P1

a.) $\phi: \mathbb{R}^2 \longrightarrow \mathbb{R}^2$

$$\begin{pmatrix} x \\ t \end{pmatrix} \longmapsto \phi\begin{pmatrix} x \\ t \end{pmatrix} = \begin{pmatrix} x+vt \\ x-vt \end{pmatrix}$$

$$= \underbrace{\begin{pmatrix} 1 & v \\ 1 & -v \end{pmatrix}}_{A} \begin{pmatrix} x \\ t \end{pmatrix}$$

Luego, $\phi$ es una transformación lineal, con $A$ invertible ($\det A = -2v \neq 0$) y con lo cual biyectiva y de clase $\mathcal{C}^{\infty}$. Con esto $g = f \circ \phi^{-1}$ está bien definida y es de clase $\mathcal{C}^2$ por ser composición de funciones $\mathcal{C}^2$

b.) $g(u,w) = f(x(u,w), t(u,w))$

$$\Rightarrow \frac{\partial g}{\partial u}(u,w) = \frac{\partial f}{\partial x}(x(u,w), t(u,w)) \cdot \frac{\partial x}{\partial u} + \frac{\partial f}{\partial t}(x(u,w), t(u,w)) \cdot \frac{\partial t}{\partial u}$$

$$\begin{pmatrix} x \\ t \end{pmatrix} = \begin{pmatrix} \frac{u+w}{2} \\ \frac{u-w}{2v} \end{pmatrix}$$

$$= \frac{\partial f}{\partial x} \cdot \frac{1}{2} + \frac{\partial f}{\partial t} \cdot \frac{1}{2v}$$

$$\frac{\partial^2 g}{\partial w \partial u} = \left(\frac{\partial^2 f}{\partial x^2}\cdot\frac{\partial x}{\partial w} + \frac{\partial^2 f}{\partial t \partial x}\cdot\frac{\partial t}{\partial w}\right)\cdot\frac{1}{2}$$
$$+\left(\frac{\partial^2 f}{\partial x \partial t}\cdot\frac{\partial x}{\partial w} + \frac{\partial^2 f}{\partial t^2}\cdot\frac{\partial t}{\partial w}\right)\cdot\frac{1}{2v}$$

$$= \left(\frac{\partial^2 f}{\partial x^2}\cdot\frac{1}{2} + \frac{\partial^2 f}{\partial t \partial x}\left(\frac{-1}{2v}\right)\right)\cdot\frac{1}{2}$$
$$+\left(\frac{\partial^2 f}{\partial x \partial t}\cdot\frac{1}{2} + \frac{\partial^2 f}{\partial t^2}\cdot\left(\frac{-1}{2v}\right)\right)\cdot\frac{1}{2v}$$
$$= \frac{1}{4}\left[\frac{\partial^2 f}{\partial x^2} - \frac{1}{v^2}\cdot\frac{\partial^2 f}{\partial t^2}\right]$$

$$= 0.$$

c.) $\frac{\partial}{\partial w}\cdot\frac{\partial g}{\partial u} = 0 \Rightarrow \frac{\partial g}{\partial u}(u,w) = \tilde{H}(u)$

$\Rightarrow g(u,w) = \hat{H}(u) + c(w)$

$\frac{\partial}{\partial u}\cdot\frac{\partial g}{\partial w} = 0 \Rightarrow \frac{\partial g}{\partial w}(u,w) = \tilde{G}(w)$

$\Rightarrow g(u,w) = \hat{G}(w) + d(u)$

$\Rightarrow g(u,w) = H(u) + G(w)$

$\Rightarrow f(x,t) = H(x+vt) + G(x-vt)$

Sol. part. $f(x,t) = \operatorname{sen}(x+vt) + \cos(x-vt)$

P2 $\quad \min c^t d \qquad c \neq 0$

$\text{s.a} \quad d^t d = 1$

$d \in \mathbb{R}^n$

i.) $\mathcal{L} = c^t d - \lambda (d^t d - 1)$

$= \sum_{i=1}^{n} c_i \cdot d_i - \lambda \left( \sum_{i=1}^{n} d_i^2 - 1 \right)$

$\frac{\partial \mathcal{L}}{\partial d_i} = 0 \iff c_i - 2\lambda d_i = 0$

$\lambda \neq 0$ (si $\lambda = 0 \Rightarrow c = 0 \rightarrow \leftarrow$)

$\iff d_i^* = \frac{c_i}{2\lambda}$

$\Rightarrow \sum_{i=1}^{n} d_i^{*2} = \frac{\sum c_i^2}{4\lambda^2} \iff \lambda^2 = \frac{1}{4} \|c\|_2^2$

$\iff \lambda = \pm \frac{1}{2} \|c\|_2$

$\Rightarrow d_i^* = \frac{c_i}{\|c\|_2} \vee d_i^* = -\frac{c_i}{\|c\|_2}$

Con lo cual $d^* = -\frac{c}{\|c\|_2}$ es un punto crítico.

ii.) Como $f(d) = c^t d$ es continua y $d^t d = 1$ es un compacto ($\Rightarrow f$ alcanza su máximo y mínimo) y $f\left(\frac{-c}{\|c\|_2}\right) = -1$

$f\left(\frac{c}{\|c\|_2}\right) = 1 \Rightarrow \frac{-c}{\|c\|_2}$ es mínimo b.) $\min f(d)$

s.a $d^t d = 1$

$d \in \mathbb{R}^n$

$$\mathcal{L} = d^t Q d + c^t d + e - \lambda (d^t d - 1)$$

$$\frac{\partial \mathcal{L}}{\partial d} = 0 \Leftrightarrow 2Qd + c - \lambda 2d = 0$$

$$\Leftrightarrow 2(Q - \lambda I)d = -c$$

$$\Leftrightarrow (Q - \lambda I)d = \frac{-c}{2} \qquad (1)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 0 \Leftrightarrow d^t d = 1 \qquad (2)$$

c.) $f: S \subseteq \mathbb{R}^n \to \mathbb{R}_+$ , $\mathcal{C}^2$

S convexo

$h(x) = \dfrac{1}{f(x)}$ $\quad x, y \in S^*$ $\quad \lambda \in [0,1] \Rightarrow \lambda x + (1-\lambda) y \in S^*$

$$h(\lambda x + (1-\lambda)y) = \frac{1}{f(\lambda x + (1-\lambda)y)}$$

f concava

$$\leq \frac{1}{\lambda f(x) + (1-\lambda) f(y)}$$

$\frac{1}{x}$ convexa

$$\leq \frac{\lambda}{f(x)} + \frac{(1-\lambda)}{f(y)}$$

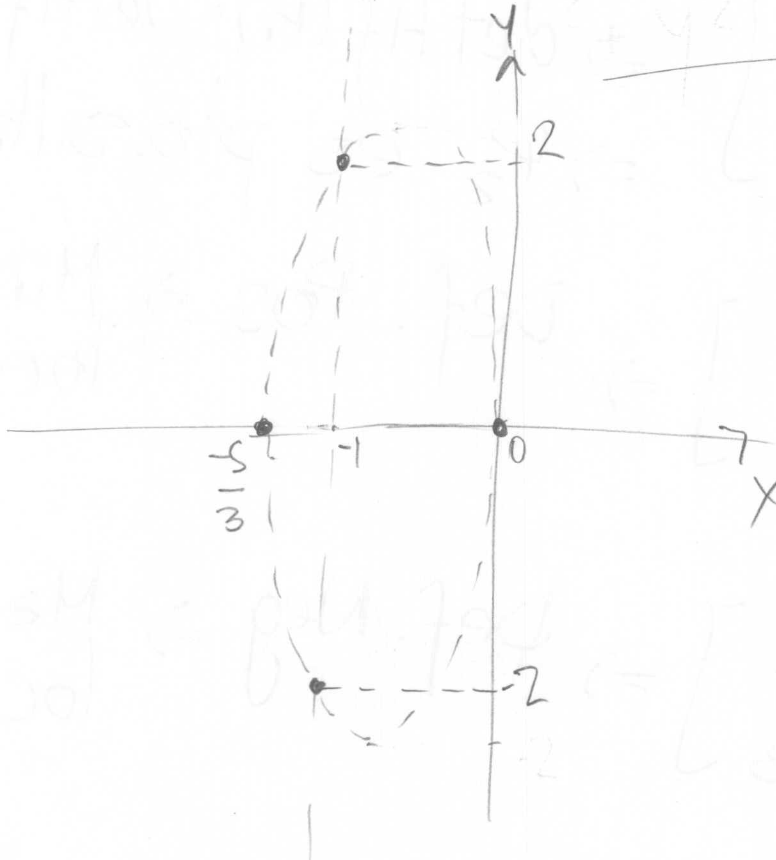
$$\leq \lambda h(x) + (1-\lambda) h(y)$$

P3

a.) $f(x,y) = 2x^3 + xy^2 + 5x^2 + y^2$

$$\nabla f(x,y) = \begin{pmatrix} 6x^2 + y^2 + 10x \\ 2xy + 2y \end{pmatrix}$$

$$\Rightarrow \nabla f(x,y) = 0 \quad \Leftrightarrow \quad \begin{array}{l} 6\left(x+\frac{5}{6}\right)^2 + y^2 = \frac{25}{36} \\ 2y(x+1) = 0 \end{array}$$




$P_1 = (-1,-2) \quad P_2 = (-1,2) \quad P_3 = (0,0) \quad P_4 = \left(-\frac{5}{3}, 0\right)$

$$Hf(x,y) = \begin{bmatrix} 12x+10 & 2y \\ 2y & 2(x+1) \end{bmatrix}$$

- $Hf(P_1) = \begin{bmatrix} -2 & -4 \\ -4 & 0 \end{bmatrix}$ $\det Hf(P_1) = +(2+\lambda)\lambda + 16 = 0$

$\Leftrightarrow 16 + 2\lambda + \lambda^2 = 0$

$\Leftrightarrow \lambda = -1 \pm \sqrt{17}$

$\Rightarrow P_1$ es pto. silla

- $Hf(P_2) = \begin{bmatrix} -2 & 4 \\ 4 & 0 \end{bmatrix}$ $\Rightarrow \det Hf(P_1) = \det Hf(P_2)$

$\Rightarrow P_2$ es pto. silla.

- $Hf(P_3) = \begin{bmatrix} 10 & 0 \\ 0 & 2 \end{bmatrix} \Rightarrow$ Def. Pos $\Rightarrow$ Min. local

- $Hf(P_4) = \begin{bmatrix} -10 & 0 \\ 0 & -\frac{4}{3} \end{bmatrix} \Rightarrow$ Def. Neg $\Rightarrow$ Max local.

b.) $P_3, P_4 \in \{(x,y) : x^2+y^2 \leq 4\}$.

veamos el borde

$$\mathcal{L} = f(x,y) - \lambda(x^2+y^2-4)$$

$$\frac{\partial \mathcal{L}}{\partial x} = 0 \Leftrightarrow 6x^2+y^2+10x-2\lambda x = 0$$

$$\frac{\partial \mathcal{L}}{\partial y} = 0 \Leftrightarrow 2y(x+1) - 2\lambda y = 0$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 0 \Leftrightarrow x^2+y^2 = 4$$

Cuyas soluciones son:

$$x_{11}^* = \left(-\frac{2}{3}, \frac{4}{3}\sqrt{2}, \frac{1}{3}\right) \qquad x_{12}^* = \left(-\frac{2}{3}, -\frac{4}{3}\sqrt{2}, \frac{1}{3}\right)$$

$$x_{21}^* = (2, 0, 11) \qquad x_{22}^* = (-2, 0, -1)$$

de donde se obtiene que

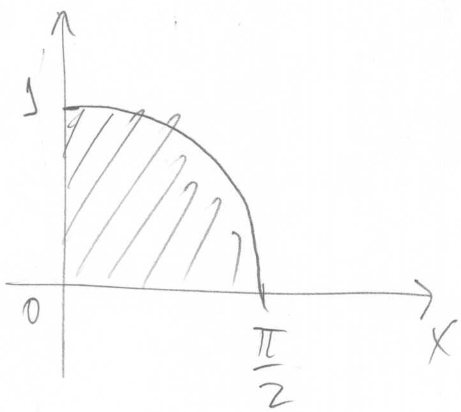
max : $(2,0)$

min : $(-2,0)$.

P4

a.) i.

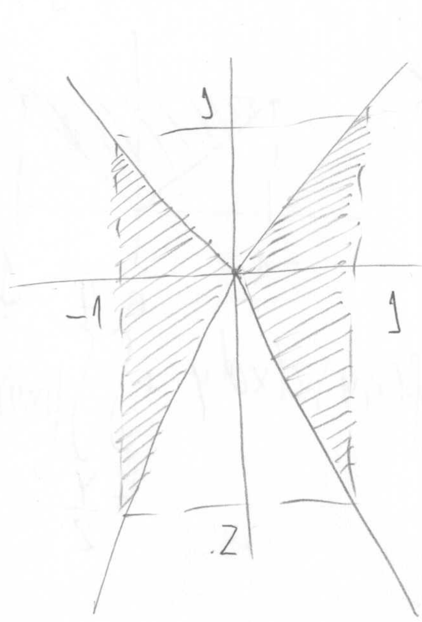
$$\int_0^{\pi/2}\int_0^{\cos x} y\,\mathrm{sen}\,x\,dy\,dx = \int_0^{\pi/2} \frac{\cos^2 x\,\mathrm{sen}\,x}{2}\,dx$$




$$= -\frac{1}{6}\int_0^{\pi/2} \frac{d}{dx}\cos^3(x)\,dx$$

$$= \frac{1}{6}$$

$$\int_{-1}^{1}\int_{-2|x|}^{|x|} e^{x+y} = \int_{-1}^{0}\int_{2x}^{-x} e^{x+y}\,dy\,dx + \int_0^1\int_{-2x}^{x} e^{x+y}\,dy\,dx$$



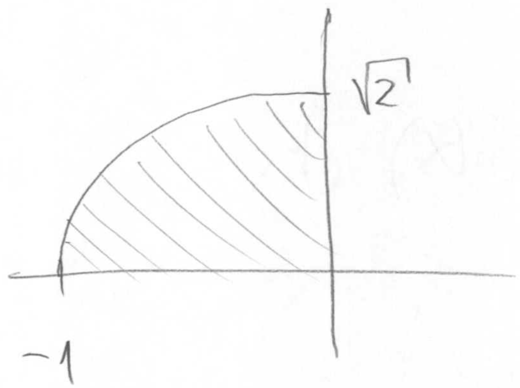

$$= \int_{-1}^{0}\left(e^{x-x} - e^{x+2x}\right)dx + \int_0^1 e^{x+x} - e^{x-2x}\,dx$$

$$= 1 - \frac{1}{3}e^{3x}\Big|_{-1}^{0} + \frac{1}{2}e^{2x}\Big|_0^1 + e^{-x}\Big|_0^1$$

$$= 1 - \frac{1}{3} + \frac{1}{3}e^{-3} + \frac{1}{2}e^{2} - \frac{1}{2} + e^{-1} - 1$$

$$= \frac{1}{3}e^{-3} + e^{-1} - \frac{5}{6} + \frac{1}{2}e^{2}$$

$$\int_{-1}^{0}\int_{0}^{2\sqrt{1-x^2}} x\,dy\,dx = \int_{-1}^{0} 2\sqrt{1-x^2}\cdot x\,dx$$

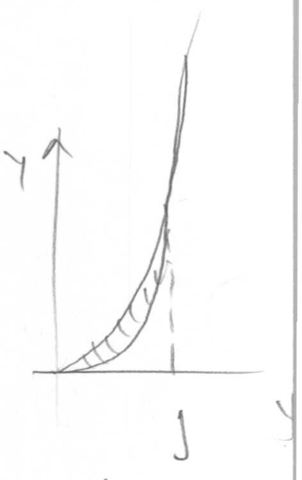



$$= -\frac{2}{3}\int_{-1}^{0}\frac{d}{dx}\left(1-x^2\right)^{3/2}dx$$

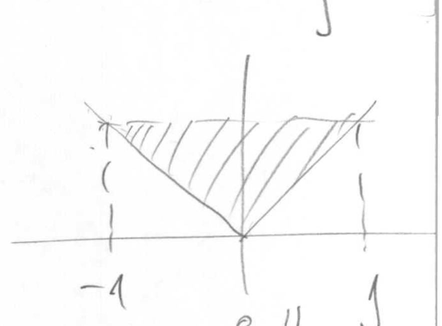
$$= -\frac{2}{3}$$

b.)

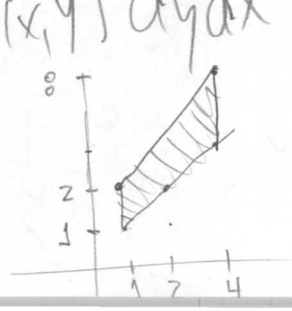
$$\int_{0}^{1}\int_{x^4}^{x^2} f(x,y)\,dy\,dx = \int_{0}^{1}\int_{\sqrt{y}}^{\sqrt[4]{y}} f(x,y)\,dx\,dy$$




$$\int_{0}^{1}\int_{-y}^{y} f(x,y)\,dx\,dy = \int_{-1}^{1}\int_{|x|}^{1} f(x,y)\,dy\,dx$$




$$\int_{1}^{4}\int_{x}^{2x} f(x,y)\,dy\,dx = \int_{1}^{2}\int_{1}^{y} f(x,y)\,dx\,dy + \int_{2}^{4}\int_{\frac{y}{2}}^{y} f(x,y)\,dx\,dy + \int_{4}^{8}\int_{\frac{y}{2}}^{4} f(x,y)\,dx\,dy$$



b.) $I = \iint_S (x+y^2)\,dx\,dy$

$x - 2x + 2$

$-x + 2$

$= 2 \iint_{S_1} (x+y^2)\,dx\,dy$

$= 2\left( \int_0^1 \int_0^x (x+y^2)\,dy\,dx + \int_1^2 \int_{2(x-1)}^{x} (x+y^2)\,dy\,dx \right)$

$= 2\left( \int_0^1 x^2 + \frac{x^3}{3}\,dx + \int_1^2 x(x-2(x-1)) + \frac{x^3}{3} - \frac{8(x-1)^3}{3} \right)$

$\vdots$

$$\boxed{I = \frac{5}{6}}$$