frac{dV}{dt} + z_0 + Y + \frac{P_0}{\gamma} + \frac{V^2}{2g} - z_1 + f\frac{L_1}{D}\left(\frac{D'}{D}\right)^2\frac{V^2}{2g} = 0$$

a)

$$\boxed{\left[\frac{L_1}{g}\left(\frac{D'}{D}\right)^2 + \frac{(z_0+Y-z_2)}{g}\right]\frac{dV}{dt} + \left[1 + f\frac{L_1}{D}\left(\frac{D'}{D}\right)^2\right]\frac{V^2}{2g} + z_0 + Y - z_1 + \frac{AY}{V_0}\frac{P_{at}}{\gamma} = 0}$$

$$\frac{L_1}{g}\left(\frac{D'}{D}\right)^2 \gg \frac{z_0+Y-z_2}{g} \qquad y \qquad \frac{dV}{dt} = \frac{dV}{dY}\cdot\frac{dY}{dt} = V\frac{dV}{dY} = \frac{1}{2}\frac{d(V^2)}{dY}$$

$$\frac{L_1}{g}\left(\frac{D'}{D}\right)^2\frac{1}{2}\frac{d(V^2)}{dY} + \left[1 + f\frac{L_1}{D}\left(\frac{D'}{D}\right)^2\right]\frac{V^2}{2g} + \left[1 + \frac{A\,P_{at}}{V_0\gamma}\right]Y + z_0 - z_1 = 0$$

$k = V^2$; $A = \frac{L_1}{2g}\left(\frac{D'}{D}\right)^2$; $B = \left[1 + f\frac{L_1}{D}\left(\frac{D'}{D}\right)^2\right]\frac{1}{2g}$; $C = 1 + \frac{A\,P_{at}}{V_0\gamma}$; $D = z_0 - z$

$$A\frac{dk}{dY} + Bk + CY + D = 0$$

Utilizando cambio de variable sugerido en el enunciado

$$X = Bk + CY + D \qquad \frac{dX}{dY} = B\frac{dk}{dY} + C$$

$$\cancel{k = \frac{X - CY - D}{B}} \qquad \frac{dk}{dY} = \frac{1}{B}\left(\frac{dX}{dY} - C\right)$$

$$\frac{A}{B}\left(\frac{dX}{dY} - C\right) + X = 0$$

$$\frac{dX}{dY} = -\frac{B}{A}X + C \Rightarrow \frac{dX}{C - \frac{B}{A}X} = dY$$

$$-\frac{A}{B}\ln\left(C - \frac{B}{A}X\right) = Y + \alpha.$$

$$C - \frac{B}{A}X = e^{-\frac{B}{A}Y - \frac{B}{A}\alpha} \Rightarrow C - \frac{B}{A}(Bk + CY + D) = e^{-\frac{B}{A}Y + \beta}$$

Hay que encontrar el valor de β, con condición de Borde.

Cuando $Y = Y_{max} \Rightarrow V = 0 \Rightarrow V^2 = 0 \Rightarrow k = 0$

$$C - \frac{B}{A}(CY_{max} + D) = e^{-\frac{B}{A}Y_{max} + \beta}$$

$$\beta = \frac{B}{A}Y_{max} + \ln\left(C - \frac{B}{A}(CY_{max} + D)\right)$$

La expresión para encontrar el nivel mínimo Y_{min}, $k = ($

b).

$$\boxed{\ln\left[C - \frac{B}{A}(CY_{min} + D)\right] = -\frac{B}{A}Y_{min} + \frac{B}{A}Y_{max} + \ln\left[C - \frac{B}{A}(CY_{max} + D)\right]}$$

$$\Downarrow$$

Y :

P3) $t=0^+$ se comienza a descargar sobre uno de los estanques el caudal $Q_e = \alpha t$

Supuestos

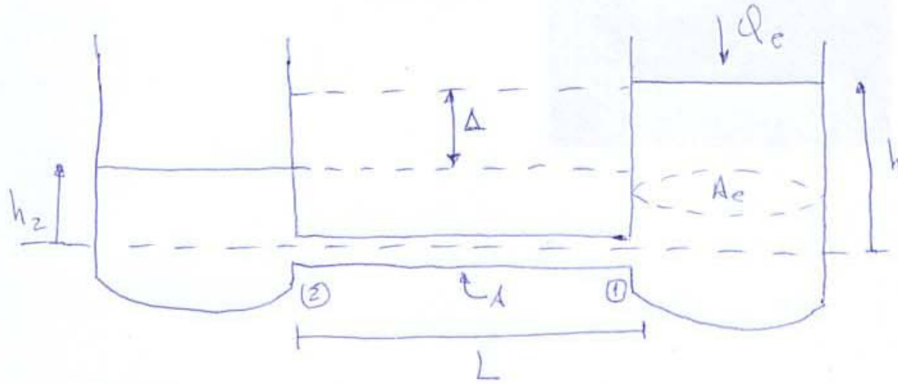
α: cte.

$\lambda_S = \lambda_f \simeq 0$

Fluido incompresible

Tubería rígida

altura de velocidad despreciable en los estanques.



$$\left.\begin{array}{l} \frac{L}{g}\frac{dv}{dt} + B_2 - B_1 = 0 \\ B_2 = h_2 \\ B_1 = h_1 \end{array}\right\} \Rightarrow \frac{L}{g}\frac{dv}{dt} + h_2 - h_1 = 0 \Rightarrow \frac{L}{g}\frac{dv}{dt} \overbrace{+ \Delta}^{h_1 - h_2} = 0 \quad (i)$$

$$\frac{\partial \forall_1}{\partial t} = Q_e - Q_s \Rightarrow \frac{\partial A_e h_1}{\partial t} = \alpha t - VA \Rightarrow A_e \frac{\partial h_1}{\partial t} = \alpha t - V A$$

$$\frac{\partial \forall_2}{\partial t} = Q_e - Q_s \Rightarrow \frac{\partial A_e h_2}{\partial t} = VA \Rightarrow A_e \frac{\partial h_2}{\partial t} = VA$$

$$A_e\left[\frac{\partial h_1}{\partial t}-\frac{\partial h_2}{\partial t}\right]=\alpha t-2VA$$

$$A_e\frac{\partial(h_1-h_2)}{\partial t}=\alpha t-2VA \Rightarrow A_e\frac{\partial \Delta}{\partial t}=\alpha t-2VA \quad \Big/ \frac{\partial}{\partial t}$$

$$A_e\frac{\partial^2\Delta}{\partial t^2}=\alpha-2A\frac{\partial V}{\partial t}$$

$$\frac{\partial V}{\partial t}=\frac{\alpha}{2A}-\frac{A_e}{2A}\frac{\partial^2\Delta}{\partial t^2} \qquad \cancel{\frac{\partial V}{\partial t}=\frac{\alpha}{2A}-\frac{A_e}{A}\frac{\partial^2\Delta}{\partial t^2}} \quad (ii)$$

ii en i

$$\frac{L}{g}\left(\frac{\alpha}{2A}-\frac{A_e}{2A}\frac{\partial^2\Delta}{\partial t^2}\right)-\Delta=0$$

$$\frac{A_e L}{2A\, g}\frac{\partial^2\Delta}{\partial t^2}+\Delta-\frac{\alpha}{2A}\frac{L}{g}=0$$

Cambio de Variable $\eta=\Delta-\frac{\alpha L}{2Ag}$ $\quad \frac{\partial^2\eta}{\partial t^2}=\frac{\partial^2\Delta}{\partial t^2}$

$$\frac{A_e}{2A}\frac{L}{g}\frac{\partial^2\eta}{\partial t^2}+\eta=0$$

$$\Rightarrow \frac{\partial^2\eta}{\partial t^2}+\frac{2g}{L}\frac{A}{A_e}\eta=0$$

$$\Rightarrow \eta(t)=C_1\cos\left(\sqrt{\frac{2g}{L}\frac{A}{A_e}}\,t\right)+C_2\operatorname{Sen}\left(\sqrt{\frac{2g}{L}\frac{A}{A_e}}\,t\right)$$

$$\eta(t)=\Delta(t)-\frac{\alpha L}{2gA} \Rightarrow \Delta(t)=\eta(t)+\frac{\alpha L}{2gA}$$

$$\Delta(t)=C_1\cos\left(\sqrt{\frac{2g}{L}\frac{A}{A_e}}\,t\right)+C_2\operatorname{Sen}\left(\sqrt{\frac{2g}{L}\frac{A}{A_e}}\,t\right)+\frac{\alpha L}{2gA}$$

Condicion inicial.

$$t=0 \quad \Delta=0 \Rightarrow \quad 0=C_1+\frac{\alpha L}{2gA} \Rightarrow C_1=-\frac{\alpha L}{2gA}$$

$t=0 \quad \dot{\Delta}=0$

$$\dot{\Delta}(t) = -C_1\sqrt{\frac{2g}{L}\frac{A}{A_e}}\,\sin\left(\sqrt{\frac{2g}{L}\frac{A}{A_e}}\,t\right) + C_2\sqrt{\frac{2g}{L}\frac{A}{A_e}}\,\cos\left(\sqrt{\frac{2g}{L}\frac{A}{A_e}}\,t\right)$$

$$\dot{\Delta}(0) = C_2\sqrt{\frac{2g}{L}\frac{A}{A_e}} = 0 \Rightarrow C_2 = 0$$

$$\boxed{\Delta(t) = \frac{\alpha L}{2gA}\left(1-\cos\left(\sqrt{\frac{2gA}{L A_e}}\,t\right)\right)}$$