Un punto base en cada problema, más lo indicado por partes si está correcta. Si hay errores el corrector juzga cuanto descontarle. Las notas entonces van de 1.0 a 7.0 en cada problema.

Problema 1

Calculamos las constantes con $\theta_1 = \pi/6, \dot{\theta}_1(0) = 0, \dot{\phi}_1(0) = \Omega = \sqrt{\frac{g}{h}}, C = \frac{1}{4}mh^2$, $A = \frac{1}{2}mh^2$, , $s = 8\sqrt{\frac{g}{h}}$

$$\begin{aligned} 2E - Cs^2 &= A\dot{\phi}^2 \sin^2\theta + A\dot{\theta}^2 + 2mgh\cos\theta = A\Omega^2\sin^2\theta_1 + 2mgh\cos\theta_1 \\ &= \frac{1}{8}mhg + mgh\sqrt{3} \qquad \text{(1 p)} \\ \alpha &= A\Omega\sin^2\theta_1 + Cs\cos\theta_1 = \\ &= \frac{1}{8}mh^{\frac{3}{2}}\sqrt{g} + mh^{\frac{3}{2}}\sqrt{g}\sqrt{3} \qquad \text{(1 p)} \end{aligned}$$

luego $f(u)$ será

$$\begin{aligned} f(u) &= \left(\frac{1}{8}mhg + mgh\sqrt{3} - 2mghu\right)\frac{1-u^2}{A} - \left(\frac{\frac{1}{8}mh^{\frac{3}{2}}\sqrt{g} + mh^{\frac{3}{2}}\sqrt{g}\sqrt{3} - Csu}{A}\right)^2 \\ &= \frac{1}{4h}g\left(-1 - 8\sqrt{3} + 16u\right)\left(-1 + u^2\right) - \left(-\frac{1}{4\sqrt{h}}\sqrt{g}\left(-1 - 8\sqrt{3} + 16u\right)\right)^2 \end{aligned}$$

$$\begin{aligned} \dot{u}^2 &= f(u) = \\ &= \frac{1}{4h}g\left(-1 - 8\sqrt{3} + 16u\right)\left(-1 + u^2\right) - \frac{1}{16h}g\left(-1 - 8\sqrt{3} + 16u\right)^2 \text{(1 p)} \end{aligned}$$

que se factoriza

$$\frac{1}{4}\left(2u - 8 + \sqrt{3}\right)\left(-1 - 8\sqrt{3} + 16u\right)\left(2u - \sqrt{3}\right) = 0 \qquad \text{(1 p)}$$

$$\begin{aligned} u_3 &= 4 - \frac{1}{2}\sqrt{3} = 3.1340, \\ u_2 &= \frac{1}{16} + \frac{1}{2}\sqrt{3} = 0.92853 \\ u_1 &= \frac{1}{2}\sqrt{3} = 0.86603 \qquad \text{(1 p)} \end{aligned}$$

$$\begin{aligned} \dot{\phi} &= \frac{1}{4\sqrt{h}}\sqrt{g}\frac{-1 - 8\sqrt{3} + 16u}{-1 + u^2} \\ \dot{\phi}(u_2) &= \frac{-1 - 8\sqrt{3} + 16u_2}{-1 + u_2^2} = 0 \qquad \text{(1 p)} \end{aligned}$$

O sea movimiento es cuspidal, $\dot{\phi}(u_2) = 0$ en $\theta_2 = 21.794^o$

Problema 2. Las matrices de rotación construidas para $\phi = \pi/6$ resultan

$$\begin{aligned} R_x(\pi/6) &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\pi/6 & -\sin\pi/6 \\ 0 & \sin\pi/6 & \cos\pi/6 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2}\sqrt{3} & -\frac{1}{2} \\ 0 & \frac{1}{2} & \frac{1}{2}\sqrt{3} \end{pmatrix}. \\ R_z(\pi/6) &= \begin{pmatrix} \cos\pi/6 & -\sin\pi/6 & 0 \\ \sin\pi/6 & \cos\pi/6 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{2}\sqrt{3} & -\frac{1}{2} & 0 \\ \frac{1}{2} & \frac{1}{2}\sqrt{3} & 0 \\ 0 & 0 & 1 \end{pmatrix} \end{aligned}$$

Para el primer orden

$$\begin{aligned} R &= R_z(\pi/6R_x(\pi/6)) = \begin{pmatrix} \frac{1}{2}\sqrt{3} & -\frac{1}{2} & 0 \\ \frac{1}{2} & \frac{1}{2}\sqrt{3} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2}\sqrt{3} & -\frac{1}{2} \\ 0 & \frac{1}{2} & \frac{1}{2}\sqrt{3} \end{pmatrix} \\ &= \begin{pmatrix} \frac{1}{2}\sqrt{3} & -\frac{1}{4}\sqrt{3} & \frac{1}{4} \\ \frac{1}{2} & \frac{3}{4} & -\frac{1}{4}\sqrt{3} \\ 0 & \frac{1}{2} & \frac{1}{2}\sqrt{3} \end{pmatrix} \qquad (1\text{ p}) \end{aligned}$$

luego

$$Tr(R) = \frac{3}{4} + \sqrt{3} = 1 + 2\cos\phi$$

$$\begin{aligned} \cos\phi &= \frac{1}{2}(\frac{3}{4} + \sqrt{3} - 1) \\ &= \frac{1}{2}\sqrt{3} - \frac{1}{8} = 0.741\,03 \Rightarrow \phi = 42.\,181^o \qquad (1\text{ p}) \end{aligned}$$

ahora

$$\begin{aligned} R - R^T &= 2\sin\phi(\hat{n}\times) \\ &= \begin{pmatrix} \frac{1}{2}\sqrt{3} & -\frac{1}{4}\sqrt{3} & \frac{1}{4} \\ \frac{1}{2} & \frac{3}{4} & -\frac{1}{4}\sqrt{3} \\ 0 & \frac{1}{2} & \frac{1}{2}\sqrt{3} \end{pmatrix} - \begin{pmatrix} \frac{1}{2}\sqrt{3} & \frac{1}{2} & 0 \\ -\frac{1}{4}\sqrt{3} & \frac{3}{4} & \frac{1}{2} \\ \frac{1}{4} & -\frac{1}{4}\sqrt{3} & \frac{1}{2}\sqrt{3} \end{pmatrix} \\ &= \begin{pmatrix} 0 & -\frac{1}{4}\sqrt{3} - \frac{1}{2} & \frac{1}{4} \\ \frac{1}{2} + \frac{1}{4}\sqrt{3} & 0 & -\frac{1}{4}\sqrt{3} - \frac{1}{2} \\ -\frac{1}{4} & \frac{1}{2} + \frac{1}{4}\sqrt{3} & 0 \end{pmatrix} = 2\sin\phi(\hat{n}\times) \end{aligned}$$

$$\begin{pmatrix} 0 & -0.933\,01 & 0.25 \\ 0.933\,01 & 0 & -0.933\,01 \\ -0.25 & 0.933\,01 & 0 \end{pmatrix} = 1.\,343\,0(\hat{n}\times)$$

$$2\sin\phi = 2\sqrt{1 - (\frac{1}{2}\sqrt{3} - \frac{1}{8})^2} = \frac{1}{4}\sqrt{15 + 8\sqrt{3}}$$

$$
\begin{aligned}
n_x &= \frac{\frac{1}{2}+\frac{1}{4}\sqrt{3}}{\frac{1}{4}\sqrt{15+8\sqrt{3}}} = 0.694\,75,\\
n_y &= \frac{\frac{1}{4}}{\frac{1}{4}\sqrt{15+8\sqrt{3}}} = 0.186\,16,\\
n_z &= \frac{\frac{1}{2}+\frac{1}{4}\sqrt{3}}{\frac{1}{4}\sqrt{15+8\sqrt{3}}} = 0.694\,75 \qquad \text{(1 p)}
\end{aligned}
$$

En el segundo orden

$$
\begin{aligned}
R &= R_x(\pi/6)R_z(\pi/6)) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2}\sqrt{3} & -\frac{1}{2} \\ 0 & \frac{1}{2} & \frac{1}{2}\sqrt{3} \end{pmatrix} \begin{pmatrix} \frac{1}{2}\sqrt{3} & -\frac{1}{2} & 0 \\ \frac{1}{2} & \frac{1}{2}\sqrt{3} & 0 \\ 0 & 0 & 1 \end{pmatrix} \\
&= \begin{pmatrix} \frac{1}{2}\sqrt{3} & -\frac{1}{2} & 0 \\ \frac{1}{4}\sqrt{3} & \frac{3}{4} & -\frac{1}{2} \\ \frac{1}{4} & \frac{1}{4}\sqrt{3} & \frac{1}{2}\sqrt{3} \end{pmatrix} \qquad \text{(1 p)}
\end{aligned}
$$

:

$$
Tr(R) = \frac{3}{4} + \sqrt{3} = 1 + 2\cos\phi
$$

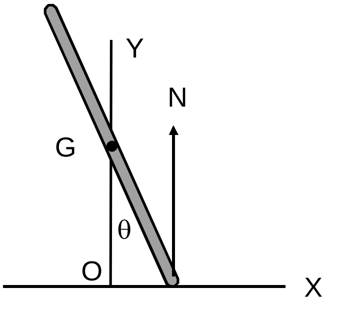
$$
\begin{aligned}
\cos\phi &= \frac{1}{2}(\frac{3}{4} + \sqrt{3} - 1) \\
&= \frac{1}{2}\sqrt{3} - \frac{1}{8} = 0.741\,03 \Rightarrow \phi = 42.\,181^o \qquad \text{(1 p)}
\end{aligned}
$$

$$
\begin{aligned}
R - R^T &= \begin{pmatrix} \frac{1}{2}\sqrt{3} & -\frac{1}{2} & 0 \\ \frac{1}{4}\sqrt{3} & \frac{3}{4} & -\frac{1}{2} \\ \frac{1}{4} & \frac{1}{4}\sqrt{3} & \frac{1}{2}\sqrt{3} \end{pmatrix} - \begin{pmatrix} \frac{1}{2}\sqrt{3} & \frac{1}{4}\sqrt{3} & \frac{1}{4} \\ -\frac{1}{2} & \frac{3}{4} & \frac{1}{4}\sqrt{3} \\ 0 & -\frac{1}{2} & \frac{1}{2}\sqrt{3} \end{pmatrix} \\
&= \begin{pmatrix} 0 & -\frac{1}{4}\sqrt{3}-\frac{1}{2} & -\frac{1}{4} \\ \frac{1}{2}+\frac{1}{4}\sqrt{3} & 0 & -\frac{1}{4}\sqrt{3}-\frac{1}{2} \\ \frac{1}{4} & \frac{1}{2}+\frac{1}{4}\sqrt{3} & 0 \end{pmatrix} = \frac{1}{4}\sqrt{15+8\sqrt{3}}(\hat{n}\times)
\end{aligned}
$$

:

$$
\begin{aligned}
n_x &= \frac{\frac{1}{2}+\frac{1}{4}\sqrt{3}}{\frac{1}{4}\sqrt{15+8\sqrt{3}}} = 0.694\,75,\\
n_y &= \frac{-\frac{1}{4}}{\frac{1}{4}\sqrt{15+8\sqrt{3}}} = -0.186\,16\\
n_z &= \frac{\frac{1}{2}+\frac{1}{4}\sqrt{3}}{\frac{1}{4}\sqrt{15+8\sqrt{3}}} = 0.694\,75 \qquad \text{(1 p)}
\end{aligned}
$$

Problema 3



Las coordenadas del centro de masa son

$$x_G = 0, \quad y_G = a\cos\theta, \quad \dot{y}_G = -a\dot{\theta}\sin\theta$$

ecuaciones

$$N - Mg = M\frac{d^2}{dt^2}a\cos\theta, \quad \Gamma_G = Na\sin\theta = \frac{1}{3}Ma^2\ddot{\theta} \qquad (1\text{ p})$$

o mejor usamos conservación de energía

$$E = \frac{1}{2}M(-a\dot{\theta}\sin\theta)^2 + \frac{1}{2}\frac{1}{3}Ma^2\dot{\theta}^2 + Mga\cos\theta = Mga \qquad (1\text{ p})$$

de donde

$$\dot{\theta}^2 = \frac{Mga(1-\cos\theta)}{\frac{1}{2}Ma^2\sin^2\theta + \frac{1}{2}\frac{1}{3}Ma^2} = 6\frac{g}{a}\frac{1-\cos\theta}{4-3\cos^2\theta} \qquad (\text{a})\ 2\text{ p})$$

la normal será

$$N = Mg + M\frac{d^2}{dt^2}a\cos\theta$$

omitiendo detalles resulta

$$\begin{aligned} N &= Mg - \frac{3Mg}{\sin\theta}\frac{d}{d\theta}(\sin^2\theta\frac{1-\cos\theta}{4-3\cos^2\theta}) \\ &= Mg\frac{4+3\cos^2\theta-6\cos\theta}{(4-3\cos^2\theta)^2} \qquad (\text{b})\ 2\text{ p}) \end{aligned}$$